DESCRIPTION OF A VERTICAL RISER BEHAVIOR IN FREQUENCY AND TIME DOMAIN

Coelho, Fabio M.
UNICAMP / FEM
fabio@dep.fem.unicamp.br

Shiguemoto, Denis A.
UNICAMP / FEM
dshiguem@dep.fem.unicamp.br

Morooka, Celso K.
UNICAMP / FEM / DEP
morooka@dep.fem.unicamp.br

Abstract. *Recent petroleum discoveries by the Brazilian oil and gas industry lead to research new technologies that allow technical and economical viability to produce petroleum in deep and ultradeep water. One of the main components of a deepwater production system is the production riser. A riser is a pipe that connects sub sea production equipments with production facilities on the deck of platforms. Sometimes, the purpose of a riser could be to export oil and gas production from the process facility to an offloading system and in sequence to the land. In general, risers installed in ultra large waterdepth need to allow high oil and gas flowrates and, sometimes this requirement is mandatory, from the economical point of view, for designing field production development of a deepwater petroleum reservoir. Many studies are available in the literature regarding viability of the use of vertical rigid production risers as an economical alternative to riser systems. In the present work, main equations for the dynamic behavior of a vertical rigid riser in terms of displacement are described and, solutions in frequency domain and time domain, respectively, are featured. Comparisons between both approaches are taken into account and, calculation results are presented and discussed to a typical riser behavior in different wave and riser conditions in order to illustrate riser displacements and to show the importance of this kind of analysis in a riser design.*

Keywords*: offshore risers, ocean waves, riser hydrodynamics*

1. Introduction

Recent discoveries by the offshore petroleum industry, mainly in Brazil, lead to develop new technologies to produce petroleum fields economically, in deep and ultra deep water depths. For instance, the recent world's record of an oilfield production from ultra deep water depth dates from July, 2004, in Gulf of Mexico that reaches 2301 meters in water depth.

An important component of an offshore petroleum production system is the riser system. A riser system makes the link between the petroleum production well on the sea bottom and the production process facilities at the sea surface. Basically, two types of riser are commonly applied for deepwater scenario: rigid and flexible riser. Flexible risers are made by several concentric tubular layers with low bending stiffness that usually allows large horizontal and vertical offset of the floating platform at the sea surface. The most used configuration of flexible riser is the catenary configuration Pesce *et al.* (2003), which has several related studies available in the literature, such as, the analytical equations proposed by Ramos Jr. *et al.* (2005), to estimate flexural structural behavior of flexible risers. Furthermore, rigid risers are generally steel pipes and, usually, have economical advantage if compared with flexible risers. But the fatigue failure life of rigid risers due to cyclic loads such as waves, currents and platform motions must be observed.

Sometimes, a riser is used not only to carry oil and gas up from a well; it can conduct water to injection into the reservoir or to export oil and gas to offloading system from the process facility. Furthermore, many studies in the literature indicate different uses for vertical production riser itself in hybrid configuration, combined, with flexible risers Pereira *et al.* (2005) and Roveri and Pessoa (2005).

In the present work, a dynamic behavior of a vertical rigid riser in terms of in line displacements is described. Fundamental equation to describe this behavior and respective numerical solutions are featured in frequency and time domain, respectively. Calculations for riser behavior in waves are carried out for both approaches and the results are compared. Important and positive aspects of each approach, frequency and time domain analysis, are depicted in the point of view of the riser design.

2. Theoretical Background

The study of dynamic behavior of vertical risers is very important for offshore petroleum system design, and usually, it can be carried out throughout two approaches: analytical and numerical methods. Analytical method usually considers uniform geometry and materials properties. Catenary equation is commonly used to model analytically flexible riser's behavior Silveira *et al.* (2003). On the other hand, numerical method allows easily changes in riser's geometry and materials properties. Solutions for numerical methods, which require solutions for a set of equations, can be obtained by solving equation of riser behavior, in matrix form, by a particular weighted residual method (Galerkin's Method), that basically consists in dividing the riser structure into small beam elements where the properties of each element are concentrated at the nodes of the element. In general, it needs an iterative process.

The adopted methodology for solution and solutions are in time or frequency domain. The riser behavior analysis can be classified in two types: deterministic and non-deterministic (stochastic). The deterministic analysis in time or frequency domain gives the riser response relative to a given excitation force. Time domain analysis is usually required when nonlinearities present in riser system are relevant. Frequency domain analysis is more utilized when nonlinearities in the riser system are possible to be simplified and linearized. Non-deterministic analysis (stochastic) is based on statistical analysis of the riser behavior in time domain, and relevant statistical parameters of riser behavior are obtained to the analysis.

In the present work, comparisons between deterministic analysis in the time and frequency domain are carried out. Results are presented in terms of displacements of the riser for different incident wave frequencies.

2.1 Linear Waves and Hydrodynamic Loads

In the present study, Linear Airy Waves Theory is adopted. Fluid particle velocities and accelerations are obtained from the linear wave equations and then hydrodynamic forces are estimated. Sinusoidal wave field, as here considered, is particularly suitable when wave height is small when compared to its length.

From the following, in Eq. (1) and Eq. (2) water particle velocity (u) and acceleration ($\dot{u}$) of incident wave that have a given height and frequency can be obtained for each point in the riser length.

$$u = \frac{gkH}{2\omega} \frac{\cosh ks}{\cosh kd} \cos\theta \tag{1}$$

$$\dot{u} = \frac{gkH}{2} \frac{\cosh ks}{\cosh kd} \sin\theta \tag{2}$$

where, H is wave height, ω is wave frequency given by the dispersion relationship ($\omega^2 = gk \tanh(kd)$), g is gravity, k is wave number, s is the distance between the sea surface and the point that velocity and acceleration are calculated, d is the water depth and θ is phase angle given by $(z - \omega t)$.

For the solution of riser displacements in frequency domain the linearized fluid viscous damping is taken into account (Morooka and Yokoo, 1992). It is well known that this procedure implies in less computational effort if compared with time domain solution. Frequency domain analysis is normally suitable for the initial stages in a riser system design process.

From values of velocity and acceleration of wave particle, hydrodynamic forces acting on each element is estimated. In following, riser mass, structural, viscous damping and stiffness matrix for each element is assembled. Finally, from the equation of riser dynamic behavior, riser displacement response are obtained for the given incident wave frequency. This procedure can be repeated for each wave frequency of interest.

In the present study, riser is considered a slender element. The hydrodynamic wave forces are calculated through the Morison's Formulation (Chakrabarti, 1987). Morison's equation is basically composed by two terms: an inertia term component and a drag component caused by water particle dynamics.

If the platform horizontal offset and the wave loads in the riser are considered, the inertia term of the hydrodynamic load in an infinitesimal segment ds of the riser could be given by:

$$df_I = C_A \rho \frac{\pi}{4} D^2 \frac{\partial u}{\partial t} ds \tag{3}$$

where, C_A represents added mass coefficient, ρ is the fluid density and D is the riser external diameter.

Then, the drag term is given by:

$$df_D = \frac{1}{2} C_D \rho D |u| u ds \tag{4}$$

with C_D as the drag coefficient.

Integration of Eq. (3) and Eq. (4) throughout overall riser length and the summation of each term results in Morison's Equation simplified for two dimensional case, as follows:

$$f_{wave} = C_A A_I(\dot{u} - \ddot{x}) + C_D A_D |u - \dot{x}|(u - \dot{x}) \tag{5}$$

where, A_I represents sectional area of the riser multiplied by fluid density and A_D is the fluid density multiplied by riser external half diameter. In the present study, the total force given by Eq. (5) in each riser element is divided and concentrated equal between two adjacent element nodes.

2.2 Frequency Domain Analysis

Horizontal displacements of the riser can be derived from the equilibrium of a riser element. The basic equation for lateral displacements of a riser considering internal forces due to produced or exported oil flow or injection water flow, environmental loads and gravity, can be described as follows:

$$\frac{\partial^2}{\partial z^2}[EI(z)\frac{\partial^2 x}{\partial z^2}] - T_e(z)\frac{\partial^2 x}{\partial z^2} - w(z)\frac{\partial x}{\partial z} + m_z(z)\frac{\partial^2 x}{\partial t^2} = f(z,t) \tag{6}$$

The left hand side terms of Eq. (6) represents horizontal reaction of the riser due to its elastic stiffness, effective tension effects, linear weight in water w, and to an inertia reaction force due to riser's acceleration, respectively. The effective tension can be obtained from the relationship between riser axial tension and forces due to riser's external and internal pressures, given by:

$$T_e(z) = T(z) + A_o(z)p_o(z) - A_i(z)p_i(z) \tag{7}$$

where *T (z)* is the riser axial tension, A_o (A_i) is riser's external (internal) sectional area, and p_o (p_i) is the external (internal) pressure of fluid. The term in the right hand side of Eq. (6) is the riser external loads.

By linearizing the viscous term in Eq. (6), and writing it in matrix representation, the equation of the riser dynamic behavior in frequency domain becomes as follows:

$$\left[-\omega^2([m]+[M]) - i\omega\left([B] + \frac{8}{3\pi}\omega\{x\}[B^*]\delta\right) + ([R]+[K])\right]\{x\}e^{i\beta} = \{f\}e^{i\alpha} \tag{8}$$

where $[M]$ and $[m]$ represents mass and added mass matrix, respectively, $[B]$ is global structural damping matrix, $[B^*]$ is linearized viscous damping matrix, $[R]$ is hydrostatic stiffness matrix, $[K]$ is stiffness matrix, $\{x\}$ riser's displacement vector, $\{f\}$ is external total forces vector, ω is incident wave frequency, δ is Kronecker's Delta, α is exciting force phase and β is displacement phase. Total external forces are calculated considering all environmental forces that act in the system. In the present work, only wave forces are taken into account.

The global matrices are obtained adding local matrices for the riser by an assembling procedure. Basically, it consists of the summation of the fourth quadrant of an element individual matrix with the second quadrant of the next element individual matrix. This process is repeated until to reach overall matrix of the system. Bellow, a representation for mass and stiffness matrix are presented:

$$M = \begin{bmatrix} m_1 & & & & \\ & \ddots & & 0 & \\ & & \ddots & & \\ & 0 & & \ddots & \\ & & & & m_n \end{bmatrix} \tag{9}$$

$$K = \begin{bmatrix} k_{11} & k_{12} & \cdots & \cdots & k_{1n} \\ \vdots & k_{22} & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ k_{n1} & k_{n2} & \cdots & \cdots & k_{nn} \end{bmatrix} \tag{10}$$

Moreover, natural frequencies and modes for riser dynamics (eigenvalues and eigenvectors) can be calculated throughout the Eq. (6). For this purpose, only mass $[M]$ matrix, stiffness $[K]$ matrix and riser's displacements $\{x\}$ in the left hand side of Eq. (6) are taken into account, and the other terms are considered as zero. Mass and stiffness matrix are considered for numerical solution of the eigenvalue problem, and eigenvalues (natural frequencies) and eigenmodes are obtained for the riser.

2.3 Time Domain Analysis

The basis of the time domain analysis of present work is shown in Ferrari and Bearman (1999), and other details have been described in previous works Martins *et al.* (2003) and Coelho *et al.* (2004). If time domain representation is compared with the frequency domain one, it can be concluded that the time domain solution represents more realistically riser dynamics; besides involved computational efforts are bigger than those in frequency domain analysis.

As well as in frequency domain, the time domain analysis uses Linear Wave Theory to estimate wave kinematics. Hydrodynamic forces are also calculated here by Morison's Equation approach. However, in this analysis, non-linearities due to hydrodynamic viscous term are accounted. Then, the equation for hydrodynamic wave loads become as follows:

$$\{f\}_{wave} = C_M A_I \dot{u} + C_D A_D |V_r| (u - \dot{x}) - C_A A_I \ddot{x} \tag{11}$$

where, $|V_r|$ is the relative velocity given by $|V_r| = \sqrt{(u - \dot{x})^2}$ for the in line case.

$$[M]_x \ddot{x} + [B]_x \dot{x} + [K]_x x = C_M A_I \frac{\partial u}{\partial t} + C_D A_D |V_r| (u - \dot{x}) - C_A A_I \ddot{x} \tag{12}$$

Equation (12) shows the equation for riser dynamic behavior in matrix form. The lumped model for riser mass and stiffness matrix is adopted. The solution for riser displacements from the Eq. (12) in time domain is calculated by applying Newmark β numerical time integration scheme.

3. Results

A typical top tensioned production riser is considered in the calculations. Pinned riser top and bottom boundary conditions were taken in the calculations. The main dimensions of the riser are shown in the Tab. 1.

Table 1: Dimensions and coefficients for the top tensioned riser

Water depth	1800.0 m
Riser length	1820.0 m
Top tension	2200.0 kN
Outside/ Inside diameter	0.25/ 0.21106 m
Riser material density	7860.0 kg/m^3
Sea water density	1025.0 kg/m^3
Riser internal fluid density	800.0 kg/m^3
Riser material Young Modulus	210.0 GPa
Drag Coefficient (C_D) / Added Mass Coefficient (C_A)	1.0 / 0.6

Table 2: Natural frequencies versus eigenmodes of the riser

Eigenmodes	1	2	3	4	5	6
Natural Frequencies	0.147189	0,2943987	0,4416514	0,5889681	0,7363701	0,8838787

Figure 1 shows calculations of the riser response in frequency domain. The riser response is shown for three different points along the riser length. As it is expected, it can be noted that the peak frequencies coincides with natural frequencies of the riser when compared with riser eigenvalue problem solution, as it can be observed from the calculated natural frequencies in Tab. 2. The structural damping was neglected in all the analysis here presented, and only the viscous fluid damping was taken into account.

Figure 2 shows comparison between calculation results in time and frequency domain for maximum riser displacement envelope. Figure 2 (a) shows maximum riser displacements with only wave loads. Figure 2 (b) presents maximum riser displacements for the only platform horizontal top motions (forced oscillation) case without wave load. In general, comparisons between frequency and time domain results have shown good agreement, as it can be observed from the Fig. 2 (a) and (b). The difference of amplitude observed in Fig. 2 (a) can be explained as the effect of viscous drag in the wave exciting force calculations accounted differently on each calculation procedure. The viscous damping term is linearized for frequency domain calculations. However, when no wave exciting force is present, as in Fig. 2 (b) differences between two calculations are very small.

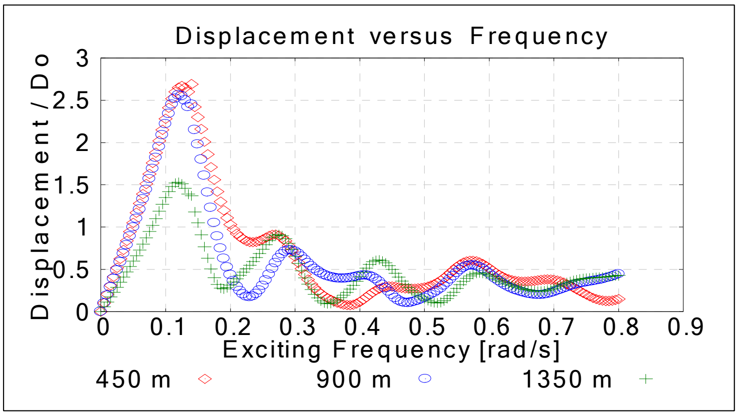
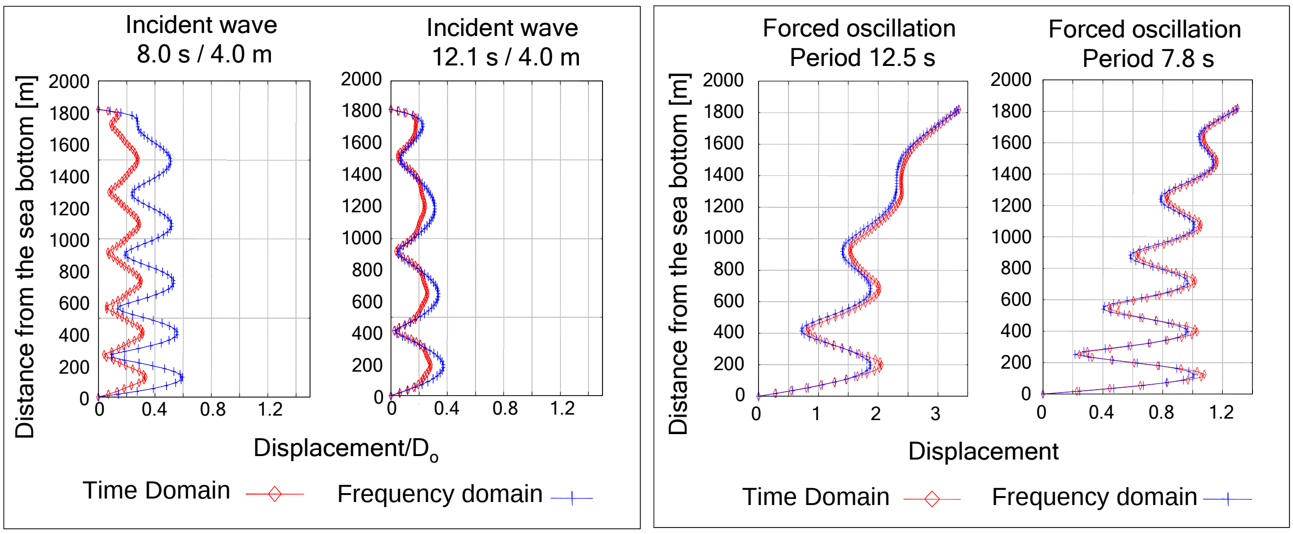


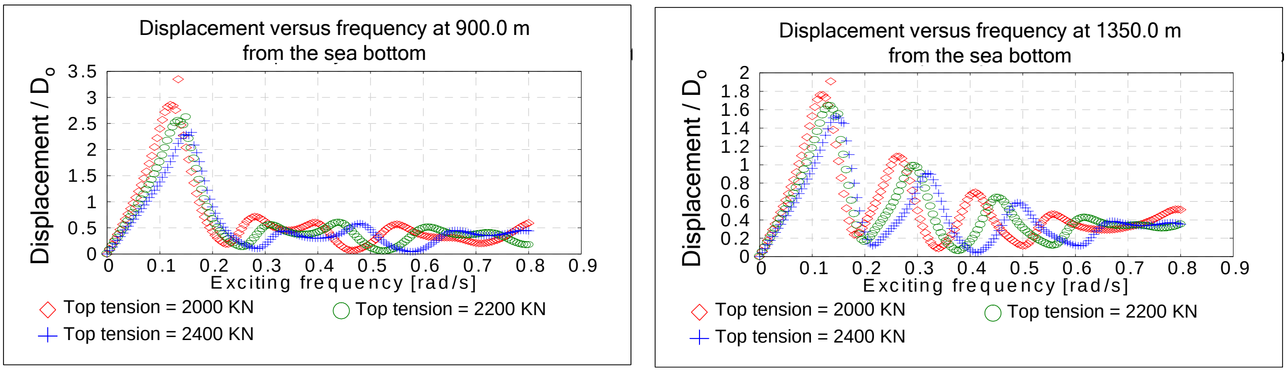
Figure 1: Displacement versus frequency for 450 m, 900 m and 1350 m above the sea bottom.



(a) (b)

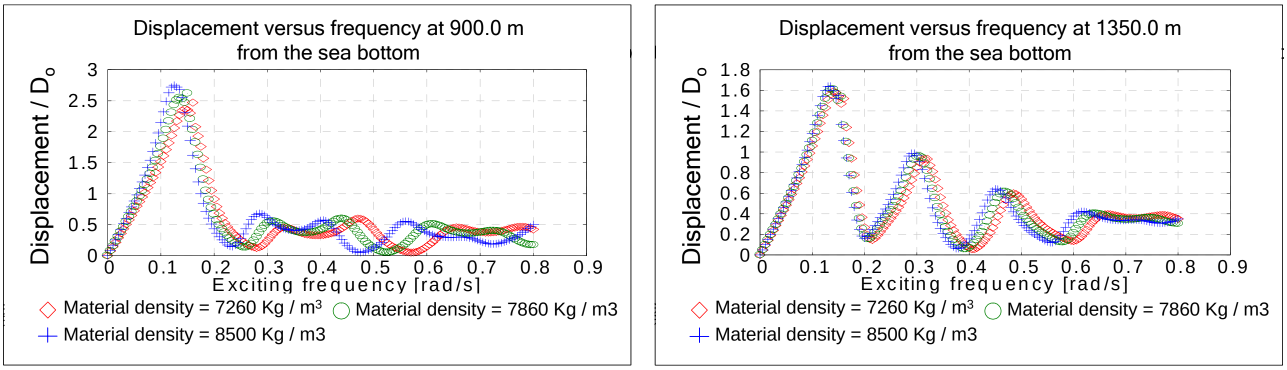
Figure 2: Displacements for the riser (a) only wave load case, and (b) only platform top horizontal motions (forced oscillation) case.

Figure 3 presents displacement against wave frequency at different points along the riser length for different riser top tensions. Figure 3 (a) shows results for riser location of 900 m from the sea bottom, and Fig. 3 (b) for 1350 m. It can be observed that variations on the riser top tension modify riser displacement response. It happens because variations of the riser top tension modify the riser geometric stiffness. In Fig. 3, it can be noted that if riser top tension increases the riser stiffness increases, and the overall riser displacement response curve moves to the right. Moreover, from Fig. 3 (a) and (b), if riser top tension increases, riser displacement along its length decreases.



(a) (b)

Figure 3: Displacements versus frequency at (a) 900.0 m and (b) 1350.0 m from the sea bottom, for different riser top tensions



(a) (b)

Figure 4: Displacements versus frequency at (a) 900.0 m and (b) 1350.0 m from the sea bottom, varying riser material density.

Similar tendency is observed from the results in Fig. 4 (a) and (b). In calculations, riser material density was changed instead of top tension. Increasing the material density, the mass of the riser also increases, and it causes a decrease in natural frequencies of the system.

4. Conclusions

In numerical simulations of a riser system many nonlinearities are usually involved, such as that introduced by water viscosity and riser geometry. The solution of riser behavior in time domain usually represents, in more appropriate manner, the nonlinearities present in a riser system. On the other hand, linearized model for riser behavior as taken in frequency domain analysis makes easy the handling and the obtaining of solutions, and involved computational efforts are, in general, less than in time domain.

In the present work, calculations for riser displacements have been performed in order to understand differences between solutions from time domain and from frequency domain analysis.

In spite of the accuracy of calculation results, comparisons with experimental data from laboratory or *in situ* measurements are fundamental to clarify riser dynamic behavior. In general, frequency domain analysis can be used in complement to time domain analysis in a designing process for production or exporting vertical riser system or further for its operation.

5. Nomenclature

H ➔ Wave Height
ω ➔ Wave frequency
u ➔ Horizontal velocity of fluid particle
$\dot{u}$ ➔ Vertical velocity of fluid particle
g ➔ Gravitational acceleration

k ➔ Wave number
s ➔ Distance between the sea surface and the point that velocity and acceleration are calculated
d ➔ Sea water level
θ ➔ Phase angle given by $(z - \omega t)$
ds ➔ Infinitesimal element of riser
C_A ➔ Added mass coefficient
ρ ➔ Fluid density
D ➔ Riser's external diameter
C_D ➔ Drag coefficient
$T_e(z)$ ➔ Effective tension
$T(z)$ ➔ Real tension
A_o ➔ Riser's external transversal section area
A_i ➔ Riser's internal transversal section area
p_o ➔ Fluid external pressure
p_i ➔ Fluid internal pressure
$[M]$ ➔ Mass matrix
$[m]$ ➔ Added mass matrix
$[B]$ ➔ Global potential damping matrix
$[B*]$ ➔ Linearized viscous damping matrix
$[R]$ ➔ Hydrostatic stiffness matrix
$[K]$ ➔ Stiffness matrix
$\{x\}$ ➔ Riser's displacements
$\{f\}$ ➔ External forces resultant vector
δ ➔ Kronecker's Delta
α ➔ Exciting force phase
β ➔ Displacement phase
T ➔ Axial tension
$\overline{m}$ ➔ Mass per unit length
L ➔ Total length
EI ➔ Young Modulus
n ➔ Number of modes
$|V_r|$ ➔ Relative Velocity

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