TECHNIQUE OF PARAMETERS AND INPUTS IDENTIFICATION IN MECHANICAL SYSTEMS

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Abstract. *In this work a technique of identification of parameters and forces based on the transformation of the system of differential equations, which conducts the dynamic behavior of the mechanical system in a system of linear equations, whose resolution is much more simple is developed. This transformation is made using functions of orhtogonal bases such as: the series of Legendre and Chebyshev. In classical techniques of identification of parameters, the identification of systems in steady state has only been possible by considering the previous knowledge of the exciting forces acting on the system, needed condition to get the unknown parameters. In this work, it is presented the development of a methodology of parameters and exciting forces identification in systems with multi-degree-of-freedom considering just the signals of the system response. For this purpose, there are two signal acquisitions to be carried out on the structure: in the first acquisition the signals of the original system are obtained and in the second acquisition the signals are obtained regarding a known variation in the parameters of the structure. This variation can be accomplished regarding an additional mass on the structure or changing the stiffness of the analyzed system. In this technique it is necessary to know, quantitatively, the change that was induced in the system so that the parameters of the system as well as the exciting forces can be identified. Computational simulation for a system with multi-degree-of-freedom are presented and the methodology is applied in a structure composed by shake tables placed in a Vibrations Laboratory.*

Keywords: *Parameters and Inputs Identification, Orthogonal Function, Legendre and Chebyshev*

1. Introduction

In view of the increase in the production processes, there are more and more demands of the industries for machines and equipment capable of performing a higher number of functions in less time and in many cases to be capable to work continuously as possible. As a consequence of this fact the systems are submitted to high dynamic forces. Normally the machines are very expensive and therefore one of the major industry concerns is to keep them operating without necessary breakdowns. In the last time, due to this constant concern, it has been verified the development of many new techniques of detection and localization of faults in mechanical systems submitted to dynamics loads. In order to guarantee continuous operation, the mechanical systems must be checked and monitored so that the faults are diagnosed and repaired as fast as possible. If the machines operate out of the normal disturbance, then a deterioration of the performance of them may occur or dangerous operation conditions may happen. In order to detect faults in mechanical systems, some methods of identification of forces or parameters using orthogonal functions have been developed since the end of 80's until the present days. These methods began with Chun (1987), Melo and Steffen (1993) by using Fourier series for structural parameters identification. They also developed the inverse methodology for identification of the forces. Pacheco (2000), in his doctor thesis, used some orthogonal functions for parameters identification through the comparisons between that functions. Pacheco and Steffen (2003) published a work where the orthogonal functions were used for identification of parameters in non linear systems. In a recent work Melo (2004) analyzed the behavior of the error found in the identification of the parameters changing the number of terms of expansion series of some orthogonal functions (Pezerat and Guyader, 2000). This work, differently, has as aim identifying forces and parameters of the mechanical systems together. In the previous mentioned works, the identification of the parameters was only possible with the previous knowledge of the inputs (forces). In this work, a methodology was developed regarding a provoked known variation in some parameters of the structure. Thus just measuring the signals before and after the known parameters variation the forces as well as the structural parameters can be identified.

2. Orthogonals Functions

A set of real functions $\phi_k(t), k = 1, 2, 3...$ is said to be orthogonal in the interval $[a,b] \in \Re$ Spiegel (1976) if:

$$\int_a^b \varphi_m(t)\varphi_n(t)d(t) = K \qquad \text{Where: } \begin{cases} K = 0 \Rightarrow m \neq n \\ K \neq 0 \Rightarrow m = n \end{cases} \tag{1}$$

The set of functions is said orthonormal if will be valid the relation:

$$\int_a^b \phi_m(t)\phi_n(t)dt = \delta_{mn} \tag{2}$$

If δ_{mn} is the Kronecker delta, the set of functions $\phi_k(t)$ is said to be orthonormal and δ_{mn} =0 if $m \neq n$ or δ_{mn} =1 if $m = n$.

If the set $\varphi_k(t)$ is orthonormal respect to the weight function *w(t)* in which *w(t) ≥ 0*, then the set of orthonormal functions is gotten through the equation:

$$\phi_k(t) = \sqrt{w(t)}\varphi_k(t), \quad \mathrm{k} = 1, 2, 3,... \tag{3}$$

Thus, it is verified the relation:

$$\int_a^b \varphi_m(t)\varphi_n(t)w(t)dt = \delta_{mn} \tag{4}$$

If a function $f(t)$ is continuous or partially continuous in the interval $[a,b]$, then $f(t)$ can be expanded in series of orthonormal functions, that is:

$$\mathrm{f}(t) = \sum_{n=1}^{\infty} c_n\phi_n(t) \tag{5}$$

Such series, called orthonormal series, constitute generalizations of the Fourier series. Admitting that the sum in Eq. (5) converges to $f(t)$, we can multiply both members for $\phi_m(t)$ and integrating them in the interval $[a,b]$. In this equation c_m are the generalized coefficients of Fourier.

The following property, related to the successive integration of the vectorial basis, holds for a set of *r* orthonormal functions in the interval $[0,t]$:

$$\underbrace{\int_0^t \cdots \int_0^t}_{n\, vezes} \{\phi(\tau)\}(d\tau) \cong [P]^n \{\phi(t)\} \tag{7}$$

Where $[P] \in \Re^{r,r}$ is a square matrix with constant elements, called operational matrix and $\{\phi_m(t)\} = \{\phi_0(t)\ \phi_1(t)\ ...\ \phi_r(t)\}^T$ is the vectorial basis of the orthonormal series. In fact, if a complete vectorial base is regarded, that is, if the series are not truncated, the relation obtained in Eq. (7) is equality. However, in the practice, it becomes not suitable, due to the high order of the matrix [*P*]. In the following sections, the vectorial basis and the operational matrix related to each type of orthogonal function considered in this paper are briefly reviewed.

Legendre polynomials

Recursive formula in the interval $t \in [0, \mathrm{t_f}]$	Operational matrix of integration
$(n+1)p_{n+1}(t) = (2n+1)\left(\frac{2t}{t_f} - 1\right)p_n(t) - np_{n-1}(t)$ $, \mathrm{n} = 1, 2, 3, \ldots, r-1$ $p_0(t) = 1$ $p_1(t) = 2t/t_f - 1$	$[P] = \frac{t_f}{2}\begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & 0 \\ -\frac{1}{3} & 0 & \frac{1}{3} & 0 & \cdots & 0 \\ 0 & -\frac{1}{5} & 0 & \frac{1}{5} & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{-1}{2r-3} & 0 & \frac{1}{2r-3} \\ 0 & 0 & \cdots & 0 & \frac{-1}{2r-1} & 0 \end{bmatrix}$

r = number of terms truncated

Chebyshev polynomials

Recursive formula in the interval $t \in [0, \mathrm{t_f}]$	Operational matrix of integration
$T_{i+1}(t) = 2\left(\frac{2t}{t_f} - 1\right)T_i(t) - T_{i-1}(t)$ $i = 1, 2, \ldots, r-1$ $T_0(t) = 1$ $T_1(t) = \frac{2t}{t_f} - 1$	$[P] = \frac{t_f}{2}\begin{bmatrix} 1 & 1 & 0 & 0 & \ldots & 0 & 0 & 0 \\ -1/4 & 0 & 1/4 & 0 & \ldots & 0 & 0 & 0 \\ -1/3 & -1/2 & 0 & 1/6 & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{(-1)^{r-1}}{(r-1)(r-3)} & 0 & 0 & 0 & \ldots & \frac{-1}{2(r-3)} & 0 & \frac{1}{2(r-1)} \\ \frac{(-1)^r}{r(r-2)} & 0 & 0 & 0 & \ldots & 0 & \frac{-1}{2(r-2)} & 0 \end{bmatrix}$

r = number of terms truncated

2.1. Identification of mechanical systems through orthogonal function

The proposed identification method can exploit either free or forced time domain responses, in terms of displacements, velocities or accelerations. Since the formulations for these three types of responses are quite similar, only the formulation for forced systems, in terms of displacements, will be presented in this work.

The development of the method starts from the equation of motion of a forced mechanical system of *N* degrees of freedom:

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{f(\mathrm{t})\} \tag{8}$$

Where [M], [C] and [K] are the inertia, damping and stiffness N-order matrices respectively; $\{x(t)\}$ is the vector of displacement time responses and $\{f(t)\}$ is the vector of exciting forces.

Integrating Eq. (8) twice in the interval [0,t], it can obtain:

$$[M]\left(\{x(t)\} - \{x(0)\} - \{\dot{x}(0)\}t\right) + [C]\left(\int_0^t \{x(\tau)\}d\tau - \{x(0)\}t\right) + [K]\int_0^t\int_0^t \{x(\tau)\}d\tau^2 = \int_0^t\int_0^t \{f(\tau)\}d\tau^2 \tag{9}$$

The signals $\{x(t)\}$ and $\{f(t)\}$ can be expanded in the truncated series of *r* orthogonal functions as follows:

$$\{x(t)\} = [X]\{\phi(t)\} \quad \text{and} \quad \{f(t)\} = [F]\{\phi(t)\} \tag{10}$$

Where: $[X] \in \Re^{N,r}$ is the matrix of the coefficients of expansion $\{x(t)\}$

$[F] \in \Re^{N,r}$ Is the matrix of the coefficients of expansion $\{f(t)\}$

Substituting Eq. (10) in Eq. (9) and applying the integral property given by Eq. (7), the following system of algebraic equations is obtained (Pacheco 2000 and Melo 2004):

$$\begin{bmatrix} [\mathrm{M}] & -[\mathrm{M}]\{x(0)\} & \left\{-[\mathrm{M}]\{\dot{x}(0)\}-[\mathrm{C}]\{x(0)\}\right\} & [\mathrm{C}] & [\mathrm{K}] \end{bmatrix} \begin{bmatrix} [X] \\ \{e\}^T \\ \{e\}^T[P] \\ [X][P] \\ [X][P]^2 \end{bmatrix} = [F][P]^2 \tag{11}$$

Admitting an induced known variation in some of the structural parameters of the system, then we have from Eq. (11) the following equation:

$$\begin{bmatrix} [\mathrm{M_M}] & -[\mathrm{M_M}]\{x_M(0)\} & \left\{-[\mathrm{M_M}]\{\dot{x}_M(0)\}-[\mathrm{C_M}]\{x_M(0)\}\right\} & [\mathrm{C_M}] & [\mathrm{K_M}] \end{bmatrix} \begin{bmatrix} [X_M] \\ \{e\}^T \\ \{e\}^T[P] \\ [X_M][P] \\ [X_M][P]^2 \end{bmatrix} = [F][P]^2 \tag{12}$$

Where $[\mathrm{M_M}]$, $[\mathrm{K_M}]$ and $[\mathrm{C_M}]$ are the inertia, stiffness and damping matrices respectively after some modification in the structural parameters to have been induced and $[X_M]$ are the expansion coefficients of the gotten output of the modified system. If it is considered that the input forces of the system do not change when the variations are induced in the parameters, then we have that:

$$\begin{aligned} &\begin{bmatrix} [\mathrm{M_M}] & -[\mathrm{M_M}]\{x_M(0)\} & \left\{-[\mathrm{M_M}]\{\dot{x}_M(0)\}-[\mathrm{C_M}]\{x_M(0)\}\right\} & [\mathrm{C_M}] & [\mathrm{K_M}] \end{bmatrix} \begin{bmatrix} [X_M] \\ \{e\}^T \\ \{e\}^T[P] \\ [X_M][P] \\ [X_M][P]^2 \end{bmatrix} = \\ &= \begin{bmatrix} [\mathrm{M}] & -[\mathrm{M}]\{x(0)\} & \left\{-[\mathrm{M}]\{\dot{x}(0)\}-[\mathrm{C}]\{x(0)\}\right\} & [\mathrm{C}] & [\mathrm{K}] \end{bmatrix} \begin{bmatrix} [X] \\ \{e\}^T \\ \{e\}^T[P] \\ [X][P] \\ [X][P]^2 \end{bmatrix} = [F][P]^2 \end{aligned} \tag{13}$$

Thus we can write that:

$$\begin{aligned} &\begin{bmatrix} [\mathrm{M}] & -[\mathrm{M}]\{x(0)\} & \left\{-[\mathrm{M}]\{\dot{x}(0)\}-[\mathrm{C}]\{x(0)\}\right\} & [\mathrm{C}] & [\mathrm{K}] \end{bmatrix} \begin{bmatrix} [X] \\ \{e\}^T \\ \{e\}^T[P] \\ [X][P] \\ [X][P]^2 \end{bmatrix} \\ &-\begin{bmatrix} [\mathrm{M_M}] & -[\mathrm{M_M}]\{x_M(0)\} & \left\{-[\mathrm{M_M}]\{\dot{x}_M(0)\}-[\mathrm{C_M}]\{x_M(0)\}\right\} & [\mathrm{C_M}] & [\mathrm{K_M}] \end{bmatrix} \begin{bmatrix} [X_M] \\ \{e\}^T \\ \{e\}^T[P] \\ [X_M][P] \\ [X_M][P]^2 \end{bmatrix} = [0] \end{aligned} \tag{14}$$

For convenient simplification, it is considered that the initial conditions of displacement and velocity are null, and this way we have:

$$\begin{bmatrix}[\mathrm{M_M}] & -[\mathrm{M_M}]\{x_M(0)\} & \{-[\mathrm{M_M}]\{\dot{x}_M(0)\}-[\mathrm{C_M}]\{x_M(0)\}\} & [\mathrm{C_M}] & [\mathrm{K_M}]\end{bmatrix}+ \\ -\begin{bmatrix}[\mathrm{M}] & -[\mathrm{M}]\{x(0)\} & \{-[\mathrm{M}]\{\dot{x}(0)\}-[\mathrm{C}]\{x(0)\}\} & [\mathrm{C}] & [\mathrm{K}]\end{bmatrix}=\begin{bmatrix}\Delta M & 0 & 0 & \Delta C & \Delta K\end{bmatrix} \tag{15}$$

Being known the variation provoked in the structure we have that $\begin{bmatrix}\Delta M & 0 & 0 & \Delta C & \Delta K\end{bmatrix}$ is known. Thus we can write:

$$\begin{bmatrix}[\mathrm{M}] & 0 & 0 & [\mathrm{C}] & [\mathrm{K}]\end{bmatrix}=\begin{bmatrix}[\mathrm{M_M}] & 0 & 0 & [\mathrm{C_M}] & [\mathrm{K_M}]\end{bmatrix}-\begin{bmatrix}\Delta M & 0 & 0 & \Delta C & \Delta K\end{bmatrix} \tag{16}$$

If we substitute the Eq. (16) into Eq. (14) and if we separate the terms after some manipulations we have:

$$\begin{bmatrix}[\mathrm{M_M}] & 0 & 0 & [\mathrm{C_M}] & [\mathrm{K_M}]\end{bmatrix}=\begin{bmatrix}\Delta M & 0 & 0 & \Delta C & \Delta K\end{bmatrix}\begin{bmatrix}[X] \\ \{e\}^T \\ \{e\}^T[P] \\ [X][P] \\ [X][P]^2\end{bmatrix}\left\{\begin{bmatrix}[X] \\ \{e\}^T \\ \{e\}^T[P] \\ [X][P] \\ [X][P]^2\end{bmatrix}-\begin{bmatrix}[X_M] \\ \{e\}^T \\ \{e\}^T[P] \\ [X_M][P] \\ [X_M][P]^2\end{bmatrix}\right\}^{-1} \tag{17}$$

This way, knowing only the output before and after of the provoked variations in the parameters of the system, the provoked variations of the matrices $\begin{bmatrix}[\mathrm{M_M}] & 0 & 0 & [\mathrm{C_M}] & [\mathrm{K_M}]\end{bmatrix}$ are determined. Substituting in Eq. (12) we can determine the coefficients of expansion of the exciting forces as:

$$[F]=\begin{bmatrix}[\mathrm{M_M}] & 0 & 0 & [\mathrm{C_M}] & [\mathrm{K_M}]\end{bmatrix}\begin{bmatrix}[X_M] \\ \{e\}^T \\ \{e\}^T[P] \\ [X_M][P] \\ [X_M][P]^2\end{bmatrix}[P]^{-2} \tag{18}$$

Substituting the Eq. (18) into Eq. (10) the exciting forces of the system can be determined.

3. Simulation

In this section an example of determination of the parameters and an unknown input in a robotic arm, as shown in Fig. 1, is presented.

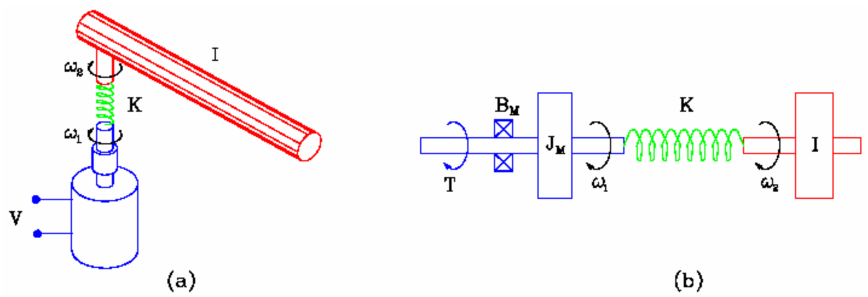

Figure 1. Flexible arm of a robot.

The model can be represented by a state equation in which the dynamic and input matrices are given by:

$$A=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K/J_M & K/J_M & -B_M/J_M & 0 \\ K/I & -K/I & 0 & 0 \end{bmatrix} \text{ and } B=\begin{bmatrix} 0 \\ 0 \\ K_e/J_M \\ 0 \end{bmatrix}$$

In which:

$\theta_1(t)$: Angular displacement of the robot arm ($\theta_1(0)$ =0°)

$\theta_2(t)$: Angular displacement in output of the reduction box ($\theta_2(0)$ =0°).

$\omega_1(t)$: Angular speed of the robot arm ($\omega_1(0)$ =0).

$\omega_2(t)$: Speed in the output of the box of reduction ($\omega_2(0)$ =0).

V: Voltage of motor armor (=sine wave with 10 V and 60 *rad/s)*

I: Inertia of the arm robot (= 0.4Kg *m2).*

K: Torsional stiffness of the spring (= 1 *N m/rad).*

J_M: Equivalent inertia of the engine including reduction box (= 0,0424 *kg m2*)

B_M: Viscous friction in the motor (= 0,0138 *N m s/rad).*

K_e: Momentum Gain for the motor (= 0,0403 *N m/V*).

The Fourth Order Runge Kutta method was used to simulate the system. It was generated 2048 points in the time interval of 0,4 *s*, in which the signals of angular displacement were used for the identification of the parameters and input of the system. As parameter variation it was added 0.001$Kg\ m^2$ and 0,01 $Kg\ m^2$ in the inertia of the arm and the inertia of the motor, respectively. In Tab. 1 the results of the parameters identification using the methods of Legendre and Chebyshev are presented. In Fig. 2 and Fig. 3 the values of the identified input is presented.

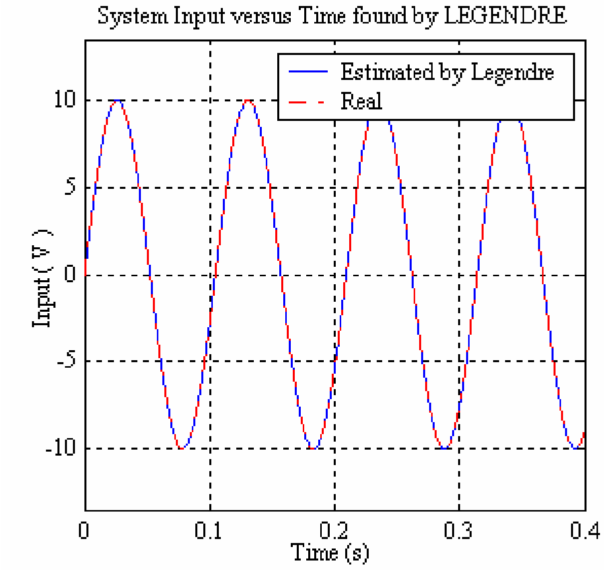

Figure (2) Input identified by Legendre

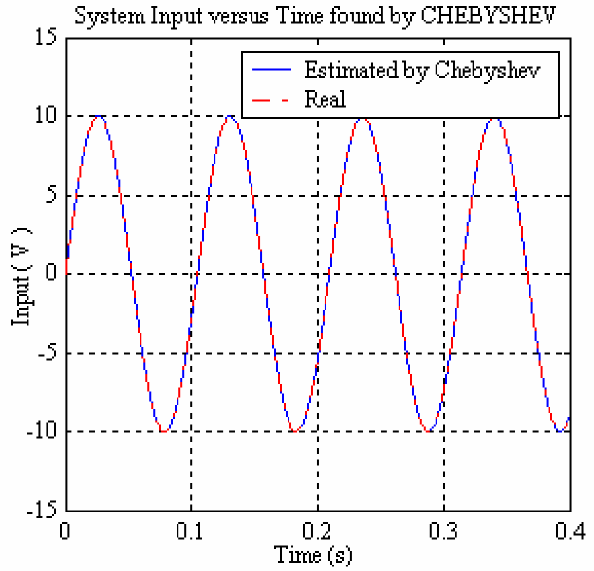

Figure (3) Input identified by Chebyshev

Table 1. Parameters Identified through the Methods of Legendre and Chebyshev.

Parameters	Theoretical value	Identified by Legendre	Error % Legendre	Identified by Chebyshev	Error % Chebyshev
$I\ (Kg\ m^2)$	0,4000	0,4000	0,0000	0,4000	0,0000
K (N m/rad)	1,0000	0,9999	0,0100	0,9998	0,0200
$J_M\ (Kg\ m^2)$	0,0424	0,0424	0,0000	0,0424	0,0000
$B_M\ (N\ m\ s/rad)$	0,0138	0,0138	0,0000	0,0138	0,0000

4. Experimental Results

A dynamic system of shake tables constituted of three horizontal plates of aluminum supported by vertical metallic stainless steel blades representing the stiffness to the system was constructed. Rubbers working together with the blades were used to simulate the viscous damping. The rubbers are fixed between the blades, as it can be seen in Fig. 4. In Tab. 2 the results are presented.

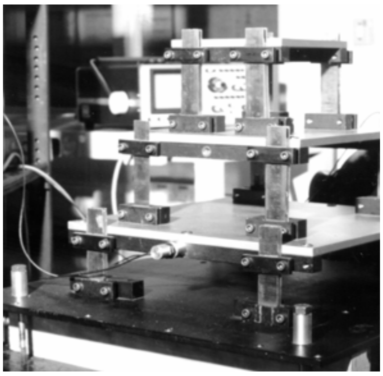

Figure 4: Vibratory System

Table 2. Space parameters of the system

table	Inferior	Intermediary	Superior
M (Kg)	6,644	4,619	1,889
K (KN/m)	274,857	114,416	104,870
C (Ns/m)	100,042	59,978	69,881

The structure was modeled as a system of three degrees of freedom with discrete parameters. The structural parameters had been determined using techniques of experimental modal analysis where of mass, damping and stiffness had been determined by taking each table and its respective supports separately as a sub-system. After setting up the system, the inferior table was excited with a harmonic force and the acquisition of the signals during 0,5 *s* and with 2048 points in this interval was performed by using DASYLAB software. Four signal channels were used in the acquisition, being three channels for acquisition of the signals of displacement and one channel for determination of the exciting force. The exciting force of the system was measured for comparison and verification of the method efficacy.

The signals used in the method were displacements; therefore the measured responses captured with accelerometers were integrated twice by using Signal Conditioner/Amplifier *Nexus* devices of the *BRUELL* Company which have this function. A small plate of 100g was added at the gravity center of each table for parameter variation and again the acquisition of the signals was carried out. From the signal responses measured before and after provoking the variation in the masses of the tables and just knowing the variation that was provoked in the structure, it can be identified the parameters and the force that acts onto the structure. Table 3 presents the identified values by using the Legendre and Chebyshev methods together with the real parameters. During the identification of the parameters it has been used 100 terms of expansion as it can be seen in the recent work (Melo *et al*, 2004). The Figs. 5 and 6 show the signals of the forces gotten by the considered methods, together with the real force (measured in the structure).

The good result gotten during the forces and the parameters identification is, in large part, due to the high simplicity and linearity of the analyzed system.

Table 3. Parameters Identified through the Methods of Legendre and Chebyshev.

Table	Parameters	Theoretical value	Identified by Legendre	Error % Legendre	Identified by Chebyshev	Error % Chebyshev
Inferior	M (Kg)	6,644	6,6364	0,114	6,631	0,196
	K (KN/m)	274,857	275,169	0,114	275,664	0,294
	C (Ns/m)	100,042	100,231	0,189	100,665	0,623
Intermediary	M (Kg)	4,619	4,596	0,498	4,563	1,212
	K (KN/m)	114,416	113,384	0,902	112,063	2,056
	C (Ns/m)	59,978	59,397	0,969	58,078	3,168
Superior	M (Kg)	1,889	1,8843	0,247	1,878	0,582
	K (KN/m)	104,870	104,017	0,813	102,782	1,991
	C (Ns/m)	69,881	69,373	0,727	68,196	2,411

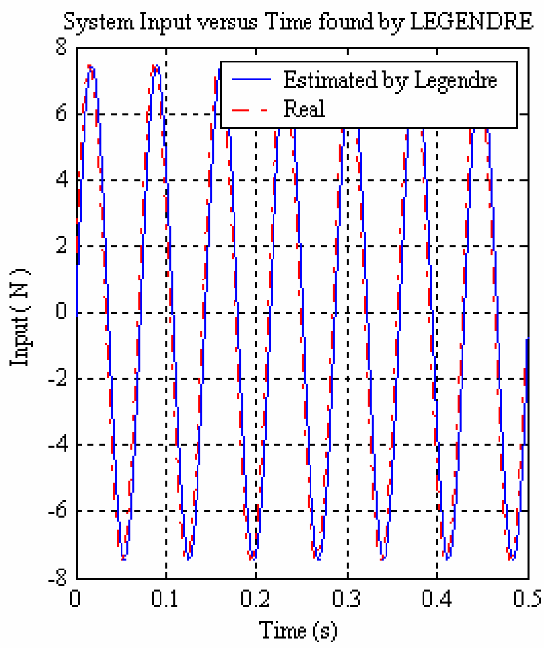

Figure (5) Input identified by Legendre.

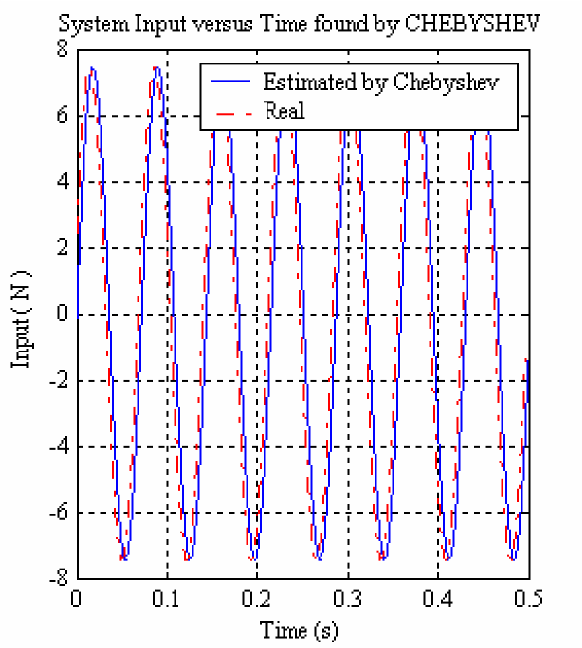

Figure (6) Input identified by Chebyshev

5. The last considerations

In this work a methodology of identification of forces and parameters of mechanical systems using orthogonal functions was developed. The method just uses the response signals and a known variation as input data. A robotic arm system was used to validate the methodology of identification of the inputs and the structural parameters. The experimental validation of the methodology was carried out through a simple system of three degrees of freedom. The error found in the identification of forces and parameter in the computational simulation and using the test rig was low. In this way, the methodology used was validated. As this methodology was not applied in complex systems yet, we don't know its robustness and sensitivity when applied on that type of systems. In future works we intend to apply this methodology in complex systems together with the methodology of the state observers, because these have the capacity to reconstruct the states not measured, and consequently, the number of sensors necessary for the identification can be reduced.

6. Acknowledgement

The authors would like to acknowledge the financial support of FAPESP (Fundação de Amparo a Pesquisa do Estado de São Paulo).

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