AN INDUCED ANISOTROPY DAMAGE MODEL APPLIED TO TWO-DIMENSIONAL SOLID ANALYSIS

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Abstract. *The work presents an investigation of the computational effectiveness and numerical validity of a recently proposed anisotropic damage model, specifically concerning two-dimensional analysis of concrete structures. In the model, the material is assumed as initial elastic isotropic medium presenting anisotropy and bimodular response (distinct elastic responses whether tension or compression stress states prevail) induced by the damage. To account for bimodularity two damage tensors governing the rigidity in tension and compression regimes, respectively, are introduced. Damage activation is expressed by two criteria indicating the initial and further evolution of damage. The local version of the model is implemented in a finite element code allowing plane analysis. Constant strain quadrilateral (4 nodes) elements are used. A finite-element analysis of a reinforced concrete beam submitted to monotonic loading is presented and critically discussed.*

Keywords*: damage mechanics, anisotropy, constitutive model*

1. Introduction

The Continuum Damage Mechanics (CDM) is a tool for the simulation of the material deterioration in equivalent continuous media due exclusively to microcracking process. A material can be simulated as a continuous medium and the influence of the internal changes caused by the microcracks are considered through scalar or tensor damage variables that decrease the initial rigidity of the equivalent medium. In the CDM, the damage effects are evidenced in the rigidity constitutive tensor. The damage implies in a reduction of several rigidity components, where the damaged material can keep its isotropic properties or to become anisotropic. In the isotropic models the damage does not affect the number of both the symmetry directions and the initial symmetry planes of the material, i. e., if the medium is initially isotropic or anisotropic with some degrees, those characteristics are preserved during the damage process.

On the other hand, the anisotropic models have the ability to change the number of both the symmetry directions and the initial symmetry planes of the material. In the last years, many constitutive models for the concrete heve been proposed in order to take into account anisotropic characteristic of that medium have been proposed.

It can be assumed that the concrete belongs to a category of materials that can be considered initially isotropic and unimodular, however they start to present different behaviours in tension and compression when damaged (Mazars *et al.*, 1990). A formulation of constitutive laws for isotropic and anisotropic elastic materials presenting different behaviours in tension and compression was proposed by Curnier *et al.* (1995) to two and three-dimensional cases. Pituba (2003) has extended this formulation in order to introduce the damage effects. A constitutive model for the concrete is then derived and briefly presented in the section 2 of this paper. Accordingly, the concrete is initially considered as an isotropic continuous medium with anisotropy and bimodularity induced by the damage. On one side the class of anisotropy induced and considered in the model (transverse isotropy) elapses from the assumption that locally the loaded concrete always presents a diffuse oriented damage distribution as appointed by experimental observations (Wiliam *et al.*, 1988 and Van Mier, 1984). It is considered that the oriented damage is responsible for the change of material characteristics presenting it as transverse isotropic medium.

Other important aspect of the model is the bimodularity induced by the damage that is captured by the definition of two damage tensors: one for dominant tension states and another one for dominant compression states.

In section 3, the proposed model is then applied in two-dimensional analysis of a reinforced concrete beam, in order to evaluate the performance of the actual version of the model presenting its potentialities and limitations. Finally, a few conclusions are discussed in section 4.

2. Damage constitutive model

The model formulation is built from the formalism presented in Pituba (2003). Moreover, the model respects the principle of energy equivalence between damaged real medium and equivalent continuous medium established in the CDM (Lemaitre, 1996). Thus, both the rigidity and flexibility constitutive tensors of the equivalent continuous medium result symmetric. Criteria for damage evolution and identification of transverse isotropy local plane are proposed.

Note that general forms to the fourth-order damage tensor $\boldsymbol{D}$ can be proposed in order to take into account the anisotropy induced by damage. In this work the definition of that tensor follows a so-called scalar form expressed as: $\boldsymbol{D} = f_j(D_i)\, \boldsymbol{M_j}$, where $f_j(D_i)$ are scalar valued functions of the damage scalar variables D_i and $\boldsymbol{M_j}$ are anisotropic tensors. In the case of this model, the particular adopted tensors to $\boldsymbol{M_j}$ are the ones that allow representing the transverse isotropy. Concerning bimodularity induced by damage, two damage tensors are defined. Then, for dominant tension states, a scalar damage tensor is proposed:

$$\boldsymbol{D_T} = f1(D_1, D_4, D_5)\,(\boldsymbol{A} \otimes \boldsymbol{A}) + 2f_2(D_4, D_5)\,[(\boldsymbol{A} \overline{\underline{\otimes}} \boldsymbol{I} + \boldsymbol{I} \overline{\underline{\otimes}} \boldsymbol{A}) - (\boldsymbol{A} \otimes \boldsymbol{A})] \tag{1}$$

where $f_1(D_1, D_4, D_5) = D_1 - 2\, f_2(D_4, D_5)$ and $f_2(D_4, D_5) = 1 - (1\text{-}D_4)\,(1\text{-}D_5)$. The variable D_1 represents the damage in direction orthogonal to the transverse isotropy local plane of the material, while D_4 is representative of the damage due to the sliding movement between the crack faces. The third damage variable, D_5, is only activated if a previous compression state accompanied by damage has occurred. In the Equation (1), the tensor $\boldsymbol{I}$ is the second-order identity tensor and the tensor $\boldsymbol{A}$, by definition, (Curnier *et al.*, 1995), is formed by dyadic product of the unit vector perpendicular to the transverse isotropy plane for himself. Those products are given in Pituba (2003).

On the other hand, for dominant compression states, it is proposed the other damage tensor:

$$\boldsymbol{D_C} = f_1^*(D_2, D_4, D_5)\,(\boldsymbol{A} \otimes \boldsymbol{A}) + f_2(D_3)\,[(\boldsymbol{I} \overline{\otimes} \boldsymbol{I}) - (\boldsymbol{A} \otimes \boldsymbol{A})] + 2f_3(D_4, D_5)\,[(\boldsymbol{A} \overline{\underline{\otimes}} \boldsymbol{I} + \boldsymbol{I} \overline{\underline{\otimes}} \boldsymbol{A}) - (\boldsymbol{A} \otimes \boldsymbol{A})] \tag{2}$$

where $f_1^*(D_2, D_4, D_5) = D_2 - 2f_3(D_4, D_5)$,$f_2(D_3) = D_3$ and $f_3(D_4, D_5) = 1 - (1\text{-}D_4)\,(1\text{-}D_5)$. Note that the compression damage tensor introduces two additional scalar variables in its composition: D_2 and D_3. The variable D_2 (damage perpendicular to the transverse isotropy local plane of the material) reduces the Young's modulus in that direction and in conjunction to D_3 (that represents the damage in the transverse isotropy plane) degrades the Poisson's ratio throughout the perpendicular planes to the one of transverse isotropy.

It must be noted that the described forms for $fj(Di)$ are appropriate in the sense that they allow capturing the damage of the shear module as well as respect the hypothesis of tangential jump null of the constitutive tensor (Curnier *et al.*, 1995). Moreover, the forms for $\boldsymbol{D_T}$ and $\boldsymbol{D_C}$ respect the energy equivalence in the sense that provide a symmetrical rigidity tensor.

Following the extended framework of Curnier proposed in Pituba (2003), the invariant form of the proposed constitutive model is given by:

$$W(\varepsilon) = \rho\psi(\varepsilon) := \begin{cases} W_-(\varepsilon) & if \quad \boldsymbol{g}\,(\varepsilon, \boldsymbol{D_T}, \boldsymbol{D_C}) < 0, \\ W_+(\varepsilon) & if \quad \boldsymbol{g}\,(\varepsilon, \boldsymbol{D_T}, \boldsymbol{D_C}) > 0, \end{cases} \quad and\ W_- = W_+ = W_0 \ \ if\ \boldsymbol{D_T} = \boldsymbol{D_C} = \boldsymbol{0} \tag{3}$$

$$W_+ = \rho\psi_+(\varepsilon) = \frac{\lambda_{11}}{2} tr^2(\varepsilon) + \mu_1 tr(\varepsilon^2) - \frac{\lambda_{22}^+(D_1, D_4, D_5)}{2} tr^2(\boldsymbol{A}\varepsilon) - \lambda_{12}^+(D_1)\, tr(\varepsilon)\, tr(\boldsymbol{A}\varepsilon) - \mu_2(D_4, D_5)\; tr(\boldsymbol{A}\varepsilon^2) \tag{4}$$

$$W_- = \rho\psi_-(\varepsilon) = \frac{\lambda_{11}}{2} tr^2(\varepsilon) + \mu_1 tr(\varepsilon^2) - \frac{\lambda_{22}^-(D_2, D_3, D_4, D_5)}{2} tr^2(\boldsymbol{A}\varepsilon) - \lambda_{12}^-(D_2, D_3) tr(\varepsilon)$$

$$tr(\boldsymbol{A}\varepsilon) - \frac{\lambda_{11}^-(D_3)}{2} tr^2(\varepsilon) - \frac{(1-2\nu_0)}{2\nu_0} \lambda_{11}^-(D_3) tr[(\boldsymbol{I} \overline{\otimes} \boldsymbol{I})\varepsilon]^2 - \mu_2(D_4, D_5)\, tr(\boldsymbol{A}\varepsilon^2) \tag{5}$$

Observe that the bimodular character is taken into account by the conditions $g\,(\varepsilon, \boldsymbol{D_T}, \boldsymbol{D_C}) > 0$ or $g\,(\varepsilon, \boldsymbol{D_T}, \boldsymbol{D_C}) < 0$, where $g\,(\varepsilon, \boldsymbol{D_T}, \boldsymbol{D_C})$ is a hypersurface that contains the origin and divides the strain space into a compression and tension sub-domains. Note also that for null values of the damage variables, the material behaves as isotropic and unimodular medium, where W_0 is the elastic potential of the undamaged material and $\lambda_{11} = \lambda_0$ and $\mu_1 = \mu_0$ are Lamè constants. The remaining parameters will only exist for no-null damage, evidencing in that way the anisotropy and bimodularity induced by damage. Those parameters are given by:

$$\lambda_{22}^+(D_1, D_4, D_5) = (\lambda_0 + 2\mu_0)(2D_1 - D_1^2) - 2\lambda_{12}^+(D_1) - 2\mu_2(D_4, D_5)$$

$$\lambda_{12}^+(D_1) = \lambda_0 D_1\,; \quad \mu_2(D_4, D_5) = 2\mu_0[1 - (1 - D_4)^2(1 - D_5)^2]$$

$$\lambda_{22}^-(D_2, D_3, D_4, D_5) = (\lambda_0 + 2\mu_0)(2D_2 - D_2^2) - 2\lambda_{12}^-(D_2, D_3) + \frac{(\nu_0 - 1)}{\nu_0}\lambda_{11}^-(D_3) - 2\mu_2(D_4, D_5)$$

$$\lambda_{12}^-(D_2, D_3) = \lambda_0[(1 - D_3)^2 - (1 - D_2)(1 - D_3)]\,; \quad \lambda_{11}^-(D_3) = \lambda_0(2D_3 - D_3^2) \tag{6}$$

The stress tensor is obtained from the gradient of the elastic potential, as follows:

$$\boldsymbol{\sigma}(\varepsilon) = \nabla_\varepsilon \rho\psi(\varepsilon) = \begin{cases} \sigma_-(\varepsilon) = \nabla_\varepsilon \rho\psi_-(\varepsilon) \ \textbf{if} \ g(\varepsilon, \boldsymbol{D_T}, \boldsymbol{D_C}) < 0, \\ \sigma_+(\varepsilon) = \nabla_\varepsilon \rho\psi_+(\varepsilon) \ \textbf{if} \ g(\varepsilon, \boldsymbol{D_T}, \boldsymbol{D_C}) > 0, \end{cases} \tag{7}$$

$$\boldsymbol{\sigma}_+(\varepsilon) = \rho\frac{\partial\boldsymbol{\psi}_+}{\partial\boldsymbol{\varepsilon}} = \lambda_{11} tr(\varepsilon)\boldsymbol{I} + 2\mu_1\boldsymbol{\varepsilon} - \lambda_{22}^+(D_1, D_4, D_5)\, tr(\boldsymbol{A\varepsilon})\boldsymbol{A} - \lambda_{12}^+(D_1)\,(tr(\varepsilon)\boldsymbol{A} + tr(\boldsymbol{A\varepsilon})\boldsymbol{I}) - \mu_2(D_4, D_5)\,(\boldsymbol{A\varepsilon} + \boldsymbol{\varepsilon A}) \tag{8}$$

$$\boldsymbol{\sigma}_-(\varepsilon) = \rho\frac{\partial\boldsymbol{\psi}_-}{\partial\boldsymbol{\varepsilon}} = \lambda_{11} tr(\varepsilon)\boldsymbol{I} + 2\mu_1\boldsymbol{\varepsilon} - \lambda_{22}^-(D_2, D_3, D_4, D_5)\, tr(\boldsymbol{A\varepsilon})\boldsymbol{A} - \lambda_{12}^-(D_2, D_3)\,(tr(\varepsilon)\boldsymbol{A} + tr(\boldsymbol{A\varepsilon})\boldsymbol{I}) - \lambda_{11}^-(D_3)\, tr(\varepsilon)\boldsymbol{I}$$
$$-\frac{(1-2\nu_0)}{\nu_0}\lambda_{11}^-(D_3)\,(\boldsymbol{I}\overline{\otimes}\boldsymbol{I})\boldsymbol{\varepsilon} - \mu_2(D_4, D_5)\,(\boldsymbol{A\varepsilon} + \boldsymbol{\varepsilon A}) \tag{9}$$

The constitutive tensor is also obtained from the elastic potential, i. e.:

$$\boldsymbol{E}(\varepsilon) := \begin{cases} E_-(\varepsilon) = \nabla_\varepsilon^2 \rho\psi_-(\varepsilon) \ if \ g(\varepsilon, D_T, D_C) < 0, \\ E_+(\varepsilon) = \nabla_\varepsilon^2 \rho\psi_+(\varepsilon) \ if \ g(\varepsilon, D_T, D_C) > 0, \end{cases} \tag{10}$$

$$\boldsymbol{E}_+(\varepsilon) = \rho\frac{\partial^2\boldsymbol{\psi}_+}{\partial\boldsymbol{\varepsilon}^2} = \boldsymbol{E_T} = \lambda_{11}[\boldsymbol{I}\otimes\boldsymbol{I}] + 2\mu_1[\boldsymbol{I}\underline{\overline{\otimes}}\boldsymbol{I}] - \lambda_{22}^+(\boldsymbol{D}_1, \boldsymbol{D}_4, \boldsymbol{D}_5)\,[\boldsymbol{A}\otimes\boldsymbol{A}] - \lambda_{12}^+(\boldsymbol{D}_1)\,[\boldsymbol{A}\otimes\boldsymbol{I} + \boldsymbol{I}\otimes\boldsymbol{A}]$$
$$-\mu_2(D_4, D_5)\,[\boldsymbol{A}\underline{\overline{\otimes}}\boldsymbol{I} + \boldsymbol{I}\underline{\overline{\otimes}}\boldsymbol{A}] \tag{11}$$

$$\boldsymbol{E}_-(\varepsilon) = \rho\frac{\partial^2\boldsymbol{\psi}_-}{\partial\boldsymbol{\varepsilon}^2} = \boldsymbol{E_C} = \lambda_{11}[\boldsymbol{I}\otimes\boldsymbol{I}] + 2\mu_1[\boldsymbol{I}\underline{\overline{\otimes}}\boldsymbol{I}] - \lambda_{22}^-(D_2, D_3, D_4, D_5)\,[A\otimes A] - \lambda_{12}^-(D_2, D_3)\,[\boldsymbol{A}\otimes\boldsymbol{I} + \boldsymbol{I}\otimes\boldsymbol{A}]$$
$$-\lambda_{11}^-(D_3)\,[\boldsymbol{I}\otimes\boldsymbol{I}] - \frac{(1-2\nu_0)}{\nu_0}\lambda_{11}^-(D_3)\,[\boldsymbol{I}\overline{\otimes}\boldsymbol{I}] - \mu_2(D_4, D_5)\,[\boldsymbol{A}\underline{\overline{\otimes}}\boldsymbol{I} + \boldsymbol{I}\underline{\overline{\otimes}}\boldsymbol{A}] \tag{12}$$

On a basis of a purely matricial interpretation, the different dyadic products appearing in Eqs.(11) and (12) have the function of allocating the material constants in certain positions of the rigidity tensors.

The variables associated to damage variables in tension (D_1, D_4) and in compression (D_2, D_3, D_5) can be understood like energy release rates during the damage evolution process. They are given by:

$$Y_1(D_1) = -\rho\frac{\partial\psi_+}{\partial D_1};\quad Y_2(D_1) = -\rho\frac{\partial\psi_-}{\partial D_2};\quad Y_3(D_3) = -\rho\frac{\partial\psi_-}{\partial D_3};\quad Y_4(D_4) = -\rho\frac{\partial\psi_+}{\partial D_4};\quad Y_5(D_5) = -\rho\frac{\partial\psi_-}{\partial D_5} \tag{13}$$

2.1. Criterion for partition of strain space

In Curnier *et al.* (1995), it is defined a hypersurface in the stress or strain space to be used for the identification of the bimodularity constitutive response. In Pituba (2003) a particular form is adopted for the hypersurface in the strain space: a hyperplane $g(\boldsymbol{\varepsilon},\boldsymbol{D})$ defined by the unit normal $\boldsymbol{N}$ and characterized by its dependence of both the strain and damage states. To simplify the presentation, the hyperplane will be here expressed as the one obtained by enforcing the direction *1* in the strain space to be perpendicular to the transverse isotropy local plane. Thus, the hyperplane is given by:

$$g(\boldsymbol{\varepsilon}, \boldsymbol{D_T}, \boldsymbol{D_C}) = \boldsymbol{N}(\boldsymbol{D_T}, \boldsymbol{D_C}) \,.\, \boldsymbol{\varepsilon^e} = \gamma_1(D_1, D_2)\ \varepsilon_V^e + \gamma_2(D_1, D_2)\ \varepsilon_{11}^e \tag{14}$$

where $\gamma_1(D_1,D_2) = \{1+H(D_2)[H(D_1)-1]\}\eta(D_1)+\{1+H(D_1)[H(D_2)-1]\}\eta(D_2)$ and $\gamma_2(D_1,D_2) = D_1+D_2$. The Heaveside functions employed above are given by:

$$H(D_i) = 1 \text{ for } D_i > 0; \qquad H(D_i) = 0 \text{ for } D_i = 0 \ (i = 1, 2) \tag{15}$$

The $\eta(D_1)$ and $\eta(D_2)$ functions are defined, respectively, for the tension and compression cases, assuming for the first one that there was no previous damage of compression affecting the present damage variable D_1 and analogously, for the second one that has not had previous damage of tension affecting variable D_2. The functions can be written as:

$$\eta(D_1) = \frac{-D_1 + \sqrt{3 - 2D_1^2}}{3};\qquad \eta(D_2) = \frac{-D_2 + \sqrt{3 - 2D_2^2}}{3} \tag{16}$$

2.2. Criteria and evolution laws of damage

As it has already been pointed out, in the model formulation the damage induces anisotropy in the concrete. Therefore, it is convenient to separate the damage criteria into two: the first one is only used to indicate damage beginning, or that the material is no longer isotropic and the second one is used for loading and unloading when the material is already considered as transverse isotropic. This second criterion identifies if there is or not evolution of the damage variables. That division is justified by the difference between the complementary elastic strain energies of isotropic and transverse isotropic material. For identifying the damage beginning it is suggested a criterion that compares the complementary elastic strain energy W_e^*, which is computed locally considering the medium as initially virgin, isotropic and purely elastic, with a certain reference value Y_{0T}, or Y_{0C}, obtained from experimental tests of uniaxial tension, or compression, respectively. Accordingly, the criterion for initial activation of damage processes in tension or compression is given by:

$$f_{T,C}(\boldsymbol{\sigma}) = W_e^* - Y_{0T,0C} < 0; \qquad f_T\, \dot{\boldsymbol{D}}_{\boldsymbol{T}} = \boldsymbol{0}\,; \qquad f_C\, \dot{\boldsymbol{D}}_{\boldsymbol{C}} = \boldsymbol{0} \tag{17}$$

then $\boldsymbol{D_T} = \boldsymbol{0}$ (i. e., $D_1 = D_4 = 0$) for dominant tension states or $\boldsymbol{D_C} = \boldsymbol{0}$ (i. e., $D_2 = D_3 = D_5 = 0$) for dominant compression states, where the material is linear elastic and isotropic. The reference values Y_{0T} and Y_{0C} are model parameters defined by $\frac{\sigma_{0T}^2}{2E_0}$ and $\frac{\sigma_{0C}^2}{2E_0}$, respectively, where σ_{0T} and σ_{0C}.are the limit elastic stresses determined in the uniaxial tension and compression regimes.

On the other hand, assuming a general situation of damaged medium for dominant tension states, the criterion for the identification of damage increments is represented by the following relationship:

$$f_T(\boldsymbol{\sigma}) = W_{e+}^* - Y_{0T}^* \le 0 \tag{18}$$

where the reference value Y_{0T}^* is defined by the maximum complementary elastic energy computed throughout the damage process up to the current state, i. e.:

$$Y_{0T}^* = MAX\,(Y_{0T}^*\,,\; W_{e+}^*) \tag{19}$$

The loading-unloading conditions are given by:

$$f_T \le 0\,,\; \dot{\boldsymbol{D}}_{\boldsymbol{T}} \ge \boldsymbol{0}\,, f_T\,\dot{\boldsymbol{D}}_{\boldsymbol{T}} = \boldsymbol{0}\,;\; \text{If } f_T = 0,\; \dot{f}_T\; \dot{\boldsymbol{D}}_{\boldsymbol{T}} = \boldsymbol{0} \tag{20}$$

It is important to notice that the damaged medium presents a transverse isotropy plane in correspondence to the current damage level. Then, the complementary elastic energy of the damaged medium is expressed in different forms, depending on whether tension or compression strain states prevail. In the case of dominant tension states ($g(\boldsymbol{\varepsilon}, \boldsymbol{D_T}, \boldsymbol{D_C}) > 0$) assuming that direction 1 in the strain space be perpendicular to the transverse isotropy local plane, it can be written:

$$W_{e+}^* = \frac{\sigma_{11}^2}{2E_0(1-D_1)^2} + \frac{(\sigma_{22}^2 + \sigma_{33}^2)}{2E_0} - \frac{\nu_0(\sigma_{11}\sigma_{22} + \sigma_{11}\sigma_{33})}{E_0(1-D_1)} - \frac{\nu_0\sigma_{22}\sigma_{33}}{E_0} + \frac{(1+\nu_0)}{E_0(1-D_4)^2(1-D_5)^2}(\sigma_{12}^2 + \sigma_{13}^2) + \frac{(1+\nu_0)}{E_0}\sigma_{23}^2 \tag{21}$$

For the damaged medium in dominant compression states, the relationships are similar to the tension case, where the complementary elastic energy is expressed in the following form:

$$W_{e-}^* = \frac{\sigma_{11}^2}{2E_0(1-D_2)^2} + \frac{(\sigma_{22}^2 + \sigma_{33}^2)}{2E_0(1-D_3)^2} - \frac{\nu_0(\sigma_{11}\sigma_{22} + \sigma_{11}\sigma_{33})}{E_0(1-D_2)(1-D_3)} - \frac{\nu_0\sigma_{22}\sigma_{33}}{E_0(1-D_3)^2} + \frac{(1+\nu_0)}{E_0(1-D_4)^2(1-D_5)^2}(\sigma_{12}^2 + \sigma_{13}^2) + \frac{(1+\nu_0)}{E_0}\sigma_{23}^2 \tag{22}$$

In the loading case, i. e., when $\dot{\boldsymbol{D}}_{\boldsymbol{T}} \ne \boldsymbol{0}$ or $\dot{\boldsymbol{D}}_{\boldsymbol{C}} \ne \boldsymbol{0}$, it is necessary to update the values of the scalar damage variables that appear in the $\boldsymbol{D_T}$ and $\boldsymbol{D_C}$ tensors, considering their evolution laws. The evolution laws of the damage variables are written as associated variables functions (Lemaitre, 1996). The associated variables are derived from the complementary energy potentials (21) e (22). For instance, for dominant tension states can be written the following expressions:

$$Y_T = \frac{\partial W^*_{e+}}{\partial D_1} + \frac{\partial W^*_{e+}}{\partial D_4} = Y_1 + Y_4 \tag{23}$$

where

$$Y_1 = \frac{\sigma_{11}^2}{E_0(1-D_1)^3} - \frac{\nu_0(\sigma_{11}\sigma_{22} + \sigma_{11}\sigma_{33})}{E_0(1-D_1)^2}; \quad Y_4 = \frac{(1+\nu_0)}{E_0(1-D_4)^3(1-D_5)^2}(2\sigma_{12}^2 + 2\sigma_{13}^2) \tag{24}$$

The variables Y_1 and Y_4 would be incorporated in the evolution laws of damage variables D_1 and D_4, respectively. In a general way, the relationships that define the associated variables may be represented by:

$$Y_{T,C} = F(\boldsymbol{\sigma}, \boldsymbol{E_0}, \boldsymbol{D_{T,C}}) \tag{25}$$

Taking also into account an implicit representation, the damage evolution laws may be given by:

$$\dot{\boldsymbol{D}}_{\boldsymbol{T,C}} = F^*(Y_{T,C}, b_{T,C}) \tag{26}$$

where $b_{T,C}$ are groups of parameters incorporated in the evolution laws of $\boldsymbol{D_T}$ or $\boldsymbol{D_C}$. Observe that in case of monotonic loading, the Eq. (26) can be integrated directly. However, the set of relationships formed by $Y_{T,C}$ and $\boldsymbol{D_{T,C}}$ leads to an implicit system whose solution can be obtained by an iterative procedure.

Considering just the case of monotonic loading, the evolution laws proposed for the scalar damage variables are resulting of fittings on experimental results and present similar characteristics to those one described in both works: Mazars (1984) and La Borderie (1991). The general form proposed is:

$$D_i = 1 - \frac{1 + A_i}{A_i + exp\left[B_i(Y_i - Y_{0i})\right]} \qquad i = 1, 5 \tag{27}$$

where A_i, B_i and Y_{0i} are parameters that must be identified. The parameters Y_{0i} are understood as initial limits for the damage activation, the same ones used in Eq. (17). The parametric identification of the model is accomplished by uniaxial tension tests in order to obtain A_1, B_1 and $Y_{01} = Y_{0T}$, by uniaxial compression tests for the identification of the parameters A_2, B_2 and Y_{02}, and finally by biaxial compression tests in order to obtain A_3, B_3 and $Y_{03} = Y_{02} = Y_{0C}$. On the other hand, the identification of the parameters for the evolution laws corresponding to the damage variables D_4 and D_5, which influence the shear concrete behavior, it won't be studied in this version of the model presented in this work because the experimental tests are not available yet to allow the parameter calibration or, even, the proposition of more realistic evolution laws. The parametric identification results of the constitutive model obtained from experimental tests varying from one to three-axial stress states in the concrete specimens are presented in Pituba (2003) and Proença and Pituba (2003).

2.3. Transverse isotropy local plane

When the damage process is activated, the formulation starts to involve the tensor $\boldsymbol{A}$ that depends on the normal to the transverse isotropy plane. Therefore, it is necessary to establish some rules to identify its location for an actual strain state. Initially, it is established a general criterion for the existence of the transverse isotropy plane. In Proença & Pituba (2003) is proposed that the transverse isotropy due to damage only arises if positive strain rates exist at least in one of the principal directions. After assuming such proposition as valid, some rules to identify its location must be defined. First of all, considering a strain state in which one of the strain rates is no-null or has sign contrary to the others, the following rule is applied:

"In the principal strain space, if two of the three strain rates are extension, shortening or null, the plane defined by them will be the transverse isotropy local plane of the material."

The uniaxial tension is an example of this case where the transverse isotropy plane is perpendicular to the tension stress direction. However, there are some cases that won't follow this rule. For example, the plane strain state in which the no-null strains have contrary signs. In this case, the first rule is not able to identify the transverse isotropy local plane of the material, so that a second rule must be applied:

"In a plane strain state, where the principal strain rates in the plane have contrary signs, the transverse isotropy local plane of the material is defined by both direction of the principal strain which is permanently null and the direction of the strain whose rate is positive."

Another particular case occurs when all principal strain rates are positive. For those states it is valid a third rule, which assumes that the direction of larger extension is perpendicular to the transverse isotropy local plane of the material. Obviously, it can be suggested criteria based on others formulations, such as for instance, the microplanes theory developed by Bazant and Ozbolt (1990).

3. Two-Dimensional Solid Analysis

The two-dimensional version of the proposed model was implemented in a finite element code for plane analysis. In those analyses, the concrete has a nonlinear behaviour, while a linear elastic behaviour for the reinforcement is admitted. It was assumed perfect bonding between the materials. The constant strain quadrilateral (4 nodes) elements were used. The loading and geometry symmetries were used in the numerical application. This numerical application was initially analyzed by Perego (1989) using a damage model proposed by Mazars (1984). The Mazars' and proposed model responses are illustrated in Fig. 2 in order to comparing. The beam geometry and its reinforcement are illustrated in Fig. 1. The concrete used in the beam has elasticity modulus E_c = 24700 MPa ; the steel has E_a = 210000 MPa. The Table 1 contains the parameter values of models employed in this analysis, for more details see Pituba (2003).

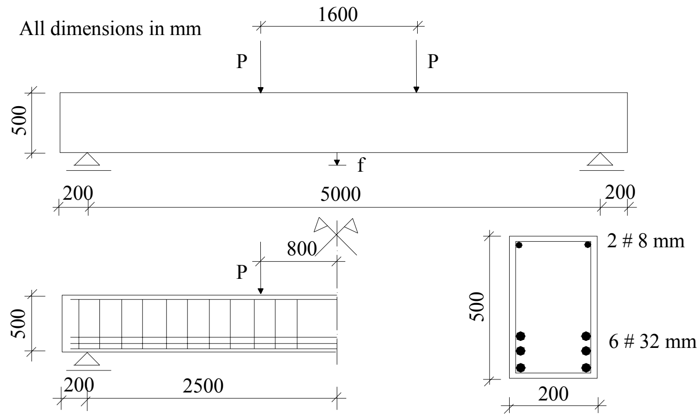

Figure 1. Geometry and reinforcement details

Table 1 – Parameters values of the Mazars' and proposed model

Mazars (1984)		Model		
Tension	Compression	Tension	Compression	
A_T = 0.995	A_C = 1.13	Y_{01}=0.72x10^{-4}MPa	Y_{02}=0.5x10^{-3}MPa	Y_{03}=0.5x10^{-3}MPa
B_T = 8000	B_C = 1643.5	A_1=50	A_2 = -0.9	A_3 = -0.6
ε_{d0}=0.000067		B_1 = 6700 MPa^{-1}	B_2 =0.4MPa^{-1}	B_3 =70000 MPa^{-1}

Some additional remarks concerning the discretization and adopted idealizations are necessary.

The hypothesis about the elastic-linear behaviour for the steel is justifiable in this numerical application because the beam has high reinforcement rate, where the collapse take place mainly due to the cracked concrete.

In the analysis with the proposed model was used a mesh with 342 elements placed in the middle plane of the beam. The beam height was subdivided into 38 layers of elements where only one layer represents all the reinforcement bars with equivalent area disposed in the geometric barycenter of the steel bars.

The Figure 2 shows the load-displacement curves. The proposed model response is in correspondence with the experimental results presented in Perego (1989). In fact, the proposed model is able to capture with accuracy the nonlinearity of the experimental curve up to the load of 220 kN. Soon after, instability arises in the numerical response due to the high damage levels. In the Perego with the Mazars' model, the numerical instability occurs about the load of 120 kN.

Note that the proposed model reduces the material rigidity in agreement with the considered direction, i. e., the damage process occurs in a selectable way. That does not occur along the Mazars' model analysis, because there is the excessive rigidity reducing in all the directions around a point considered in the material. This process presents numerical problems related to ill-conditioning of the equation system in an intermediate load level.

The instability process deserves a plausible explanation. In fact, the numerical model tries to reproduce a situation of strongly localized damage, which it would be equivalent to the formation of an intensive cracking zone. Physically, that zone occurs indeed among the applied forces.

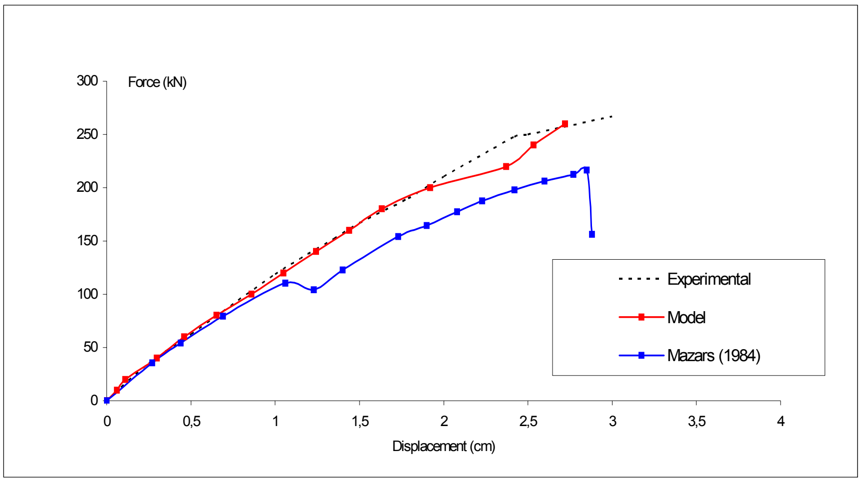

Figure 2 – Beam 6#32.0 mm: numerical and experimental results

The Figure 3 shows the value distributions of the damage variables in some loading stages.

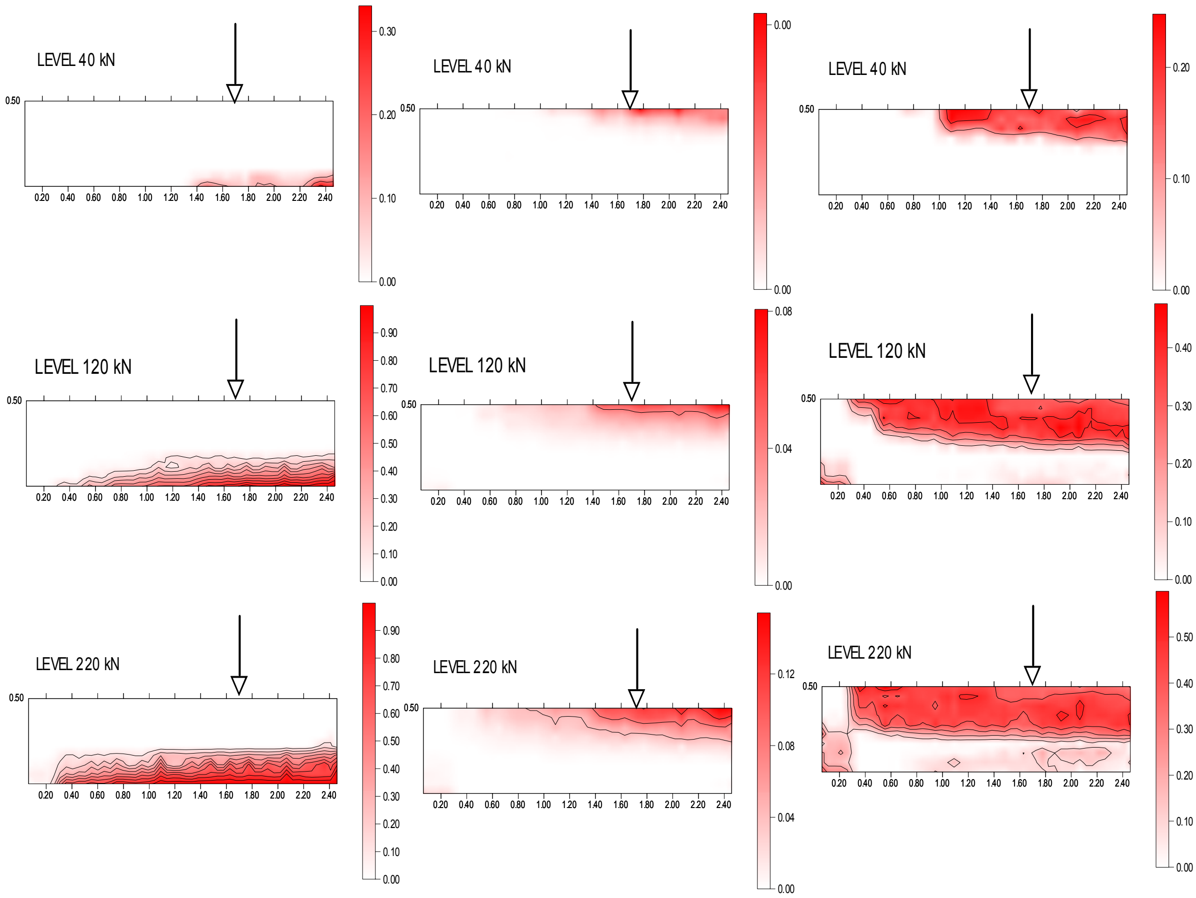

Figure 3. D_1, D_2 and D_3 damage distribution.

It must be noted that to around 15% of the ultimate load, it is observed a considerable damage process in tension in the bottom zone of the beam. For successive loading levels, the damage zone in tension increases of bottom upward and towards to the support zone. In the top zone of the beam, damage variable in compression D_2 starts to present values around 50% of the ultimate load. Near to load of 220 kN, it is observed reasonable values of the damage variable D_3 including a indicative of a certain process of damage localization in the beam support. It is also noted a large area in the bottom of the beam with values near to unit for the damage variable D_1. This evolution of damage process is in correspondence with the form described in Perego (1989).

Finally, the proposed model applied in plane analyses of reinforced concrete structures has been shown to be efficient and accurate for the usual cases. However, some numerical problems would arise in the beam analyses with low reinforcement rates, where the cracking zone is more localized (Pituba, 2003). Therefore, it can be pointed out that the damage localization is a possible restrictive situation of the proposed model employment in the plane analyses.

4. Conclusions

The study concerns to the employment of an anisotropic damage model to two-dimensional solid analysis.

In the first part of this work, a damage constitutive model accounting for induced anisotropy and bimodular elastic response was presented. A criterion was proposed in order to characterize the division of strain space. Energy criteria were introduced for identification of the damage beginning and its evolution processes. Besides, rules for the determination of the transverse isotropy local plane of the material were proposed. Note that other criterion can be proposed based on microplane theory. This idea can be used in the improvement of the constitutive model in future works. Moreover, constitutive model with orthotropy and bimodularity induced by damage can be derived of the extended formulation proposed in Pituba (2003) in order to evaluate the potentialities in the analysis of reinforced concrete structures. However, it must be observed that the parametric identification could become infeasible at the practical point of view.

It was shown in the numerical application that the proposed model has shown quite efficient when dealing with high reinforcement rate beams. However, it must be observed that beams with low reinforcement rates can evidence some numerical problems due to plane analysis. In theses cases the cracking process starts to present a localized distribution limiting the model employment. In order to overcome numerical problems a non-local version of the model can be proposed and implemented in a computational code, for instance, with so-called Generalized Finite Element Method.

On the other hand, it is believed that it can be important the identification of evolution laws for damage variables D_4 and D_5 which influence the shear concrete behaviour. This asset is available by the proposed model, but it was not explored in this work.

Finally, the proposed model has shown some advantages of its employment related to the isotropic models, such as the selective stiffness deterioration. This should be more evident in three-dimensional analyses, what encourage us to proceed with this formulation and its improvements suggested above to deal with more complex structures in future works.

5. Acknowledgements

The authors wish to thank to FAPESP (Fundação de Amparo à Pesquisa do Estado de São Paulo) for the financial support.

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