

## Crack Growth Retardation Effects in Civil Airplane Structures

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*The accurate prediction of fatigue crack growth has long been recognized as of critical importance for the design of aeronautical structures. Modern transport airplanes have to be designed to meet damage tolerance requirements, established by international certification agencies. Under the damage tolerant design philosophy, the fatigue lives of structural members are predicted using Fracture Mechanics concepts, more specifically crack propagation calculations. Maintenance procedures are also dependent upon crack growth predictions, since inspection intervals are defined from propagation results combined with specific information on the selected NDE techniques to be used. Therefore, reliable predictions of fatigue crack propagation have great impact on the safe and economical operation of modern aircraft.*

*The prediction of fatigue crack propagation under complex, variable amplitude loading requires the modeling of the so-called retardation effects, which are basically due to the plastic zone ahead of the crack tip caused by overloads. This paper discusses the application of such models to predict crack growth in structural components of a transport airplane. A review of the traditional crack growth retardation models is presented. In order to assess the accuracy of the models current available in the literature, numerical simulations are performed and the results are compared to published data. Then, selected retardation models are used to predict crack growth in typical aeronautical components. The results are validated against experimental data from crack propagation tests in coupon specimens submitted to the standard MiniTWIST load spectrum.*

**Keywords:** Crack Propagation, Fracture Mechanics, Load Sequence Effects, Crack Retardation, TWIST

### 1. Introduction

Due to the nature of the load spectra to which commercial aircraft are subjected, the load sequence effects have a great influence on the total life of a structural component. Usually, in the lack of a better understanding of this technology, the evaluation of crack propagation performed on aircraft by the industry does not take in account these load sequence effects. Such effects are known as crack retardation when they cause the reduction of the crack propagation rate. In most cases this approach is too conservative as has published Brot *et al* (2002). However, in some cases, if these sequence effects are not taken in account, it can lead to non-conservative results, affecting directly the safe operation of the aircraft as has noted McClung *et al* (2002) for compressive dominated load spectrum.

Several models can be found in the literature in order to predict this behavior by changing the crack propagation rates in order to simulate the effect of load sequence. These models are generally based on adjustable parameters driven by the material characteristics and mainly by the applied load spectra according to Zhang *et al* (1987). The models are based on different formulations which can be as simple as considering a general retardation factor to be applied to the crack opening stress intensity factor or as complicated as taking into account the elasto-plastic behavior ahead of the crack caused by the overloads and the underloads.

### 2. Crack retardation models

Actual aircraft operation load usually results in a combination of high and low stresses with variable stress ratios. Any load cycle influence is affected by its history and can be described by two effects:

#### Retardation:

Retardation occurs after an overload peak, when the crack tip plastic zone caused by a load cycle is inside the crack tip plastic zone caused by the last load cycle, reducing the crack propagation rate.

#### Underload Effect:

The underload effect occurs when a compressive load (or even tension load) is lower than the lowest load found since the last overload. This effect causes the reduction of the retardation effects.

### 2.1 Wheeler Retardation Model

The Wheeler retardation model (*Wheeler et al* 1972) includes a correction parameter  $C_p$  to be added to the damage accumulation equation in order to slow crack growth after an overload which results in Eq.(1)

$$a_r = a_0 + \sum_{i=1}^r C_p f(\Delta K) \quad (1)$$

where  $a$  is the crack size and  $f(\Delta K)$  is a function of the stress intensity factor range, defined as  $K_{\max} - K_{\min}$ .

The  $C_p$  parameter is bounded between 0 and 1 (starting at 0 just after an overload until it reaches the value of 1.0 in a late cycle). The  $C_p$  parameter is representative of a plastic zone evaluated according to a plane strain state at the crack tip according to Eq. (2)

$$r_0 = \frac{1}{6\pi} \left( \frac{K}{\sigma_y} \right)^2 \quad (2)$$

where  $\sigma_y$  is the yield stress and  $K$  is the stress intensity factor (SIF)

The  $C_p$  parameter is evaluated as shown in Eq. (3)

$$C_p = \left( \frac{Z_i}{Z_{ol} + a_{ol} - a_i} \right)^m \quad (3)$$

where:

$$C_p = 1 \Rightarrow Z_i + a_i \geq Z_{ol} + a_{ol} \quad (4)$$

The plastic zone size  $Z$  in the Wheeler model is evaluated at Eq.(2).  $M$  is an experimental parameter and provides the model the capacity to match experimental crack growth tests. All other parameters are defined on Figure 1

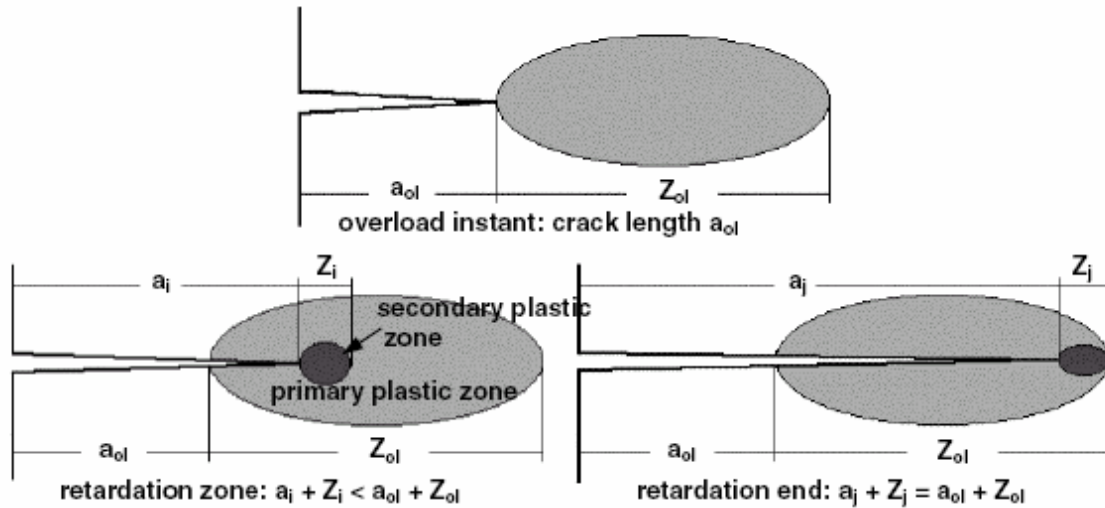


Figure 1: Plastic Zone Interaction (adapted from *Ebner et al, 2004*)

## 2.2 Willenborg Model

On the Willenborg model, (*Willenborg et al, 1971*) the retardation effect is considered directly on the  $R_{eff}$  function. An effective value of  $\Delta K$  is evaluated from the crack tip residual stress after the application of an overload. Once the  $\Delta K_{eff}$  is obtained it is used to evaluate an effective value of the stress ratio  $R$  ( $R_{eff}$ ). There is no need of empirical factors.

On this model the current SIF (Stress Intensity Factor) is reduced by a residual stress factor  $K_{RW}$ , evaluated by Eq. (5)

$$K_{RW} = K_{ol} \sqrt{\frac{(Z_{ol} + a_{ol} - a_i)}{Z_{ol}}} - K_{max} \quad (5)$$

where  $K_{ol}$  is defined as the maximum overload SIF and the other parameters are defined in Figure 1.

The plastic zone in this model is evaluated according to a plane stress state (Eq.6)

$$r_0 = \frac{1}{2\pi} \left( \frac{K}{\sigma_y} \right)^2 \quad (6)$$

The retardation effect is considered until the crack tip ( $a_i$ ) reaches the plastic zone limit ( $Z_{ol} + a_{ol}$ ), reducing the  $K_{RW}$  term to zero, canceling the SIF reduction effect. Both maximum and minimum SIF ( $K_{max}$  and  $K_{min}$ ) are affected by the  $K_{RW}$  correction, resulting in  $K_{eff,max}$  and  $K_{eff,min}$  which are used to evaluate  $R_{eff}$  (which for negative values is considered as 0, thus making  $\Delta K_{eff} = K_{eff,max}$ )

## 2.3 Generalized Willenborg (GW)

This model is based on the *Gallagher et al (1974)* modification of the Willenborg model. The main difference of this model is the substitution of  $K_{RW}$  parameter by a  $K_R^G$  parameter evaluated according to Eq. (7)

$$K_R^G = \Phi K_{RW} \quad (7)$$

where:

$$\Phi = \frac{1 - \frac{\Delta K_{th}}{\Delta K}}{(R_{SO} - 1)} \quad (8)$$

being  $R_{SO}$  the shut off value of the  $K_{max}^{ol} / K_{max}$  ratio, which can be experimentally adjusted. When the  $R_{SO}$  value is exceeded, there is crack arrest and  $K_{eff,max}$  is evaluated as Eq (9)

$$K_{eff,max} = \frac{\Delta K_{th}}{(1 - R)} \quad (9)$$

## 2.4 Modified Generalized Willenborg (MGW)

The MGW (NASGRO, 2002) is a modification of the GW model in order to take in account the underload effects on retardation caused by compressive loads (or even tension loads).

On the MGW the  $K_{eff,min}$  is evaluated after Eq.(10):

$$K_{eff,min} = \begin{cases} (K_{min} - K_R^G), & \text{if } K_{min} > K_R^G \\ 0 & , \text{ if } 0 < K_{min} \leq K_R^G \\ K_{min} & , \text{ if } K_{min} \leq 0 \end{cases} \quad (10)$$

As can be seen on Eq.(10),  $\Delta K_{eff}$  and  $R_{eff}$  are higher than the ones evaluated at the GW, therefore leading to a smaller retardation consideration.

However, the underload effects are only evaluated at the underload. In order to evaluate the underload effects after the underload the factor  $\Phi_{MGW}$  is used instead of the factor  $\Phi$  of the GW model and is defined at Eq.(11)

$$\Phi_{MGW} = \begin{cases} \min \left( 1, \frac{2,523\phi_0}{1 + 3,5(0,25 - R_{ul})^{0,6}} \right), & \text{if } R_{ul} < 0,25 \\ 1,0 & , \text{ if } R_{ul} > 0,25 \end{cases} \quad (11)$$

where  $R_{ul}$  is defined by Eq.(12):

$$R_{UL} = \frac{S_{UL}}{S_{OL}} \quad (12)$$

The factor  $\Phi_0$  is the factor  $\Phi_{MGW}$  at  $R_{ul}=0$  and is dependent on the material, and has to be experimentally evaluated. Values for  $\Phi_0$  can be found in the literature [NASGRO, 2002 and CASTRO *et al*, 2001] between 0.2 and 0.8.

The MGW model predicts reduction of retardation effects for loads up to 25% of the overload.

## 2.5 Strip Yield Model

The Strip Yield model (De Koning *et al*, 1997) is a mechanical model based on the hypothesis that a crack propagates through a plastic zone at the crack tip and the plastic deformation on this area will lead to load sequence effects such as retardation.

On the Strip Yield model,  $K_{op}$  is evaluated by a crack opening model based on the Strip Yield model (Dugdale *et al*, 1960)], modified to consider the plastic deformed material ahead of the crack. In this model all plastic deformation is considered to be in an infinitesimal strip along the crack path. The material within the strip is represented by a series of finite-width rigid-perfectly plastic bar elements.

The original Strip Yield model was only defined for a plane stress state, therefore in order to consider a more general stress state, the local yield stress ( $\sigma_y$ ) is multiplied by a restriction factor  $\alpha$  with values between 1.15 and 2.5 unless in compression cases where  $\alpha$  is considered as 1.0. The  $\alpha$  factor can be considered in two different approaches. The first one considers  $\alpha$  constant along the plastic zone elements and the second one considers a parabolic variation of  $\alpha$  along the plastic zone elements defined by finite element analysis. This present work considers the first approach, as all comparative analyses performed have shown no significant difference between the two hypotheses.

## 3. Comparison with Test Results

In order to evaluate the load sequence effects of a typical commercial aircraft spectra on a structural component made of a typical aeronautical alloy the retardation models have been evaluated using experimental data obtained from laboratory tests.

Two tests were performed using a MTS-810 test machine at CTA/IAE/ASA lab with a variable load spectra based on the MiniTWIST methodology (Lowak *et al*, 1979) with a reference stress of 31.2 Mpa in accordance with ASTM E647-002000). In order to reduce test time, the test frequency was set to 20Hz (this value was reduced to 1Hz at the compression cycles in order to avoid pin-slob).

The MiniTWIST load spectra is a short version of the TWIST (Jonge *et al*, 1973) load spectra which was developed in order to create a standardized load sequence for fatigue test programs. TWIST stands for **T**ransport **A**ircraft **W**ing **S**tructure and it based on the lower wing skin at the wing root due to its critical aspects. This methodology is based on the mean stress level in flight which was obtained by measuring several different commercial aircraft real wing loads over a wide range of weights, wing loads, cruising speeds and design flight distances.

The applied load spectra had 62.246 cycles divided in 10 blocks of 6.000 cycles (except for block 10 with 8.246 cycles). All blocks have been executed in sequence.

All measurements were made at an interval of 6.000 cycles until 170.000 cycles where the faster crack propagation required inspections intervals of 3.000 cycles. The crack was measured on both sides of the test specimen with a precision on 0,5mm (achieved by a magnifying glass and a graduated scale). All measurements were performed with test running in order to be able to see crack fully opened. Both sides of the crack have grown in a similar way.

The test specimens A and B were compact test specimen (C(T)) with Chevron notch, machined from Al 7050-T76511 alloy with the following dimensions, according to Figure 2:

- W = 80 mm
- D = 20 mm
- t = 5 mm
- $c_0$  = 18 mm

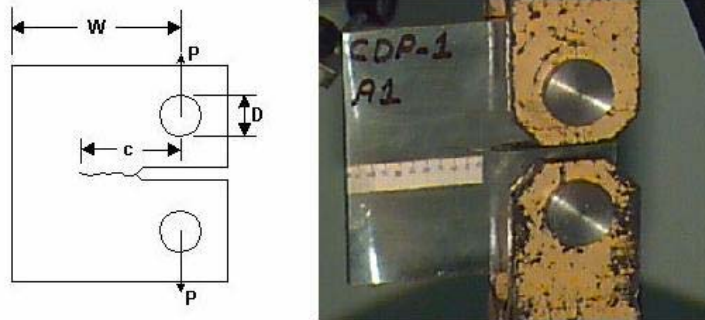


Figure 2: Test Specimen

The test results were evaluated using NASGRO software for all retardation models, except for the Wheeler model were the CRACK2000 (Mello *et al*, 1997) software has been used (this retardation model is not implemented at NASGRO). The results can be found on Figure 3, Figure 4 and Table 1.

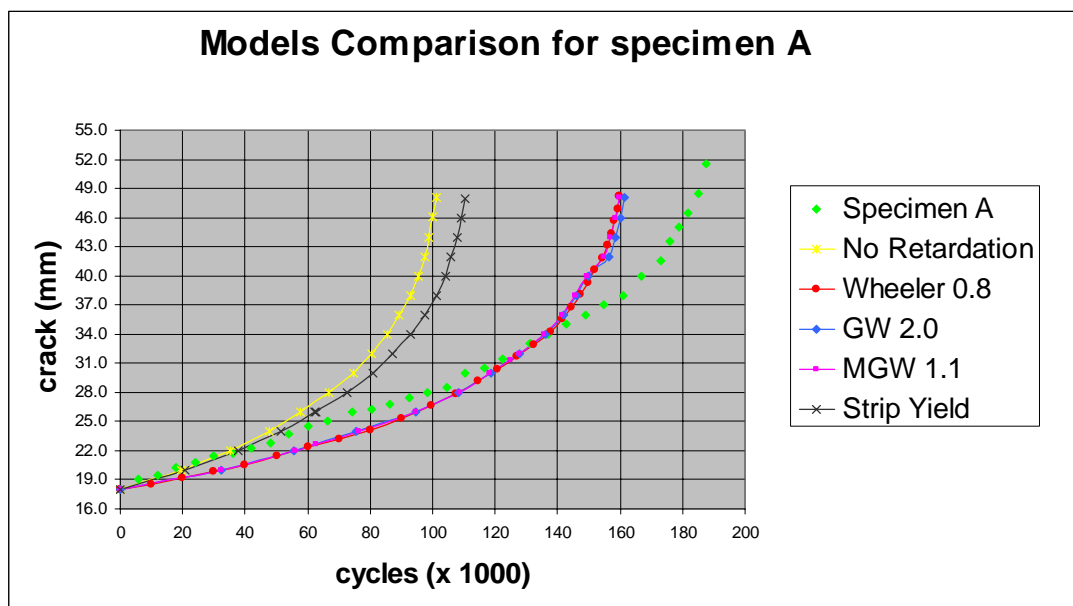


Figure 3: Models Comparison for Specimen A

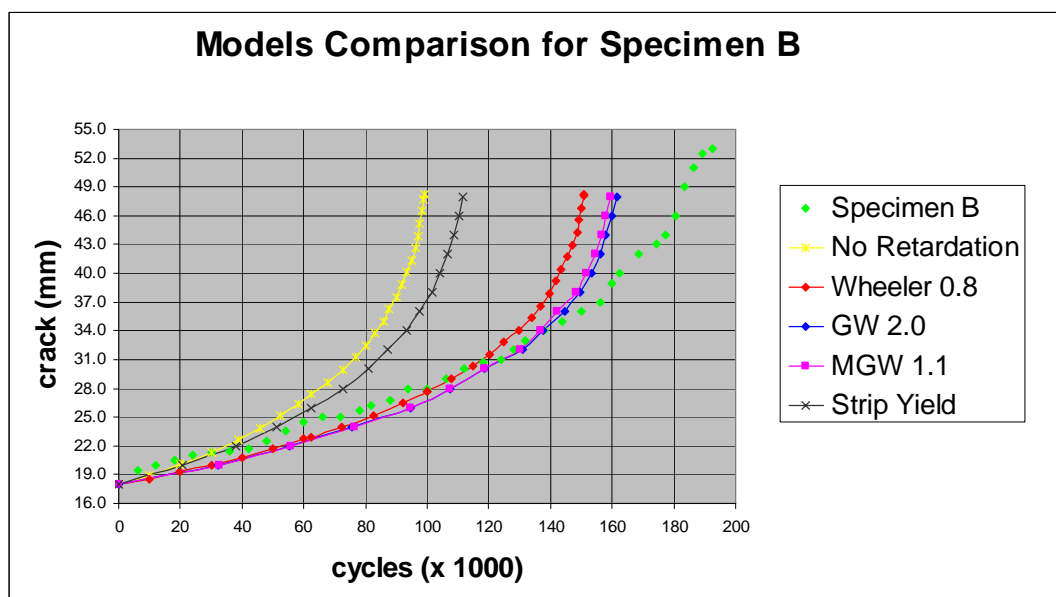


Figure 4: Models Comparison for Specimen B

**Table 1: Test Results summary**

	Test	no retardation	error (%)	Wheeler	error (%)
CDP A	184936	101153	45.30	159695	13.65
CDP B	183428	101299	44.77	150792	17.79
	Test	GW	error (%)	Strip Yield	error (%)
CDP A	184936	161470	12.69	110248.4	40.39
CDP B	183428	161564	11.92	111478	39.23
	Test	MGW	error (%)		
CDP A	184936	159657	13.67		
CDP B	183428	159391	13.10		

#### 4. Conclusions

From the results presented in Figure 3, Figure 4 and Table 1, the following conclusions can be made:

→ The Strip Yield which has no calibration parameter was found to be too conservative for the performed test conditions

→ The models based on the plastic zone ahead of the crack tip (GW, MGW and Wheeler) were found to be accurate in evaluating the retardation effects, having very similar propagation curves.

→ The underloads effects consideration performed by the MGW models do not show any significant difference regarding the other models (GW and Wheeler)

#### 5. References

- American Society for Testing and Materials, "Standard Test Method for Measurement of Fatigue Crack Growth Rates", ASTM E 647-00, United States, 2000
- BROT, A., MATIAS, C., "An Evaluation of Several Retardation Models For Crack Growth Prediction Under Spectrum Loading", 2002 USAF Aircraft Structural Integrity Program Conference, Georgia, 2002.
- BUNCH, J. O., TRAMMELL, R. T., TANOUYE, P. A., "Structural Life Analysis Methods Used on the B-2 Bomber", Northrop, Adv. Fatigue Lifetime Pred. Techn.: 3rd Vol., A96-26758 06-39, ASTM, pp. 220-247, 1996
- CASTRO, J. T. P, MEGGIOLARO, M. A., "Comparison of Load Interaction Models in Fatigue Crack Propagation", COBEM 2001
- DE KONING, A. U., TEN HOEVE, H. J., HENDRIKSEN, T. K., "The Description of Crack Growth on the Basis of the Strip-yield Model for Computation of Crack Opening Loads", the Crack Tip Stretch and Strain Rates, National Aerospace Laboratory (NLR) Report: NLR-TP-97511L, Netherlands, 1997.
- DUGDALE, D.S., "Yielding of Steel Sheets Containing Slits", Journal of Mechanics and Physics of Solids, Vol 8, 1960, pp. 100-104
- EBNER, E.C., "Efeitos da Seqüência de Carregamentos na Propagação de Trincas em Estruturas Aeronauticas Comerciais", ITA, Sao Paulo, 2004
- GALLAGHER, J.P., "A Generalized Development of Yield Zone Models", AFFDL-TM-74-28-FBR, Wright Patterson Air Force Laboratory, Dayton, Ohio, 1974
- JONGE, J.B., SCHÜTZ, D., LOWAK, H., SCHIJVE, J., "A Standardized Load Sequence for Flight simulation Tests on Transport Aircraft Wing Structures", National Aerospace Laboratory (NLR), Report : NLR TR 73029 U, Netherlands, 1973

- LOWAK, H., JONGUE, J. B., FRANZ, J., SCHÜTZ, D., “MINITWIST – A Shortened Version of Twist”, National Aerospace Laboratory (NLR), Report : NLR MP 79018 U, Netherlands, 1979.
- McCLUNG, R. C., McMASTER, F.J., FEIGER, J. H., “Comparison of Analytical Crack Closure Models and Experimental Results Under Flight Spectrum Loading”, ASTM/SF2M Symposium on Fatigue Testing and Analysis Under Variable Amplitude Loading Conditions, Tours, 2002
- MELLO, Jr. A., “Crack 2000 Program: Software for Practical Fracture Mechanics and Damage Tolerance Analysis”, CTA, São José dos Campos, SP – Brasil, 1997
- NASGRO – “Fracture Mechanics and Fatigue Crack Growth Analysis Software”, Reference Manual, Southwest Research Institute, San Antonio, Texas, May, 2002
- WALKER, K., “The Effect of Stress Ratio During Crack Propagation and Fatigue for 2024-T3 and 7075-T6 Aluminum”, ASTM STP 462, American Society for Testing and Materials, Philadelphia, 1970
- WHEELER, O.E., “Spectrum Loading and Crack Growth”, J. of Basic Engineering, pp.181-186, 1972
- WILLENBORG, J., ENGLE, R. M., WOOD, H. A., “Crack Growth Retardation Model Using an Effective Stress Concept”, Air Force Flight Dyn. Lab., Wright Patterson Air Force Base, Dayton, Ohio, TM 71-1-FBR
- ZHANG, S., MARISSSEN, R., SCHULTE, K., TRAUTMANN, K. K., NOWACK, H., SCHIJVE, J., “Crack Propagation Studies on Al 7475 on the Basis of Constant Amplitude and Selective Variable Amplitude Loading Histories”, Fatigue Fract. Engng. Mater. Struct. Vol. 10, No. 4, pp. 315-332, 1987

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## **7. Responsibility notice**

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