DIAGNOSE OF FAULTS IN MECHANICAL SYSTEMS WITH UNKNOWN INPUTS USING PROPORTIONAL AND INTEGRAL STATE OBSERVERS

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Abstract. *In this work a methodology of fault analysis in mechanical systems using robust state observers is presented. The input is either unknown or only partially known. The observers are used to reconstruct the states not measured directly. This type of observer has great use in systems where the forces or disturbances cannot be measured or are very difficult to measure. A study of existing observers with unknown input is presented. Proportional and Integral observers are used and in this work, the gain of the observer is determined by the Kalman Filter gain and this is a opitmum gain determined by algebraic Riccati equation. The methodology developed is applied to a truss structure and a flexible robot arm through numerical simulations.*

Keywords**:** *State Observers, Kalman Filter, Analysis of faults*

1. Introduction

With the increase in production process, are more and more demands of the industries for machines and equipment capable of executing a greater number of functions in less time and in many cases to be capable to act continuously. This leads to the fact of that these systems are submitted the high dynamic forces. Normally these mechanisms are expensive and therefore one of the major concerns of industry is to keep its equipment functioning without necessary breakdowns.With this constant concern, in the last times, we verify much development of new techniques of detection and localization of faults in mechanical systems submitted dynamics loads. In order to guarantee continuo operation of the mechanical systems, they must be supervised and monitored so that the faults are diagnosed and repaired as fast as possible, if not so the disturbance in normal operation can to take the one's a deterioration of the performance of the system or the dangerous situations. Robust observers can reconstruct the states not measured or estimate the motion of the system that can't be measured directly. Thus, faults can to be detected without knowledge of the motion at many points in the system by being able to monitor them through the reconstruction of the states. The existing methodologies using state observers are usually used in control problems and detection of possible faults in sensors and in instruments. In this work the state observers are used to faults detection in mechanical systems, using a Proportional and Integral (PI) observer to estimate the unknown inputs. The Kalman filter observer is used to localization and quantification of the faults. In previous works the identification of the faults using only state observers was possible with the previous knowledge of the inputs, in this work the unknown input will be found using PI observers.

It is physical and economically unviable, in some control systems, for transducers to be placed to measure all the variables of state. When analyzing the methodology of state observers, his found that some possess the capacity to reconstruct the inaccessible states, however, the necessary condition for this reconstruction is that the states are observed, Luenberger (1964), Luenberger, (1971) and Marano (2002). In the observers described by Luenberger the gain is determined through algorithms of allocation of eigenvalues and eigenvectors of the observer matrix with a certain criterion (for example, for the minimization of the sensitivity of the eigenvalues allocation with parameter variation) (Melo, 2004). A careful analysis must be made so that the speed of estimation, determined for the eigenvalues, is not very great so that sensitivity to the noise in the sensor also is not great. This type of observer corresponds to a deterministic observer. The problem of the noise in the sensor of course leads a stochastic observer who not only handles better the noise in the sensor (Muscolino, Cacciola, and Impollonia, 2003), but also is characterized by having a gain that is optimized as a certain criterion as it will be seen ahead. The optimized observer, or stochastic observer is known as Kalman-Bucy (KF) filter (Valer, 1999). The filter of Kalman has demonstrated to be useful in many applications (such as in navigation, identification of parameters and filter of signals); however, the interest here is its application with ends to faults detection.

2. Method of the State Observers

The state observers where all the inputs of the system must be known and available have great utility in the case where only one input to the control system is known. In the cases where the system is submitted the unknown inputs or disturbance which can not be measured or the measurement is very difficult or simply impossible, the performance of observer can be very poor. In this work was developed a faults diagnose methodology using observers of state in which its input is considered unknown or partially unknown in which the Proportional and Integral observer is used to estimate the unknown inputs, in which, the gain of this observer is determined for the gain given by the Kalman Filter. After the identification of the unknown inputs these are used for the detection of possible faults that are occurring in the systems. For this, the Kalman Filter was used to generate unknown states.

A very convenient representation for systems with these characteristics is as indicated for the following equation:

$$S:\begin{cases}\dot{x}(t) = A\,x(t) + B\,u(t) + B_d\ v_d(t)\\ y(t) = C\,x(t)\end{cases} \tag{1}$$

In which:

$x(t)$ is a state vector *n x 1*, $u(t)$ is a input vector *r x 1*, $y(t)$ is a output vector *m x 1*, $v_d(t)$ is a disturbance vector (unknown input) *p x 1*, A is the system matrix *n x n* (dynamic matrix), B is the distribution matrix *n x r* (input matrix), C is the measures matrix *m x n* and B_d is the matrix distribution of disturbance *p x n*, where *n* the order of the system, *r* the number of inputs *u(t)*, *m* the number of outputs *y(t)* and *p* the number of disturbance $v_d\,(t)$.

The estimate problem to the state of a linear and invariant system in the time with known and unknown inputs has been subjects of researching in the last decades and with considerable importance because in real system, there are many situations where the disturbance are present or some inputs are inaccessible, then a conventional observer which all the inputs are known can not be used. This way, an observer capable of estimate the state for linear system with partially unknown inputs, not sensible to disturbance, can be of great utility.

The idea is projecting an observer who is capable of estimate the disturbance v_d. The Fig. 1 suggests the function of this observer.

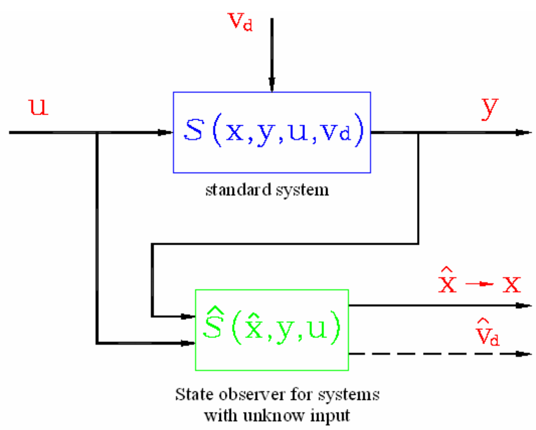

Figure 1. Observer with unknown inputs

2.1. Modeling the Observer with Unknown Inputs

In according this approach, we verify that dynamics of the disturbance vector satisfies the differential equation following:

$$v_d(t) = c_d\,w(t) \tag{2}$$

$$\dot{w}(t) = A_d\,w(t) \tag{3}$$

Which *w* represents the disturbance state contained in the matrix A_d and the matrix C_d indicates like the disturbance is dependent of this state. The choice of these matrices depends on the kind of the disturbance. Thus, for example, in the case where the disturbance v_d *is* constant, a convenient choice for this is that matrix A_d = 0 and C_d = *I* (I is a identity matrix). Arranging the Eq. (1) with (2) and (3) we get a increased model of state:

$$S_a : \begin{cases} \dot{x}_a(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{w}(t) \end{bmatrix} = \begin{bmatrix} A & B_d C_d \\ 0 & A_d \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ v_d(t) \end{bmatrix} \end{cases} \tag{4}$$

It is verified in the equation above that w is not controllable through of u. But, in general, it is observable (Valer, 1999) and with this, is possible to project an observer for this system that estimate as variable x as w. Thus, an observer of full order for this new system will be:

$$\begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{w}}(t) \end{bmatrix} = \begin{bmatrix} A & B_d C_d \\ 0 & A_d \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{w}(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \left(y(t) - C\,\hat{x}(t) \right) \tag{5}$$

The matrix $K = \begin{bmatrix} K_1^T & K_2^T \end{bmatrix}^T$ guarantees stability to the observer, as a result:

$$\hat{S}_{ed/ds} : \begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + B_d C_d \hat{w}(t) + K_1 (y(t) - C\hat{x}(t)) \\ \dot{\hat{w}}(t) = A_d \hat{w}(t) + K_2 (y(t) - C\hat{x}(t)) \end{cases} \tag{6}$$

2.2. Proportional-Integral Observer

When the spectrum of disturbance does not contain high frequencies, the observer of the section (2.1) can be used considering $A_d = 0$ and $C_d = I$ getting a simplification in the model. In this case the corresponding part to the estimation the disturbance vector becomes in a bank of integrators and the corresponding part to the estimation the state vector becomes in proportional and integral to the residual: $y(t) - C\hat{x}(t)$. This observer is called proportional-integral or PI and has superior properties comparing with the proportional full order observer. The proportional-integral observer is capable estimate any disturbance (constant, linear and nonlinear) but it has to be slower than the constant of time of integral action and the number of measurements can't be minor that the number of disturbance. Increasing the integral gain is possible to reject the faster disturbance. However, this has a negative effect decreasing the stability of the observer. Using the Eq. (6), we have for the case of proportional-integral observer:

$$\hat{S}_{pi} : \begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + B_d v_d(t) + K_P (y(t) - C\hat{x}(t)) \\ \dot{\hat{v}}_d(t) = K_I (y(t) - C\hat{x}(t)) \end{cases} \tag{7}$$

Or equivalent: $$\hat{S}_{pi} : \left\{ \dot{\hat{x}}_a(t) = A_a \hat{x}_a(t) + B_a u(t) + K_a (y(t) - C_a \hat{x}_a(t)) \right. \tag{8}$$

In which: $\hat{x}_a = \begin{bmatrix} \hat{x} \\ v_d \end{bmatrix}$, $A_a = \begin{bmatrix} A & B_d \\ 0 & 0 \end{bmatrix}$, $B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}$, $C_a = \begin{bmatrix} C & 0 \end{bmatrix}$, $K_a = \begin{bmatrix} K_P \\ K_I \end{bmatrix}$

The necessary and enough condition for the existence of the observer is that the pair (A_a, C_a) has been, in the least, observable, thus it is possible to place the eigenvalues of the following matrix of the complex plan:

$$\hat{A}_a = A_a - K_a C_a = \begin{bmatrix} A - K_P C & B_d \\ -K_I C & 0 \end{bmatrix} \tag{9}$$

In this work the gain of observer PI is determined by the gain gotten for Kalman Filter presented in section (2.3).

2.2.1. Example: Non-linear force estimated by PI observer

Following is presented an example of determination of an unknown input in a robotic arm as shown in Fig. 2.

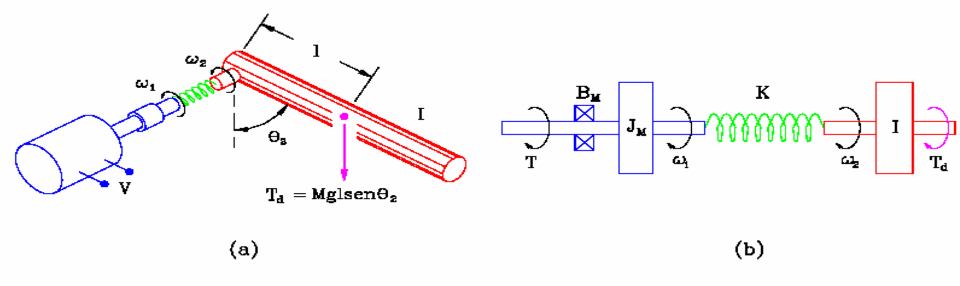

Figure 2. Flexible robot arm with an unknown input.

A mathematical model can be represented by the state equation:

$$S:\begin{cases}\dot{x}(t)=A\,x(t)+B\,u(t)+B_d\;v_d(t)\\ y(t)=C\,x(t)\end{cases} \tag{10}$$

Where:

$$A=\begin{bmatrix}0&0&1&0\\0&0&0&1\\-K/J_M&K/J_M&-B_M/J_M&0\\K/I&-K/I&0&0\end{bmatrix},\; B=\begin{bmatrix}0&0\\0&0\\K_e/J_M&0\\0&1/I\end{bmatrix},\; C=\begin{bmatrix}1&0&0&0\\0&1&0&0\end{bmatrix},\, x(t)=[\theta_1\quad\theta_2\quad\omega_1\quad\omega_2]^T,$$

$$B_d=[0\quad 0\quad 0\quad 1]^T \text{ and } y(t)=[\theta_1\quad\theta_2]^T$$

In which:

$\theta_1(t)$: Angular displacement (robot arm) - $\theta_1(0)$ =15°.

$\theta_2(t)$: Angular displacement in output of the reduction box - $\theta_2(0)$ =15°.

$\omega_1(t)$: Angular speed (robot arm) $\omega_1(0)$ = 0 *rad* s^{-1}.

$\omega_2(t)$: Speed in the output of the reduction box - $\omega_2(0)$ =0 *rad* s^{-1}.

V(t): Voltage of motor DC armor - square shaped Wave of 5 V and 3 *rad* s^{-1} ($u(t)=[V(t)\quad 0]^T$).

I: Inertia of the arm robot - 0.4Kg m^2.

K: Torsional stiffness of the spring - 1 *N m/rad*.

J_M: Inertia equivalent of the motor including reduction box - 0,0424 *kg* m^2.

B_M: Viscous friction - 0,0138 *N m s/rad*.

K_e: Momentum Gain - 0,0403 *N m/V*.

T_d(t): Nonlinear force from the weight of the arm (v_d(t) =T_d(t)).

To simulate the system was used the Runge Kutta method, in which it is considered as output unknown a nonlinear force from the weight of the arm and equal the T_d(t) = M g l sin ($\theta_1(t)$) with M = 1 *Kg*, g = 9,8 *Kg ms* $^{-2}$ and l = 0,3 *m*. The variable of state estimated by PI observer does not consider the force nonlinear and the PI observer can estimate this force (disturbance). For PI observer, the nonlinear force is considered as being an interferential input to the system. In the Fig. 3 is presented the real input and the estimate for the observer.

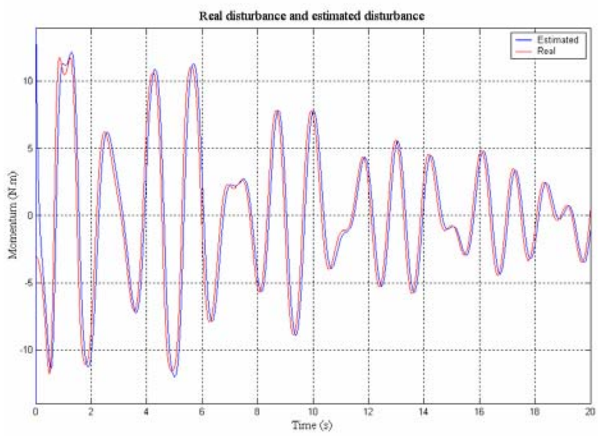

Figure 3. Unknown input estimated through observer PI.

2.3. General structure of the State Observer: Kalman Filter

Considering an invariant and time-observable linear system:

$$\bar{S}:\begin{cases}\dot{x}(t) = A\,x(t) + B\,u(t) + L\,\xi(t)\\ y(t) = C\,x(t) + \eta(t)\end{cases} \tag{11}$$

Where: *x(t)* is the state vector *n x 1*, *u(t)* is the input vector *p x 1*, *y(t)* is the output vector *k x 1*, *A* is the system matrix *n x n* (dynamic matrix), *B* is the distribution matrix *n x p* (inputs matrix), *C* is the measures matrix *k x n*, where *n* is the system order, *p* is the number of inputs u(t), and *k* the number of outputs *y(t).* The vector ξ is called noise of excitement in the state and represents a disturbance in the system and the vector η is called noise in the sensor, (Inouye and Suga, 1999). Due to stochastic nature of the vectors ξ e η, in the Kalman Filter, they have certain statistical properties, corresponding to the white Gaussian noise, stationary (invariant in the time) and not correlated between itself. Mathematically, we have:

$$\mathrm{E}[\xi(t)] = 0,\ \mathrm{E}[\eta(t)] = 0,\ \forall t \tag{12}$$

$$\mathrm{E}\left[\xi(t)\xi^T(t)\right] = \Xi\delta(t-\tau),\ \mathrm{E}\left[\eta(t)\eta^T(t)\right] = \Theta\delta(t-\tau) \tag{13}$$

$$\mathrm{E}\left[\xi(t)\eta^T(\tau)\right] = \mathrm{E}\left[\eta(t)\xi^T(\tau)\right] = 0 \tag{14}$$

In which $\mathrm{E}[\bullet]$ denotes the expected value (the expected value $\mathrm{E}[\bullet]$ mathematically is defined as the function density of probability $fdp(x)$ given for: $\int_{-\infty}^{+\infty} x fdp(x)dx$), $\delta(t-\tau)$ is the Dirac delta (impulse in $t=\tau$). The symmetric positive definite matrices Ξ and Θ are defined for the noise intensities ξ and η, respectively.

$$\Xi = \Xi^T \geq 0,\ \Theta = \Theta^T > 0 \tag{15}$$

The Eq. (12) is a characteristic of the white noise and means that the expected value (mean) is zero in any instant of time. The Eq. (13) also indicates a characteristic of the white noise: it is full unexpected already that is not correlated for any $t \neq \tau$. Finally, the Eq. (14) indicates that the noises ξ and η are not correlated between it. An important observation is that continuous white noises not exist in the reality, they are abstractions, and with these considerations we have easier deductions being the limit for a stochastic process. In the domain frequency, the white noise corresponds to stochastic process with a constant spectral density power (with value Ξ and Θ corresponding to the noises ξ and η respectively) for all the frequencies.

Given the assumptions mentioned above, the problem of optimum estimate of the state vector x in presence of white noises (as vectors of state as the measured variable) can be formulated to find optimum value (filter of Kalman) that it generates an estimate $\bar{x}$ for the real state vector x, so that minimizes the covariance of the error estimation $e(t) = \bar{x}(t) - x(t)$:

$$\Im_{KF} = \mathrm{E}\left[e(t)e^T(t)\right] \tag{16}$$

In according to restriction Eq. (11), Kalman and Bucy had proved that the best structure for the Kalman filter (among all the possible structures, linear and nonlinear) when the dynamics of the system is linear and the noises are white and Gaussians is the following one:

$$\bar{S}_{KF}:\left\{\dot{\bar{x}}(t) = A\ \bar{x}(t) + B\ u(t) + \mathrm{K}_{KF}\left(y(t) - C\ \bar{x}(t)\right)\right. \tag{17}$$

where K_{KF} is the matrix of the state observer, $\{\bar{x}(t)\}$ is the state vector of the observer.

2.3.1 Filter Algebraic Riccati Equation (Fare)

The solution of the optimization problem can be found in literature. Since in the present work the main interest is the application of the control methodologies, we present here without test the solution for this problem. For this admitted that the vector of initial state is not correlated to the noises ξ and η :

$$\mathrm{E}\left[x(0)\ \eta^T(t)\right] = \mathrm{E}\left[x(0)\ \xi^T(t)\right] = 0 \tag{18}$$

The optimum gain K_{KF} for the Kalman filter is given by the following relation:

$$\mathrm{K}_{KF} = S_{KF} C^T \Theta^{-1} \tag{19}$$

In which S_{KF} is defined like a symmetrical and positive matrix satisfying the Riccati equation of for the Kalman filter (FARE):

$$S_{KF} A^T + A\ S_{KF} + L\ \Xi\ L^T - S_{KF} C^T \Theta^{-1} C\ S_{KF} = 0 \tag{20}$$

2.4. Project of State Observers

The project of a system is presented in the Fig. 4 functioning with state observers, where if verifies the known excitement force *u(t)*, unknown inputs v_d *(t)*, the measured outputs *y(t)*, the observers PI used to identify the unknown input, the global and robust observers to the parameters subject to faults s1…, sn and a unit of logical decision. The observer of global state is responsible for the detection of the fault, the robust state observer, is responsible for the location of the same one. The global observer is a copy of the original system, and analyzes all the system detecting possible faults. The robust state observer can detect the fault if this occurs in the parameter for which it was projected. We have to project a bank of robust observer, each one in relation to a parameter to be monitored, to become possible a good location of fault.

When the system is functioning adequately, without indications of faults, the observer of global state answers equal the real system. When one component of the system in question starts to fail, the state observer feels the influence of this process quickly. The objective is using this effect of the state observer to locate and to quantify the fault in the mechanical system. The global and robust observers are modeled, in this work, using the methodology of the Kalman Filter because with this the noise in the system is better worked. They are put in a bank of observers and the RMS values of the differences between the real signs in displacement (measured) and the generated for the observers are analyzed in a unit of logical decision that it analyzes the trend of the progression of the fault and sets in motion, when will be necessary, an alarm system. The alarm system can be initialized when we have a parameter variation. This is on line process and the model of observer PI must be changed during all the process where the fault is occurring, with possibility to identify the disturbance with good accuracy.

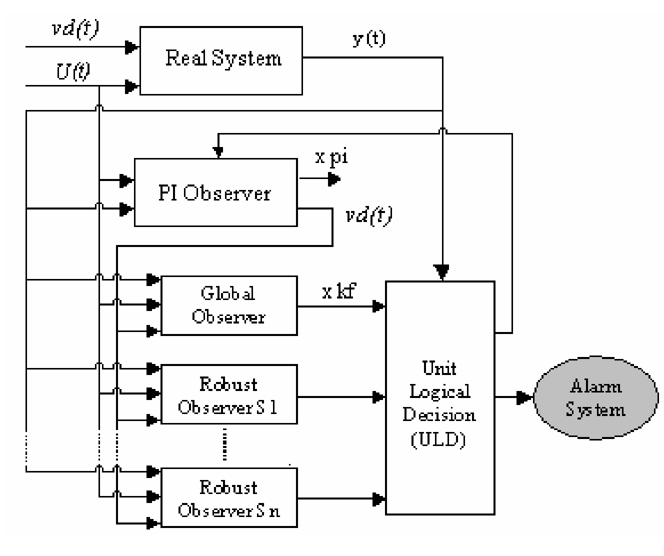

Figure 4. System of Robust Observation.

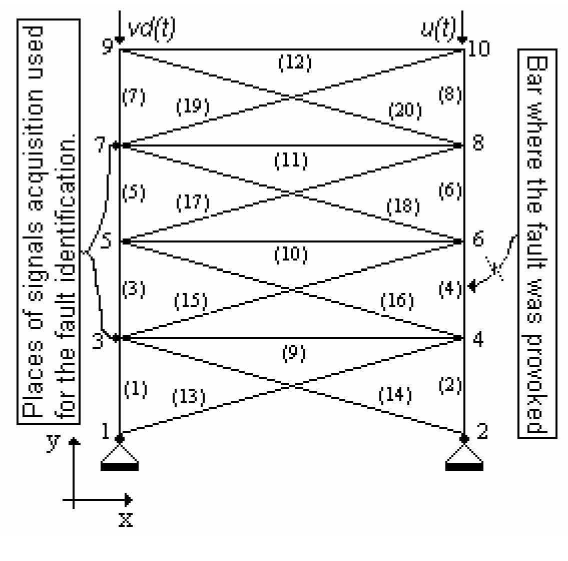

Figure 5. Truss Structure with 20 bars

2.5. Simulation and results for a truss structure with 20 bars

To validate the methodology of identification and location of faults applied the mechanical systems using the state observer, filter of Kalman, with forces unknown identified through observers PI, was simulated a truss structure with 20 bars as shown in the Fig. 5. For this we used the finite elements method to be able to simulate the structure, where each bar represents a composite element for two joins and each join has two degrees of freedom (d.o.f) being displacement in x and y. Considering that the structure has restrictions in the joins one and two, we have, a system with 16 dof, as shown in the figure below, the system was excited in the join 9 and 10 in the direction of y with harmonic forces of 300 N and 500 N and frequencies of 250 rad/s and 3700 rad/s respectively. The force applied in join 9 is considered unknown and will be determined by observer PI.

All the elements that compose the truss are isoperimetric with the following properties: ρ = 7850 kg/m3, E = 200GPa, height = 2 cm, width = 3 cm. All the bars in the x direction have 2 m and in the y direction have length equal 0,5 m. We considered during the simulation a low proportional damping the matrix of mass and the stiffness of the system given for: *C=1.00e-10*K+1.00e-04*M*. The output of this system was gotten through the method of Runge-Kutta of fourth order with 4096 points in the interval of 0,2 s, of this form, was used only the output of the displacement in the direction x in joins 3 and 7, with this we initiated the identification and location process of the fault which the structure was submitted simulating to fault we used a reduction in the area of bar four of 30%. To validate the robustness of the Kalman Filter for presence of noises in the signs, it was added, to the input, a white noise with energy equal 5% of the value of the energy of the input sign *u(t)*.

The bank of robust observers is generated for the parameters subjected to the faults with percentile variation of 10% in the area of each bar. Was considered, in this work, that all the bars of the system are susceptive the occurrence of a possible fault. In the Fig. 6 and 7 the are presented the reverse values of differences RMS found between the “measured” sign in the structure and the signs generated for the global observers (0% of fault) and for the robust observers, reducing in 10% the value of each subject parameter to fault, in the Fig. 6 the truss does not present fault, already in the Fig. 7 the fault is considered during the simulation. It is verified in Fig. 6 that the biggest values found were 0% of fault, thus the structure did not present fault. In Fig. 7 could be located and be quantified a fault provoked in bar 4 with 30% of reduction in this parameter.

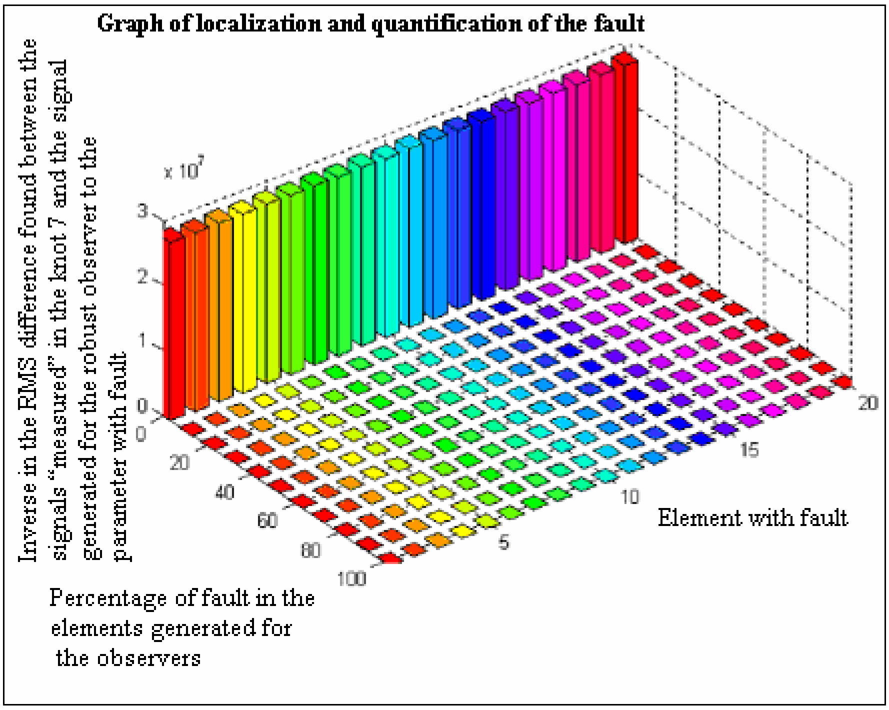

Figure 6: Robust observer bank generated with variation of 10% in each parameter for the system without fault.

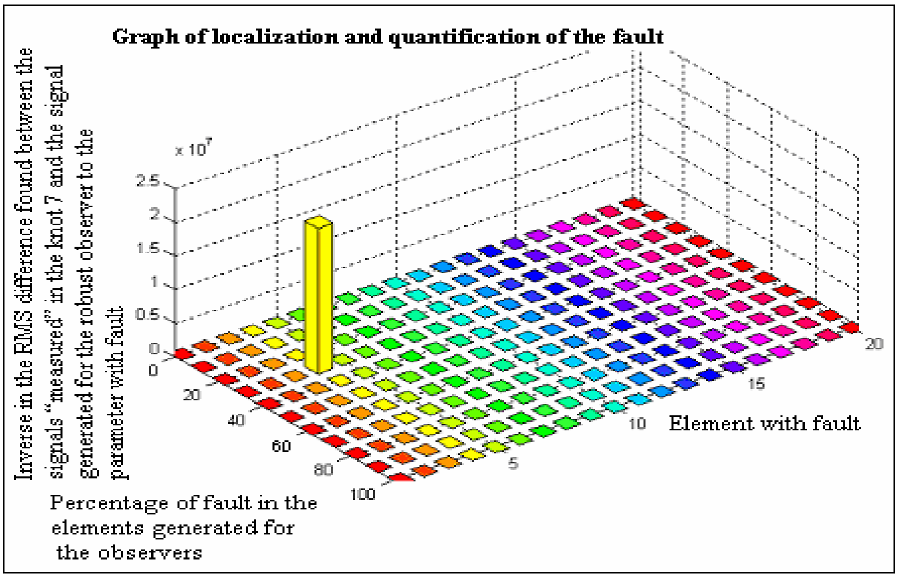

Figure 7: Robust observer bank generated with variation of 10% in each parameter for the system with a provoked fault of 30% in the bar.

3. Conclusion

In this work was developed a methodology of faults diagnose using state observers. It was used The Kalman Filter to the construction of observers bank, and in this case the observer needs that all the inputs are known or considering white noise like the only interference used to project the Kalman Filter. In this in case, neither all inputs were known, for this, in this work, the inputs were estimated using the Proportional and Integral observer. With this gain the states identified by PI observer reject the inputs unknown. It was presented a robotic arm, in which, it was possible to identify the external force due the mass of the arm. An example of fault diagnose using a truss structure with 20 bars in which were considered two inputs, being that one was unknown and identified by PI observer. When we consider the computational time necessary for assembly of robust observers bank to the parameters subject to faults, this time is relatively high, but in the practical the state observers bank is assembled only one time. With this, during the acquisition of on-line signs in the structure, is not necessary to have the observer bank because it was constructed before. With this, we conclude state observers methodology can be used to detect faults in on-line system.

4. Acknowledgement

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