BIOMECHANICAL MODEL FOR NET JOINT TORQUES OF PEDALING

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Abstract. *In present work, a biomechanical model to estimate net joint torques during a cycle of pedaling is proposed. The model was formulated as a bidimentional, eight-bar and three degrees of freedom linkage with two closed loops. Each bar represents crank, stationary bar, thigh, shank and foot for right and left legs. Initialy, a kinematic analysis was performed to calculate position, velocity and acceleration for dependant and generalized angular coordinates. Then, expressions of position, velocity and acceleration for centers of mass, of each limb's segment was developed as functions of angular coordinates. The dynamic analysis was performed using a Newton-Euler approach where a free body diagram was constructed to illustrate net joint torques, as well as internal and external forces. The equations corresponding the summation of forces and moments of each segment was arranged into a linear system which was solved repeatedly for each position of the linkage (e.g crank angle varying from 0º to 360º in one, to one degree). Model dimensional and inertial parameters, as well as measured pedal forces were taken from published data of other authors. The net joint torque curves for hip, knee and ankle shows a good correspondence between other author's results. This work is a first step in a more elaborated biomechanical model of pedaling, where muscle contraction dynamics is considered and the net joint torques takes part in the objective function.*

Keywords*: biomechanics, kinematic analysis, inverse dynamics, pedaling*

1. Introduction

Understanding and modeling bicycle biomechanics is important for several reasons. The availability of such models could lead to: (i) prevention of overuse injuries in knee incurred from pedaling activity, (ii) a better use of stationary ergometer as a form of physical therapy, (iii) an improvement of performance in competitions (Hull and Jorge, 1985). Specialy, net joint torques are frequently computed to provide insights into the motor control of biomechanical systems and could ultimately be used to develop more effective rehabilitation strategies for pacients with motor deficit (Runge and Zajac, 1995). Recently, Menegaldo et al. (2005) has used net joint torques in a cost function to estimate the muscle forces in musculoskeletal systems based on the inverse dynamics of a multi-body system associated optimal control problem.

The objective of the work reported in this article was to develop an analytical/numerical procedure ~~to~~ for determining the kinematics of angular variables over a complete crank cycle, and then use this procedure to compute net joint torques based on an inverse dynamics technique. The model was tested with measured data available at literature. The model presented here, after validated, will be used as input in a work that is still in progress, whose objective is to estimate muscular forces via optimal control with a moment based objective function.

2. Methods

The kinematic model was developed following the theory presented by Dougthy (1988). It was formulated as a bidimentional, eigth-bar and three degrees of freedom (D.O.F.) linkage with 2 closed loops. Each bar represents crank, stationary bar, thigh, shank and foot for right and left legs. The system has three independent generalized coordinates: cranck angle (q_1), right ankle angle (q_2) and left ankle angle (q_3). Is was assumed that q_1 varies linearly with respect to time, with a constant velocity of 60 rpm, while q_2 varies accoding to data presented by Redfield and Hull (1986a). Figure 1 shows the length of each bar (L_1, L_2,…., L_7) and the vertical and horizontal distance from seat to crank axis (D_1, D_2, repectively).

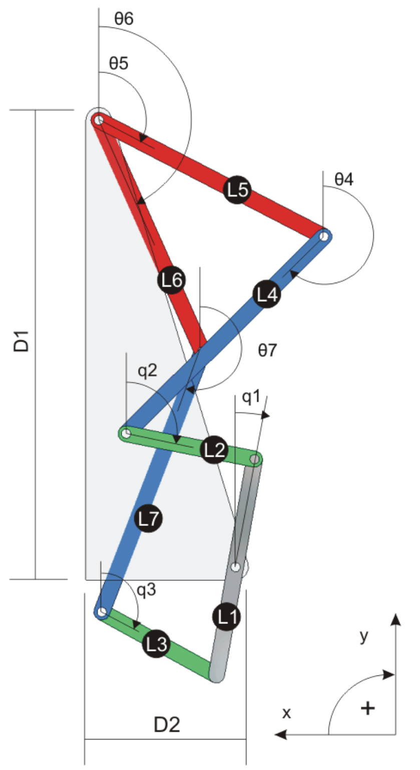

Figure 1: Generalized and dependent coordinates and dimensions

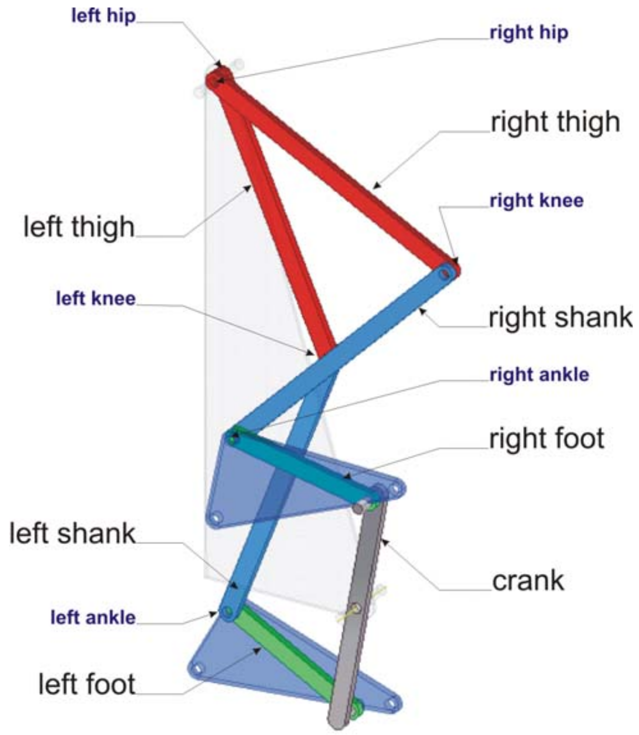

Figure 2: Nomenclature

2.1. Position Analisys

According to Figure 1, it is possible to write four equations, namely, position loop functions, representing horizontal and vertical projections for right and left legs.

$$f_1 = +(L_1/2)\operatorname{sen}(q_1) - L_2\operatorname{sen}(q_2) - L_4\operatorname{sen}(\theta_4) - L_5\operatorname{sen}(\theta_5) + D_2 = 0 \tag{1}$$

$$f_2 = +(L_1/2)\cos(q_1) - L_2\cos(q_2) - L_4\cos(\theta_4) - L_5\cos(\theta_5) - D_1 = 0 \tag{2}$$

$$f_3 = -(L_1/2)\operatorname{sen}(q_1) - L_3\operatorname{sen}(q_3) - L_6\operatorname{sen}(\theta_6) - L_7\operatorname{sen}(\theta_7) + D_2 = 0 \tag{3}$$

$$f_4 = -(L_1/2)\cos(q_1) - L_3\cos(q_3) - L_6\cos(\theta_6) - L_7\cos(\theta_7) - D_1 = 0 \tag{4}$$

For this system, with three generalized coordinates, it is necessary to provide three angular informations for each one of them. At first, we can impose only q_1 angle; q_2 and q_3 should be experimentaly measured. Redfield and Hull (1986a) measured the ankle angle as a function of crank angle, which can be fitted by a curve like:

$$\beta(\theta) = A_1\operatorname{sen}(\theta) + A_2\cos(\theta) + A_3 \tag{5}$$

where θ is the crank angle and A_1, A_2, A_3, are constants to be determinated. Using the graph presented at Figure 2 of Redfield and Hull's (1986a) paper, and taking into account the differences between both systems of coordinates; Eq.(7) was derived to calculate right ankle angle (q_2) as a function of q_1. The curve of q_3 angle (Eq. 8) has the same shape of q_2 curve shifted by 180º.

$$q_1(t) = 2\pi \cdot t \qquad \text{[rad/s]} \tag{6}$$

$$q_2(q1(t)) = -(7\pi/45)\operatorname{sen}(q_1(t)) - (\pi/45)\cos(q_1(t)) + (11\pi/18) \qquad \text{[rad/s]} \tag{7}$$

$$q_3(q_1(t)) = -(7\pi/45)\operatorname{sen}(q_1(t)+\pi) - (\pi/45)\cos(q_1(t)+\pi) + (11\pi/18) \qquad \text{[rad/s]} \tag{8}$$

Making these assumptions, only one degree of freedom needs to be defined, since Eq. (7) and Eq. (8) are compatibility equations that can be used to reduce de number of D.O.F. However, these equations relates q_2 and q_3 with q_1, only for the particular case mesured by Redfield and Hull (1986a). Equations (7) and (8) may vary with cadence, load, pedaling technique etc. Treating the system as 3-D.O.F. problem, it should be possible to use data from other authors, in order to compare results, as well as use our own measeured data in future works.

To complete position analisys, it is necessary to solve, for θ4 to θ7 variables, the system of equations by equaling Eqs. (1) to (4), to zero. By doing so, a non-linear system of equations arises, where a Newton-Raphson's algorithm was used to solve it for each position of the mechanism.

2.2. Velocity Analysis

Velocities for the generalized coordinates are obtained by taking the derivative of Eqs. (6) to (8) with respect to time. Velocity of dependent coordinates are calculated solving Eq. (9) which can be expanded into Eq. (10).

$$[J]\cdot\{\dot{\theta}\} = -[B]\cdot\{\dot{q}\} \tag{9}$$

$$\begin{bmatrix} \dfrac{\partial f1}{\partial\theta 4} & \cdots & \dfrac{\partial f1}{\partial\theta 7} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f4}{\partial\theta 4} & \cdots & \dfrac{\partial f4}{\partial\theta 7} \end{bmatrix} \cdot \begin{Bmatrix} \dot{\theta}4 \\ \dot{\theta}5 \\ \dot{\theta}6 \\ \dot{\theta}7 \end{Bmatrix} = -\begin{bmatrix} \dfrac{\partial f1}{\partial q1} & \cdots & \dfrac{\partial f1}{\partial q3} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f4}{\partial q1} & \cdots & \dfrac{\partial f4}{\partial q3} \end{bmatrix} \cdot \begin{Bmatrix} \dot{q}1 \\ \dot{q}2 \\ \dot{q}3 \end{Bmatrix} \tag{10}$$

Matrices [J] and [B] are formed differentiating the position loop functions (Eqs. (1) to (4)), with respect to dependent and generalized coordinates, respectively. The solution of Eq. (9) can also be expressed in terms of [K] matrix (Eq. (11)), where [K] is defined as Eq. (12), which is designated *Velocity Coefficient Matrix*.

$$\{\dot{\theta}\} = [K]\cdot\{\dot{q}\} \qquad \text{or} \qquad \begin{Bmatrix} \dot{\theta}4 \\ \dot{\theta}5 \\ \dot{\theta}6 \\ \dot{\theta}7 \end{Bmatrix} = \begin{bmatrix} K_{\theta 4q1} & \cdots & K_{\theta 4q3} \\ \vdots & \ddots & \vdots \\ K_{\theta 7q1} & \cdots & K_{\theta 7q3} \end{bmatrix} \cdot \begin{Bmatrix} \dot{q}1 \\ \dot{q}2 \\ \dot{q}3 \end{Bmatrix} \tag{11}$$

$$[K] = -[J]^{-1}[B] \tag{12}$$

Velocity coefficient matrix plays an important role in velocity analysis. It means that dependent velocities can be expressed as a linear combinations of the generalized velocity coordinates, e.g.:

$$\dot{\theta}4 = K_{\theta 4q1}\cdot\dot{q}1 + K_{\theta 4q2}\cdot\dot{q}2 + K_{\theta 4q3}\cdot\dot{q}3$$

It is important to stress that the values of [K] are recalculated of each position of the mechanism. The elements of [K] are numerical approximations for dθ/dq, at each instant of time and angular position. Consequently, dθ/dt = (dθ/dq)(dq/dt).

2.3. Acceleration analysis

The accelerations of generalized coordinates are obtained by differentiating twice Eqs. (6) to (8). Through differentiating Eq. (11), one can show that the unknown accelerations of dependent coordinates is found by Eq.(13). In Eq. (13), matrices $[L_1]$, $[L_2]$ and $[L_3]$ represent the derivatives of the velocity coefficient matrix [K], with respect to q_1, q_2 and q_3, respectively (Eq. (14)).

$$\{\ddot{\theta}\} = \dot{q}1[L1]\{\dot{q}\} + \dot{q}2[L2]\{\dot{q}\} + \dot{q}3[L3]\{\dot{q}\} + [K]\{\ddot{q}\} \tag{13}$$

$$[L_i] = \frac{\partial[K]}{\partial q_i}\text{, for i = 1, 2, 3 (number of generalized coordinates)} \tag{14}$$

Equation (14) can be evaluated numericaly implementing the formula of Eq. (15), which can be derived premultiplying both sides of Eq. (12) by [J] and applying the chain rule.

$$[L_i] = [J]^{-1} \cdot \left(\frac{\partial [B]}{\partial q_i} - \frac{\partial [J]}{\partial q_i} \cdot [K] \right), \text{ for i = 1, 2, 3} \tag{15}$$

2.3. Inverse Dynamics

Inverse dynamics consists of finding forces (or moments) required to reproduce a pre-determined kinematics. In this work it is accomplished by solving the linear system resulting from Newton-Euler's equations of each limb for each position of the linkage. After the kinematical analysis is performed, the variables that appear on the right side of Eqs. (16), (17) and (18) can be calculated. Some of them are input parameters taken from literature (mass and moment of inertia), while others were calculated in the kinematical analysis (generalized and dependant coordinates accelerations) or can be derived from free body diagram and kinematic analysis (linear accelerations of centers of mass) Table 1 shows a summary of known variables of right hand side of Eqs. (16), (17) and (18).

$$\sum Fx = m_i \ddot{X}_i, \quad \text{for i = 1…7} \tag{16}$$

$$\sum Fy = m_i \ddot{Y}_i, \quad \text{for i = 1…7} \tag{17}$$

$$\sum M_{CG} = I_i \ddot{q}_i, \quad \text{for i = 1…3;} \qquad \text{and} \qquad \sum M_{CG} = I_i \ddot{\theta}_i, \quad \text{for i = 4…7} \tag{18}$$

The left side of Newton-Euler's equations expresses the sum of external forces and moments acting on each limb. Some of them are unknown (net joint torques and internal reaction forces) while others are known (pedal reaction forces).

Table 2 shows a summary of variables of left hand side of Eqs. (16), (17) and (18). It also shows two types of pedal reaction forces. A coordinate transformation was necessary because Redfield and Hull (1986a) measurement apparatus recorded pedal reaction forces in a normal/tangencial basis while this formulation is in a horizontal/vertical basis. So, we had to write compatibility Equations (19)…(22).

$$R_{12x} = Fn_2 \,\text{sen}(\pi/2 - q_2) + Ft_2 \cos(\pi/2 - q_2) \tag{19}$$

$$R_{12y} = -Fn_2 \cos(\pi/2 - q_2) + Ft_2 \,\text{sen}(\pi/2 - q_2) \tag{20}$$

$$R_{13x} = Fn_3 \,\text{sen}(\pi/2 - q_3) + Ft_3 \cos(\pi/2 - q_3) \tag{21}$$

$$R_{13y} = -Fn_3 \cos(\pi/2 - q_3) + Ft_3 \,\text{sen}(\pi/2 - q_3) \tag{22}$$

After Newton-Euler equations are written and properly assembled in matrices and vectors, the steps required to solve inverse dynamics problem are:

1. Run the kinematical analysis to find position, velocities and accelerations of generalized and dependent coordinates.
2. Calculate linear acceleration of centers of mass from kinematic analysis.
3. Evaluate the compatibility equations to find horizontal and vertical pedal reaction forces
4. Solve a 14x14 linear system with the summation of forces in horizontal and vertical directions, namely: R_{1x}, R_{24x}, R_{37x}, R_{45x}, R_{5x}, R_{6x}, R_{67x}, R_{1y}, R_{24y}, R_{37y}, R_{45y}, R_{5y}, R_{6y}, R_{67y}.
5. Solve a 7x7 linear system with the summation of moments around center of mass of each limb, using the solution of previous linear system as input.
6. Increase generalized coordinates by a Δq_i (e.g.: 1º) and go to step 1.

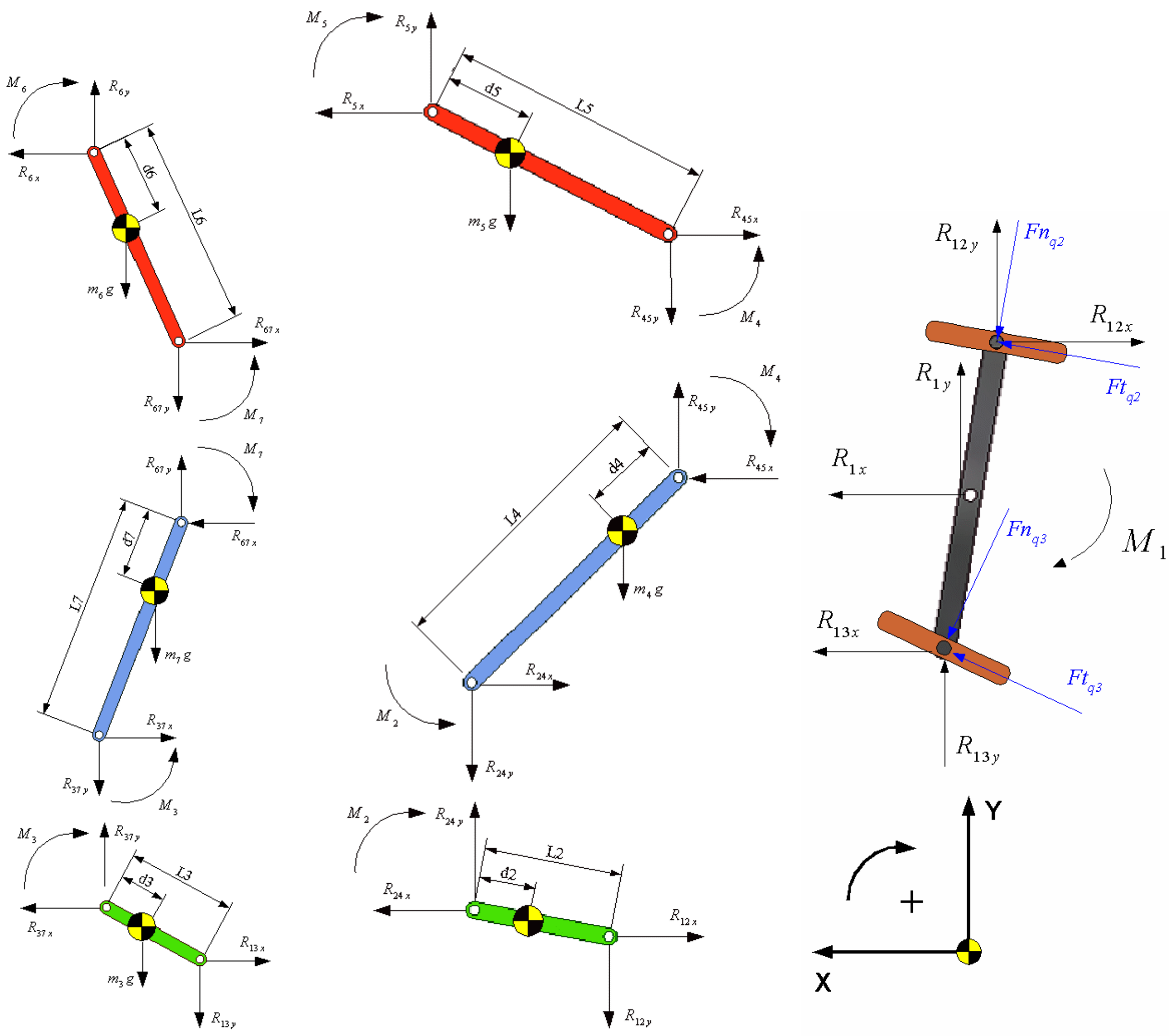

Figure 3: Free-body diagram. (a) left leg, (b) right leg, (c) pedal, crank, and system of coordinates.

Table 1: Summary of variables of right hand side of Newton-Euler equations (known variables)

Symbol	Variable	Source
m_i , i=1...7	Mass of the limb I	(Gonzalez and Hull, 1989)
I_i , i=1...7	Centroidal moment of inercia of limb I	(Gonzalez and Hull, 1989)
$\ddot{q}_i$, i=1...3	Accelerations of generalized coordinates / center of mass angular acelerations of the crank and feet.	Can be obtained differentiating twice Eqs.(6), (7), (8).
$\ddot{\theta}_i$, i=4...7	Accelerations of dependent coordinates / center of mass angular acelerations of the both shank and thigh.	Eq. (13)
$\ddot{X}_i$ and $\ddot{Y}_i$, i=1...7	Horizontal and vertical linear acceleration of each limb's center of mass.	Can be obtained differentiating twice positon loop equations of centers of mass (not shown)
d_i , i=1...7	Distance of each limb's centers of mass to its proximal joint.	(Gonzalez and Hull, 1989)

Table 2: Summary of variables of left hand side of Newton-Euler equations (unknowns)

Symbol	Variable	Source
Fn_2, Ft_2, Fn_3, Ft_3	Pedal normal and tangencial reaction force	Redfield and Hull (1986a)
$R_{12x}, R_{12y}, R_{13x}, R_{13y}$	Pedal horizontal and vertical reaction force	Eqs.(19)…(22)
R_{ijx}, R_{ijy}, $i \geq 2,\ j \geq 4,\ j > i$	Internal reaction force of limb i with limb j.	Figure 3
M_i, $i=1\ldots7$	Net joint torque at joint I	Figure 3

3. Results

Kinematics results are shown at Figure 4 and Figure 5. It was not possible to find a complete set of curves describing kinematics of pedaling for a similar biomechanical model, except for Hull and Jorge, (1985); who shows acceleration curves for hip, knee and ankle. Accelerations curves of Figures 4 and 5 clearly exhibit two peaks for each curve. A detailed comparison of those curves with Figure 5a of Hull and Jorge's (1985) article shows a very good correspondence of peak's phasing and a regular amplitude correspondence. The mean peak position error (MPPE) was ±12%, while the mean peak amplitude error (MPAE) was about 43%. MPAE was defined taking the peak amplitude difference between Hull and Jorge's (1985) and peak amplitude calculated in this work, divided by Hull and Jorge's (1985) peak amplitude; averaged for hip, knee and ankle curves. MPPE was defined in a similar way to MPAE, substituting amplitude by the angle where the main peak occurs.

Recalling that 0º crank angle is at top dead center, Figure 4 clearly shows that the maximum velocity of hip angle occurs at 100º, when crank is almost horizontal and between 200º and 320º this velocity is nearly constant. Knee velocity curve has a sinusoidal shape with a maximum at 180º.

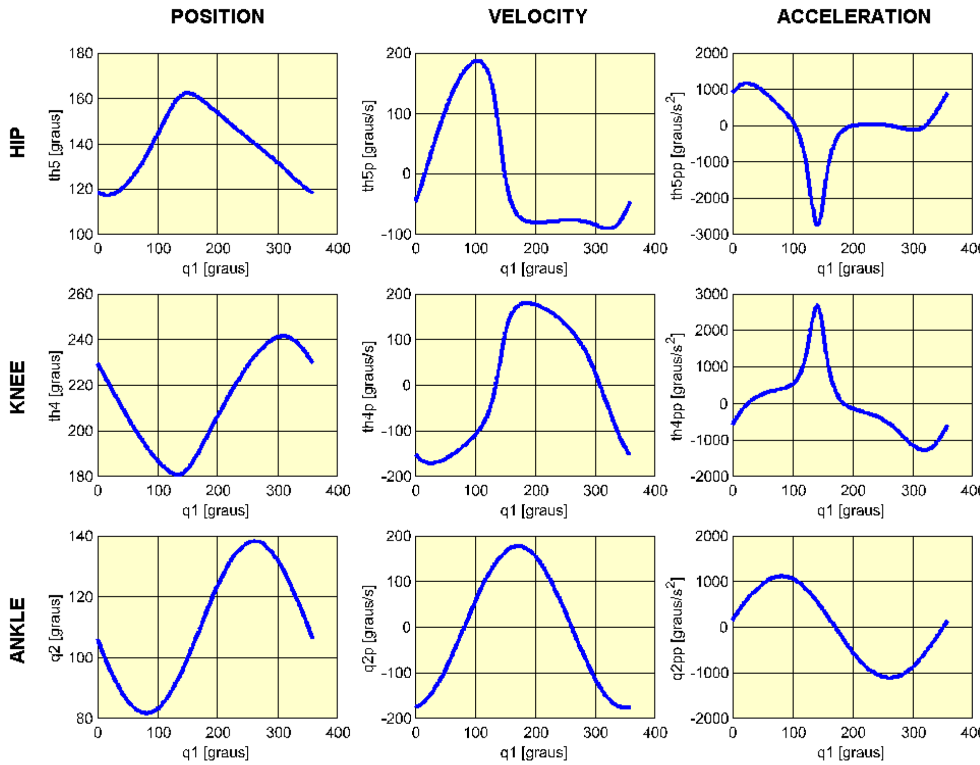

Figure 4: Right leg kinematics results

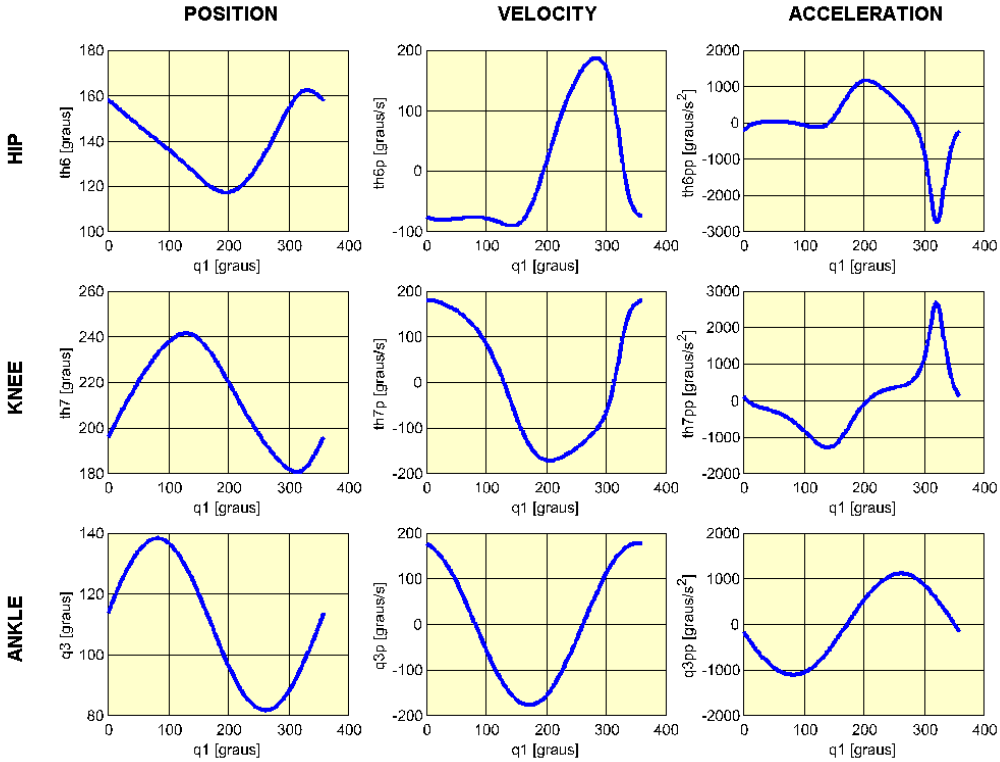

Figure 5: Left leg kinematics results

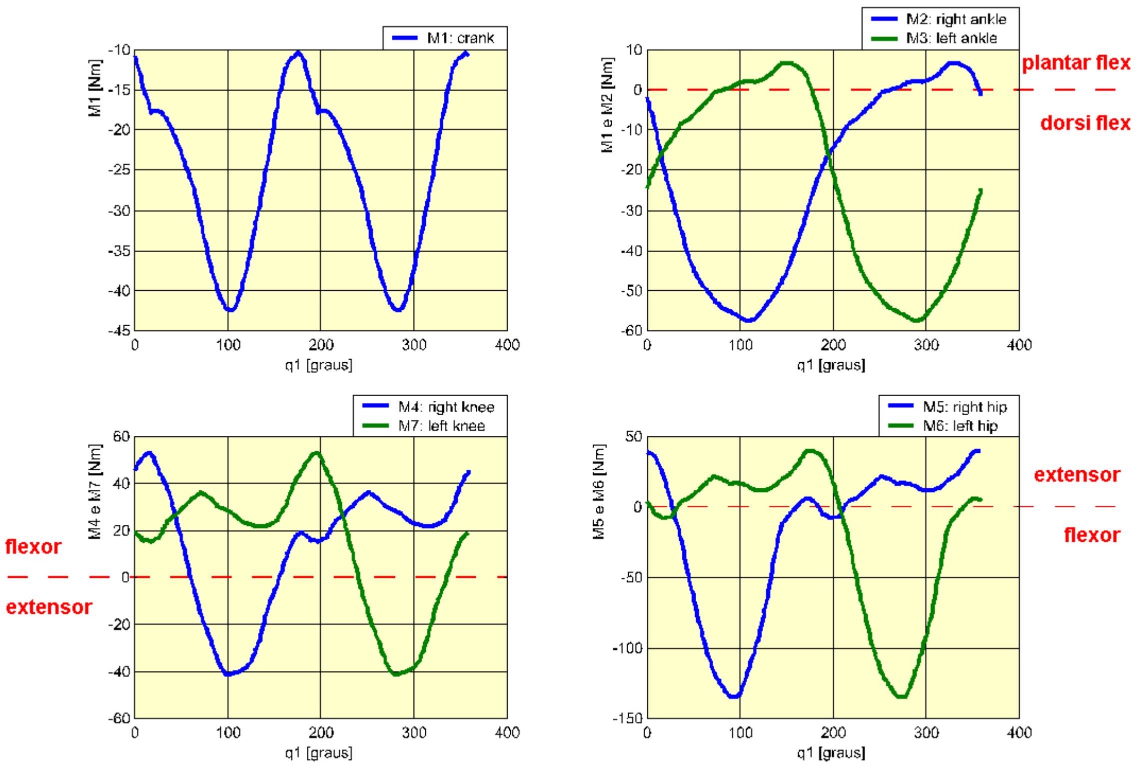

Figure 6: Net joint torques

For total net joint torques, more sources of data were available in the literature. Several authors (Hull and Jorge, 1985; Redfield and Hull, 1986a; Redfield and Hull, 1986b; Kautz and Hull, 1995; Neptune and Hull, 1998) show net joint torque curves calculated by different ways. A detailed comparison between Figure 6, and net joint torques obtained by the cited authors, cannot be easily carried-out, since each one adopted a different system of coordinates. Even

though, the positioning of peaks in hip, knee and ankle differ around ±30% of the a mean positioning between the five cited references; while the modulus of peak amplitude presented a deviation from a mean, of 56% for the ankle, 30% for the knee and 160% for the hip.
Surprisingly, right hip moment was negative (flexor moment) between 0º and 90º. This was not expected because during this phase of pedaling, the thigh goes downwards, which caracterizes an extension movement. This is possibly due to a misinterpretation of system of coordinates adopted by Redfield and Hull (1986a) to measure pedal reaction forces. A comparison between pedal reaction forces between Redfield and Hull (1986a) and Gregor, (1985) (who explicitly identifies the coordinate system adopted) disclose this problem. Another unexpected result, but least important, was the discontinuity found at crank torque curve (Figure 6), near 20º and 200º. This can be interpreted as reading errors due to transforming measured pedal forces graphics of Redfield and Hull's (1986a) paper into numerical data.

2.3. Concluding remarks

This paper has shown the derivation of equations for constructing a biomechanical model for calculation of net joint torques. The presented results has some drawbacks: (i) we were obliged to collect input data from two different authors; (ii) raw measured data was not available, so it was necessary to make visual estimations to transform published graphics to numerical data; (iii) ambiguity of data presented by some references. Besides, it is usual in biomechanical analysis to find large differences in calculated and/or measured physiological quantities, specialy among different subjects. However, attained results are reasonably similar to the ones found in literature. Once this model is validated, it can be used for processing experimental data obtained by means of ergometer apparatus and the resulting net joint torque curves could be used for a before/after comparison of injured subjects submited to rehabilitation programs or atletes in training protocol.

3. Acknowledgements

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5. Responsibility notice