THEORICAL ANALYSIS OF THE CHANNEL FLOW WITH RADIATION IN PARTICIPATING MEDIA THROUGH THE USE OF THE GENERALIZED INTEGRAL TRANSFORM TECHNIQUE

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Abstract. The problem of channel flow with convection heat transfer between parallel plane plates at a prescribed temperature is of great interest for researchers due to its vast aplicability to industry and other engineering sectors. This investigation determines the transient temperature distribution for the coupling conduction-radiation for an absorbing, emitting, scattering, thermally developing slug flow between two parallel plates. The temperature distribution field is obtained through the use of GITT (Generalized Integral Transform Technique), while the radiative contribution is determined by the Galerkin method of analytic approach. The results are presented in tables and graphs, and the analysis is verified in terms of stability, convergence and computacional cost. The effects of several parameters, such as the single scattering albedo, optical thickness of the medium and conduction-radiation coupling factor, are also studied.

Keywords: GITT, Galerkin method, coupled conduction-radiation, forced convection, optical thickness.

1. Introduction

The study of heat transference, taking in consideration the interaction of forced convection and radiation, is important in several areas of engineering, especially when it involves high temperature processes. Overlapping of radiation is found, for example, in cooling of hot locations when the heat is carried to cooler regions. When the transport medium is participant, the radiation has strong influence on the temperature field and local heat transference tax, as evidenced by the Nusselt number.

According to Özişik (1973), in the case of forced convection in a participating fluid flowing at temperatures encountered in engineering applications, the continuity and the momentum equations remain unchanged. However, the energy equation contains an additional term: the divergence of the net radiative heat flux vector.

The present investigation is aimed at investigating the effect of radiation in heat transference and temperature distribution on the region of a participant fluid in thermal development flowing inside a channel, which absorbs, emits and scatters isotropically. For such, the Generalized Integral Transformed Technique was applied, as defined by Mikhailov and Özişik (1994), Cotta (1993) and Santos et al (2001), to solve the energy equation. The Garlekin method, also used by Özişik and Yener (1982), and Yener and Tari (1998), was applied to compute the radiative effect simultaneously.

2. Problem Formulation

Consider an absorbing, emitting and scattering, gray, incompressible, constant-property fluid in developing slug flow between two infinite parallel plates at a distance 2L apart that emits and reflects diffusely the radiation. Figure 1 shows the geometry and the coordinate system. Here the interaction of the radiation is considered.

According to Özişik (1973), the temperature distribution in the medium satisfies the energy equation:

$$\rho C_P u_m \frac{\partial T(x, y)}{\partial x} = k \frac{\partial^2 T(x, y)}{\partial y^2} - \frac{\partial q^r}{\partial y}, \text{ in } 0 \le y \le L, x \ge 0$$
(1)

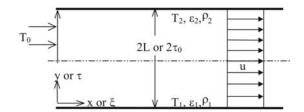


Figure 1: Thermally developed laminar flow between parallel plates with radiation.

For such problem it is considered that the fluid with temperature T_0 enters a channel whose superficies are maintained at prescribed temperatures. The following can then be inferred:

Boundary Conditions:

$$T(x,y) = T_1 \quad \text{at y = 0}$$

$$T(x, y) = T_2 \quad \text{at y = 2L}$$

Inlet condition:

$$T(x, y) = T_0 \qquad \text{at } x = 0 \tag{4}$$

and the divergent radiative heat flow:

$$\frac{\partial q^r}{\partial y} = (I - \boldsymbol{\varpi})\beta \left[4n^2 \sigma T^4(x, t) - 2\pi \int_{-1}^{1} I(x, \mu) d\mu \right]$$
 (5)

where ϖ is the single scattering albedo, β is the extinction coefficient for the medium (m⁻¹), n is the refraction index, and σ is the Stefan-Boltzmann constant.

3. Analysis

Dimensionless form:

The problem is rewritten in dimensionless form by using the following dimensionless groups:

$$\tau = \beta y \qquad \qquad \xi = \frac{k\beta^2 x}{\rho u_m C_p} = 16\tau_0^2 \frac{x/D_e}{Re \cdot Pr} \qquad \qquad \theta(\xi, \tau) = \frac{T(x, y)}{T_w} \tag{6}$$

$$Q_R = \frac{q^r}{4n^2\sigma T_r^4} \qquad N = \frac{k\beta}{4n^2\sigma T_r^4} \qquad \qquad \phi(\tau,\mu) = \frac{\pi I(\tau,\mu)}{n^2\sigma T_r^4}$$
 (7)

By making use of the groups above, the energy equation and the related boundary conditions can be rewritten as:

$$\frac{\partial \theta(\tau, \xi)}{\partial \xi} = \frac{\partial^2 \theta(\tau, \xi)}{\partial \tau^2} - \frac{1}{N} \frac{\partial Q_R}{\partial \tau} \qquad \text{in } 0 \le \tau \le \tau_0 \text{ and } \xi \ge 0$$
(8)

$$\theta(\tau, \xi) = \theta_1 \qquad \text{at } \tau = 0 \tag{9}$$

$$\theta(\tau, \xi) = \theta_2 \qquad \text{at } \tau = 2\tau_0 \tag{10}$$

$$\theta(\tau, \xi) = \theta_0 \qquad \text{at } \xi = 0 \tag{11}$$

with the divergent of the heat flow for radiation in the dimensionless form given as:

$$\frac{\partial Q_R}{\partial \tau} = (I - \varpi) \Big[\theta^4(\tau, \xi) - G(\tau) \Big] \tag{12}$$

where $G(\tau) = \frac{1}{2} \int_{-1}^{1} \varphi(\tau, \mu') d\mu'$ is the dimensionless incident radiation (which is solved by the Galerkin method) and

 $\varphi(\tau,\mu')$ satisfies the dimensionless radiative transfer equation below, considering the isotropic scattering of the radiation.

$$\mu \frac{\partial \varphi(\tau, \mu)}{\partial \tau} + \varphi(\tau, \mu) = (I - \varpi)\theta^{-1}(\tau, \xi) + \frac{\varpi}{2} \int_{-1}^{1} \varphi(\tau, \mu') d\mu', \quad 0 < \tau < \tau_{\theta}, \quad -1 \le \mu \le 1$$
(13)

$$\Phi^{+}(0,\mu) = \varepsilon_{1}\theta_{1}^{4} + 2\rho_{1}\int_{0}^{1}\Phi^{-}(0,-\mu')\mu'd\mu', \quad \mu > 0$$
(14)

$$\varphi^{-}(\tau_{0},-\mu) = \varepsilon_{2}\theta_{2}^{4} + 2\rho_{2}\int_{0}^{1} \varphi^{+}(\tau_{0},\mu')\mu'd\mu', \quad \mu > 0$$

$$\tag{15}$$

where μ is the cosine of the angle formed between the positive axis of y and the direction of the intensity of the radiation ρ_i , and ε_i (i=1,2) represents the reflectivity and the emissivity of the boundary surface, respectively. Many analytical and numerical methods were developed in order to solve the radiative transfer equation – for example, Leite (2004) used the discreet ordinate method, while Siewert and Thomas (1990) used the P_N method.

The energy radiative transfer equations are coupled to the high temperature term to the fourth potency. The determination of the temperature distribution and other greatnesses of interest in engineering are highly dependable on the solution of these two equations. The resolution procedure accomplished in this investigation allows the simultaneous determination of thermal field, radiative heat flow, incident radiation and radiative intensity in any point of the medium.

The problem defined by equations (8-11) can be readly solved by the classical integral transform technique. However, in order to enhance convergence, a filtering strategy is used as the first step in the solution procedure. The simplest choice of a filtering solution for the temperature field is extracted from the pure problem:

$$\theta(\tau, \xi) = \theta_F(\tau) + \theta_H(\tau, \xi) + \theta_R(\tau, \xi) \tag{16}$$

Substitution of the filter into the original problem (8), (9), (10) and (11) yields the following set of equations:

Problem 1:

$$\frac{d^2\theta_F(\tau)}{d\tau^2} = 0 \qquad \text{in } 0 < \tau < 2\tau_0 \tag{17}$$

$$\theta_F(\tau) = \theta_I \qquad \text{at } \tau = 0$$
 (18)

$$\theta_{E}(\tau) = \theta_{\gamma}$$
 at $\tau = 2\tau_{0}$ (19)

And the solution of these is:

$$\theta_F(\tau) = \left(\frac{\theta_2 - \theta_I}{2\tau_0}\right) \cdot \tau + \theta_I \tag{20}$$

Problem 2:

$$\frac{\partial \theta_H(\tau, \xi)}{\partial \xi} = \frac{\partial^2 \theta_H(\tau, \xi)}{\partial \tau^2} \quad \text{in } 0 < \tau < 2\tau_0, \quad \xi \ge 0$$
 (21)

$$\theta_{\mu}(\tau, \xi) = 0$$
 at $\tau = 0$ (22)

$$\theta_H(\tau, \xi) = 0$$
 at $\tau = 2\tau_0$ (23)

$$\theta_H(\tau, \xi) = \theta_0 - \theta_F(\tau) \quad \text{in } 0 \le \tau \le 2\tau_0, \quad \xi = 0 \tag{24}$$

The solution for problem 2 is obtained by following the formalism in the Generalized Integral Transform Technique. The first step consists in choosing an auxiliary eigenfunction problem, which will be used in the integral transformation of the equation system to be solved. The present choice corresponds to the classical Sturm-Liouville problem below:

$$\frac{\partial^2 \psi_i(\tau)}{\partial \tau^2} + \gamma_i^2 \psi_i(\tau) = 0 , \quad \text{in } 0 < \tau < 2\tau_0$$
 (25)

with boundary conditions:

$$\psi_i(\tau) = 0 , \qquad \text{at } \tau = 0 \tag{26}$$

$$\psi_i(\tau) = 0 , \quad \text{at } \tau = 2\tau_0 \tag{27}$$

The problem described by equations (25-27) was solved by the classic method of separation of variables, where the eigenvalues γ_i , and related eigenfunctions $\psi_i(\tau)$ are known as:

$$\psi_i(\tau) = \sin(\gamma_i \tau) \tag{28}$$

satisfying the following ortogonality property:

$$\int_{0}^{\tau_{s}} \psi_{i}(\tau) \psi_{j}(\tau) d\tau = \begin{cases} 0, & \text{if } i \neq j \\ N_{i}(\gamma_{i}), & \text{if } i = j \end{cases}$$
(29)

The eigenvalues γ_i that appear in equation (28) and the norms $N_i(\gamma_i)$ of the eigenfunctions are given respectively as:

$$\gamma_i = \frac{i\pi}{2\tau_0} \qquad \qquad N_i(\gamma_i) = \tau_0 \tag{30}$$

According to the ortogonality properties of the eigenfunctions, it can be written that:

$$\theta_H(\tau,\xi) = \sum_{i=1}^{N_c} \frac{\psi_i(\tau)\overline{\theta}_{Hi}(\xi)}{N_i^{1/2}}$$
 Inversion Formula (31)

$$\overline{\theta}_{Hi}(\xi) = \frac{1}{N_{*}^{1/2}} \int_{0}^{2\tau_{0}} \psi_{i}(\tau) \theta_{H}(\tau, \xi) d\tau \qquad \text{Integral Transform}$$
 (32)

The integral transformation of (21) is now accomplished by applying the operator $\frac{1}{N_i^{1/2}} \int_0^{\tau_0} \psi_i(\tau) d\tau$ (after using

boundary conditions (26-27) and (22-23)):

$$\overline{\theta}_{Hi}(\xi) = \frac{1}{N_i^{1/2}} \int_0^{2\tau_0} \psi_i(\tau) [\theta_0 - \theta_F(\tau)] d\tau \cdot e^{-\gamma_i^2 \xi}$$
(33)

Problem 3:

$$\frac{\partial \theta_R(\tau, \xi)}{\partial \xi} = \frac{\partial^2 \theta_R(\tau, \xi)}{\partial \tau^2} - \frac{1}{N} \frac{\partial Q_R}{\partial \tau} \qquad \text{in } 0 < \tau < 2\tau_0, \quad \xi > 0$$
(34)

$$\theta_{R}(\tau,\xi) = 0 \quad \text{at } \tau = 0 \tag{35}$$

$$\theta_R(\tau, \zeta) = 0$$
 at $\tau = 2\tau_0$ (36)

$$\theta_R(\tau,\xi) = 0 \qquad \text{in } 0 \le \tau \le 2\tau_0, \quad \xi = 0 \tag{37}$$

and therefore
$$\frac{\partial Q_R}{\partial \tau} = (I - \varpi) \Big[\Big[\theta_F(\tau) + \theta_H(\tau, \xi) + \theta_R(\tau, \xi) \Big]^4 - G(\tau) \Big\}$$
 (38)

Problem 3, defined by equations (34-38), when being solved by the Generalized Integral Transform Technique (GITT), presents the same auxiliary eigenvalue problem as defined for problem 2. Therefore, problem 3 presents the same eigenfunctions, norms, eigenvalues and inverse transformed pair as problem 2. After the integral transformation, it results:

$$\frac{d\overline{\theta}_{P_i}}{d\xi} = -\gamma_i^2 \overline{\theta}_{P_i}(\xi) - \frac{(l-\varpi)}{N} \overline{G}_i(\xi)$$
(39)

Where $\overline{G}_{i}(\xi)$ represents the integral transform of the radiative heat flow divergent, given as:

$$\overline{G}_{i}(\xi) = \frac{1}{N_{i}^{1/2}} \int_{0}^{\tau_{0}} \psi_{i}(\tau) \left\{ \left[\theta_{F}(\tau) + \theta_{H}(\tau, \xi) + \theta_{R}(\tau, \xi) \right]^{i} - G(\tau) \right\}$$

$$(40)$$

The ordinary differential equation system, given by equation (39) must satisfy the following transformed inlet condition:

$$\overline{\theta}_{Pi}(0) = 0 \tag{41}$$

The incident radiation, $G(\tau)$, that appears in equation (40) is obtained by the Garlekin method and equals:

$$G(\tau) = \frac{1}{2} \left\{ \varepsilon_{l} \theta_{l}^{4} E_{2}(\tau) + 2\rho_{l} E_{2}(\tau) K_{l} + \varepsilon_{2} \theta_{2}^{4} E_{2}(2\tau_{0} - \tau) + 2\rho_{2} E_{2}(2\tau_{0} - \tau) K_{2} + (1 - \varpi) \int_{0}^{2\tau_{0}} \theta^{4}(\tau', \xi) E_{l}(|\tau - \tau'|) d\tau' + \varpi \sum_{m=0}^{M} c_{m} \left\{ (-1)^{m+1} m! E_{m+2}(\tau) + \sum_{j=0}^{m} \frac{m! \left[l + (-1)^{j} \right]}{(m-j)! (j+1)} \tau^{n-j} - \sum_{j=0}^{m} \frac{m!}{(m-j)!} (2\tau_{0})^{m-j} E_{j+2}(2\tau_{0} - \tau) \right\} \right\}$$

$$(42)$$

where c_m are the expansion coefficients for the representation of the incident radiation.

$$K_{1} = \beta^{*} \left\{ \alpha_{2} E_{3}(2\tau_{0}) \varepsilon \, \beta_{1}^{4} + E_{3}(2\tau_{0}) \varepsilon_{2} \theta_{2}^{4} + (1 - \varpi) \int_{0}^{2\tau_{*}} \theta^{4}(\tau', \zeta) \left[E_{2}(\tau') + \alpha_{2} E_{2}(2\tau_{0} - \tau') \right] d\tau' + \varpi \sum_{m=0}^{M} c_{m} \left(T_{m} + \alpha_{2} T_{m}^{*} \right) \right\}$$

$$(43)$$

$$K_{2} = \beta^{*} \left\{ \alpha_{l} E_{3}(2\tau_{0}) \varepsilon_{2} \theta_{2}^{4} + E_{3}(2\tau_{0}) \varepsilon_{l} \theta_{l}^{4} + (1 - \varpi) \int_{0}^{2\tau_{o}} \theta^{4}(\tau', \xi) \left[\alpha_{l} E_{2}(\tau') + E_{2}(2\tau_{0} - \tau') \right] d\tau' + \varpi \sum_{m=0}^{M} c_{m} \left(\alpha_{l} T_{m} + T_{m}^{*} \right) \right\}$$

$$(44)$$

with:

$$\alpha_1 = 2\rho_1 E_3(2\tau_0) \qquad \qquad \alpha_2 = 2\rho_2 E_3(2\tau_0) \qquad \qquad \beta^* = \frac{1}{1 - \alpha_1 \alpha_2} \tag{45}$$

The terms T_m e T_m^* are presented by Cengel (1984) and $E_n(z)$ is the integral of exponential functions of the type:

$$E_n(z) = \int_0^1 \eta^{n-2} e^{-z/\eta} d\eta$$
 (46)

According to Özişik (1973), once the temperature distribution of the body is determined, the local Nusselt number in the dimensionless form can be calculated by:

$$Nu = \frac{4\tau_0}{I - \theta_m(\xi)} \left[-\frac{\partial \theta(\tau, \xi)}{\partial \tau} + \frac{Q_R(\tau, \xi)}{N} \right]_{\tau = 0}$$
(47)

with $\theta_m(\xi)$ representing the average dimensionless temperature and $Q_R(\tau,\xi)$ the liquid flow of radiative heat. The latter is given by:

$$Q_{R}(\tau,\xi) = \frac{1}{2} \left\{ \left(\varepsilon_{I} \theta_{I}^{4} + 2\rho_{I} K_{I} \right) E_{3}(\tau) + (I - \varpi) \int_{0}^{\tau} \theta^{4}(\tau',\xi) E_{2}(\tau - \tau') d\tau' + \varpi \sum_{m=0}^{M} c_{m} m! \left[(-I)^{m+I} E_{m+3}(\tau) - \sum_{j=0}^{m} \frac{\tau^{m-j}}{(m-j)!} \cdot \frac{(-I)^{j+I}}{j+2} \right] - \left\langle \left(\varepsilon_{2} \theta_{2}^{4} + 2\rho_{I} K_{2} \right) E_{3}(2\tau_{0} - \tau) + (I - \varpi) \int_{\tau}^{2\tau_{0}} \theta^{4}(\tau',\xi) E_{2}(\tau' - \tau) d\tau' + \varpi \sum_{m=0}^{M} c_{m} m! \sum_{j=0}^{m} \frac{I}{(m-j)!} \left[\frac{\tau^{m-j}}{j+2} - (2\tau_{0})^{m-j} E_{j+3}(\tau_{0} - \tau) \right] \right\rangle \right\}$$

$$(48)$$

4 Results and Discussion

The resulting system of ordinary differential equations, submitted to the inlet transformed condition, was solved by computacional code written in programing language Fortran using Fortran Powerstation 4.0 software and implemented in a Pentium III-450Mhz personal microprocessor. The results are presented in tables and graphs, with the analysis taking into account the quality obtained in terms of convergence and stability of the solution.

Table 1 shows the values obtained for the temperature field at various positions along the plate. For the simulation, the following radiative properties were considered: ϖ =0.5, τ_0 =1.0, N=0.1, ε_1 = ε_2 =1.0, θ_1 = θ_2 =1.0 and θ_0 =0.0. It can be observed that the results converge in the fourth decimal house for a truncation in the order of 25 eigenvalues (Nc=25) in any point of the medium.

Figure 2 presents the effect of the conduction-radiation parameter (N) on the temperature profile in several dimensionless positions (ξ). The cases of N=10 and N=0.1 characterize, respectively, weak and reasonably strong radiation effects. It can be observed from the resulting graphs that the increase in the effect of radiation causes a "flattening" of the curves and a decrease in the temperature gradients of the channel walls. This phenomenon occurs due to an increase of the heat propagation velocity in the medium. Also, the effect of radiation is less pronounced for small values of ξ .

$\tau/2\tau_0$	Nc=10	Nc=15	Nc=20	Nc=25	Nc=30	Nc=35	Nc=40
0.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	0.1762	0.1791	0.1791	0.1791	0.1790	0.1791	0.1791
0.2	0.0290	0.0247	0.0247	0.0247	0.0247	0.0247	0.0247
0.3	0.0124	0.0169	0.0169	0.0169	0.0169	0.0169	0.0169
0.4	0.1927	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150
0.5	0.0101	0.0143	0.0143	0.0143	0.0143	0.0143	0.0143
0.6	0.1927	0.0150	0.0150	0.0150	0.0150	0.0150	0.0150
0.7	0.0124	0.0169	0.0169	0.0169	0.0169	0.0169	0.0169
0.8	0.0290	0.0247	0.0247	0.0247	0.0247	0.0247	0.0247
0.9	0.1762	0.1791	0.1791	0.1791	0.1790	0.1790	0.1790
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 1 Convergence analysis of the temperature profile.

Figure 3 shows the effect of albedo in single scattering for the local Nusselt number. The curve for a pure diffusing medium (ϖ =1) represents the case of a non-interaction between conduction/convection and radiation; in this case, the value of the Nusselt number decreases with the axial distance to an assintotic value of π^2 . For ϖ smaller than 1, the Nusselt number increases to decreasing values of ϖ , becoming a maximum when the medium is purely absorbing (ϖ =0).

The effect of the conduction-radiation parameter (N) on the local Nusselt number is shown in Figure 4 (for the cases where albedo ϖ =0 and ϖ =0.9). When the conduction is dominating (N=10), the effect of the albedo is neglectable, as shown by superposition of the curves. In this case, the heat transfer mechanism can be considered to occur only by conduction and convection. On the other hand, an increase in influence of radiation (N=0.1) leads to an increase in the local Nusselt number in all axial positions along the boundary surface.

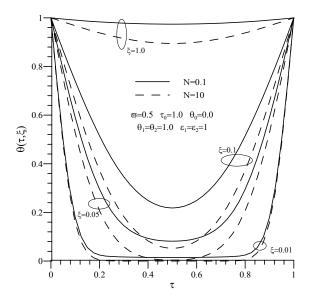


Figure 2: Effect of ξ and N on temperature.

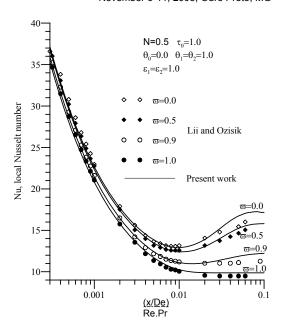


Figure 3: Effects of N on local Nusselt number and comparison data.

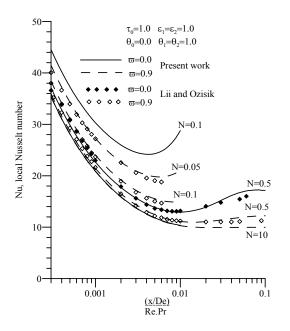


Figure 4: Effects of N on local Nusselt number and comparison data.

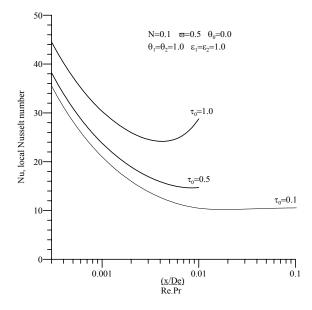


Figure 5: Effects de τ_0 on local Nusselt number.

In Figure 5, the effect of medium optical thickness on the local Nusselt number is analyzed. For bodies optically thin $(\tau_0 \le 0.1)$, the effect of radiation is small and can be neglected without significantly affecting the precision of the results obtained. However, as the value of τ_0 increases, the greater the effect that radiating energy has on the temperature distribution in the medium and, consequently, the values of the local Nusselt number.

5 Conclusion

The simultaneous application of GITT and Garlekin method was determined to be efficient in the study of problems involving heat transfer in participating media for the conduction-convection-radiation coupling, as the results obtained were in high agreement with the results of Lii Özişik (1973). Furthermore, the applied methodology allows an analytical and systematic treatment of problems presenting a high mathematical complexity.

When the effect of the radiation becomes predominant in a heat transfer process, the complete thermally developed flow condition is not reached. Therefore, the inclusion of the radiation effect term in the equation is imperative as its neglect will result in significant errors in results obtained.

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