ENHANCING TIME-FREQUENCY REPRESENTATIONS FOR FAULT ANALYSIS IN GEARED SYSTEMS

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Abstract. The knowledgement of the fault development phenomenon in geared systems has been considered an important task not only in the design project but also in the maintenance of it. Usually, the vibration signal generated by these systems are multi-component signal and not stationary signal due to transient forces that appears from the superficial contact between the teeth and some type of imperfection, which is developed in the form of vibrations that excite several resonance frequencies of the system. The classic spectral analysis applied to the fault detection has shown limitations, especially concerning in the appearance of spurious components that can contaminate the spectrum and difficult the interpretation. To minimize these problems, this work investigates two time-frequency representations, which can supply a new discernment about the interpretation of these signals, evidencing the physical phenomenon caused by transient vibrations. The aim of this work is to establish some parameters that allow comparing, in the time-frequency representation, the Spectrogram and the Pseudo Wigner-Ville to apply in the fault analysis in gear systems and also to enhance the fault signal due to the residual signals. Some experimental tests, using three different conditions, are performed with these gear signals: toothless gear, scratched gear and normal gear. It was verified that the Pseudo Wigner-Ville technique is a powerful tool to analyze faults in geared systems.

Keywords: Joint Time-Frequency Representations, Fault Analysis, Vibration Signals, Geared Systems.

1. Introduction

The main fault in geared systems is usually related with problems that appear in the tooth, in the axis, or combination of these. The fault concerning the tooth can be originated from wear, scratch and crack and concerning the axis, the fault is primarily due to the misalignment and the unbalanced of it. Currently, the use of condition monitoring to study faults in machines is showing to be an important tool, especially when it uses physical parameters such as vibration, acoustic, electric and thermal signals. However, some specialists consider the vibration signal parameter as the most representative for detecting faults (Staszewski, Worden and Tomlinson, 1997). From all the rotative machines present in the industry, the geared system is the most important, especially in the automotive and aeronautic industries. The monitoring of these machines can be done analyzing the signal vibration just in the first step of the project (quality control), as mentioned by Menegatti and Duarte (1999), or in the maintenance of it (condition monitoring) in according to Lima (1985) and McFadden (2000).

The present work focuses a brief bibliographical revision of detection techniques and a diagnosis of faults in gears. It is being discussed the characterization of the models using the vibration signal parameter in a pair of normal gear, the spectral components concerning meshing frequency and its harmonics, the lateral bands, the spurious components and its sources, and finally, the characteristics of the signals modified by the introduction of defects such as cracks, pitting, etc. that produce a more complex vibration signal. Sequentially, it is applied two join time-frequency representations, a Spectrogram and a Pseudo Wigner-Ville, to analyze both computational and experimental signal vibration with some typical faults in order to find the viability of this technique to diagnose faults in geared systems.

2. Faults in geared systems

In the process of converting the applied force in useful work, appears impure forces due to geometric imperfections and bad contact between surfaces in movement, which generate vibrations and noises that allow analyzing the condition machine. The main imperfection when operating a gear is the profile deviation of the original tooth and the source of this deflection is due to static and dynamic forces and an error in the geometric design of the tooth, which is accentuated by the wear process.

- Load Effects: The tooth deflection by result the static and dynamic load reproduce a vibration signal with characteristics of stepped nature, it occur from the flexibility fluctuation in the time between meshing tooth pairs. In

according to Randall (1982), its variation will be large for spur gears, and too much large for helical gears. For this reason, were verified that is possible to identify the effect of the load from vibration signal, observing the frequency components such as, the meshing frequency and its harmonics for each pair of geared tooth.

- Machining Errors Fault's machine can generate two groups of vibratory components: the first group indicates the meshing frequency and its harmonics of the whole theeth and the second group, involving only individual tooth, shows ghost components or random variations and normally they have a low spectrum level spread over a large number of harmonics.
- Uniform Wear: Frequencies components caused by wear were explained properly by Randall (1982). If the wear were considered uniform for all the teeth, this fault will appear in meshing frequency and its harmonics, however in smaller amplitude than the effects caused from deflection of the tooth, that occur in same components of frequency. However an appreciable wear can to change the meshing frequency much more than the deflection by the load, in this case, the effect of wear will be more pronounced in the higher harmonics of the meshing frequency.
- **Amplitude Modulation Effect**: The vibration signal from geared system, stated here, were constituted of meshing frequencies, its harmonics and spurious frequencies. In fact, the load on the tooth can to fluctuate, if it varies, the amplitude from signal change, resulting in a amplitude modulation. The most common sources that generate amplitude modulation is eccentricity of gear related with the rotation speed (frequency modulated) this gear around the meshing frequency (modulating frequency).
- Frequency Modulation Effect: Situations where the rotation frequency is variable and the spacing among the teeth not uniforms, occur the frequency modulation. Actually, when occur increase the modulation by variation the contact pressure among the teeth, exists simultaneous the variation the torque, consequently, flotation the angular speed, that provokes frequency modulation. It appear into spectrum with increased amplitude the families of lateral bands and equal spacing the frequency modulating, the same frequency that induze the amplitude modulation (Randall, 1982).
- Damaged teeth: Punctual faults present at the teeth of gears are: pitting, scratched teeth and broken teeth. They cause a located loss of stiffness the teeth, causing increase the phase modulation, and also the amplitude modulation, during the period of meshing the damaged tooth, they are evident when appear a increase of lateral bands in spectrum. When these defects are of great extension, an abrupt change in the force on the tooth occurs, that can to excite some resonance frequencies of the system (Arato Jr. and Silva, 2001).

3. Faults detection techniques of geared systems for vibration analysis

3.1. Methods of Analysis in Time and Frequency Domain

The techniques on time domain signal process from mechanical systems and return a simple value, indicating the state of "health" the component. The general indicartor commonly used are: RMS Value, Pick Value, Kurtose and Factor K (equal to multiplication RMS Value and Kurtose (Silva, 1999). Specific indicators are deliberate by James and Limmer (2000), as such FMO, FM4 values and its hybrid, NA4. FMO value is result the division Pick Value and adds the quadratic averages harmonics the meshing frequency. FM4 value is equal Kurtose from the signal, after removed harmonics the meshing frequency and first order side bands. NA4 value is equal Kurtose from the signal, removed only the frequencies related to harmonics the meshing frequency, and the Kurtose is calculated for an average updated in the time.

The technique on frequency domain is based in application of concepts the Fourier Transform, modified under the implementations of FFT algorithms form. It analysis consists in comparing current spectra and spectra that characterizes the system without defect or reference. But, conform several searching its difficult to detection incipient faults, because the faults in mechanical systems usually occur in low frequencies and can be easily polluted for frequencies from signals of other machines and noises of low frequency that will be present in the measure, according to Lima (1985) and Silva (1999).

3.2. Methods of Joint Time-Frequency Analysis

The time-frequency representations (TRFs) provide some temporal information and some spectral information, simultaneously. Thus, TFRs are used for the analysis of signals containing multiple time-varying frequencies.

One form of TFR can be formulated by the multiplicative comparison of a signal with itself, expanded in different directions about each point in time. Such formulations are known as quadratic TFRs because the representation is quadratic in the signal. This formulation was first described by Eugene Wigner in 1932 in the context of quantum mechanics and, later, reformulated as a general TFR by Ville in 1948 to form what is now known as the Wigner-Ville distribution. Unfortunately, although quadratic TFRs offer perfect temporal and spectral resolutions simultaneously, the quadratic nature of the transforms creates cross-terms whenever multiple frequencies are superimposed. This was partly addressed by the development of the Choi-Williams distribution in 1989 but most recent applications of TFRs have turned to linear methods.

Currently, the distributions are being developed as a result of the choice of a arbitrary function called kernel, that carry the distribution to attend restrictions. Starting from a general equation deduced by Cohen, new distributions were created with particulars properties, such as the Choi-Williams, Zao-atlas-Marks and Cohen-Posch distributions. New distributions there are surging, using the concepts of linear functions families as it is the case Wavelets transform, that allows an analysis multi-resolution (Boashash, 1992).

The main time-frequency distributions that will be analyzed, can be calculate in the following way:

- Short Time Fourier Transform (STFT): In the continuous case, a window function, which is nonzero for only a short period of time, is multiplied by the function to be transformed. The Fourier transform of the resulting signal is taken as the window is slid along the time axis, resulting in a two-dimensional representation of the signal. Mathematically, this is written as:

$$STFT[t,w] = \int_{-\infty}^{\infty} x(\tau(*w(\tau-t)e^{-jw\tau}d\tau)) d\tau$$
 (1)

where w(t) is the window function, commonly a lone cosinusoid or gaussian "hill" centered around zero, and x(t) is the signal to be transformed. STFT(t,w) is then a complex function representing the phase and magnitude of the signal over time and frequency.

In the discrete time case, the data to be transformed is broken up into chunks (which usually overlap each other). Each chunk is Fourier transformed, and the complex result is added to a matrix, which records magnitude and phase for each point in time and frequency. This can be written as:

$$STFT[n,w] = \sum_{m=-\infty}^{\infty} x[n+m]w[m]e^{-jwm}$$
(2)

likewise, with signal x[n] and window w[n]. In this case, n is discrete and omega is continuous, but in most typical applications the STFT is performed on a computer using the Fast Fourier Transform

- Wigner-Ville Distribution (WVD): The Wigner-Ville distribution is expressed for:

$$W[t, f] = \int_{-\infty}^{\infty} e^{-j2\pi f t} s^* (t - \frac{1}{2}\tau) s(t + \frac{1}{2}\tau) d\tau$$
(3)

where s^* is the complex conjugated from signal in time s(t). Wang and McFadden (1993) evaluated that WVD and it not shown very efficient for detection faults in geared systems, because the signal have characteristics multicomponents which appearance in the interference terms caused by bilineary this distribution, presenting difficulty in interpretation the time-frequency representations.

- Pseudo-Wigner-Ville Distribution (PWVD): For to suppress the crossed terms in WVD, two methods are used. The first, consist in application of a window slipping in the time domain, before to calculate to WVD. The second method, consist in combing the WVD for a moving function in the time-frequency plan, using an exponential window the Gaussian type, as proposed by Shin and Jeon (1993):

$$G(t,\omega) = \frac{1}{2\pi\sigma_{t}\sigma_{\omega}} e^{\left[t^{2}/2\sigma_{t}^{2}\right)+(\omega^{2}/2\sigma_{\omega}^{2})\right]}$$

$$W[t,f] = \int_{-\infty}^{\infty} e^{-j2\pi/t} s^{*} \left(t - \frac{1}{2}\tau\right) s(t + \frac{1}{2}\tau) d\tau$$

$$(4)$$

Where $\sigma_{_{l}}$, $\sigma_{_{o}} > 0$ e $\sigma_{_{l}}$, $\sigma_{_{o}} \ge 1/2$, are parameters related to time and frequency resolutions. In both cases, are obtained the PWVD, that seeks to reduce interferences and to avoid the negative values. Staszewski, Worden and Tomlinson (1997) has used PWVD combing with a Hamming window over the signal in time and observed that this, increase the capacity the distribution in faults detection in spur gear, because offer an significant reduction of the interferences terms.

4. Models of vibration signals for geared systems

The models presented in this work are based on the description by Arato Jr. and Silva (2001), that consider a total signal equal to adds the following components:

- Speed Rotation Frequency: It can be represented by an senoidal excitation with frequency equal to speed the axis of rotation, f_r . The value Y_a is amplitude, and the component $Y_g(t)$ is given by expression:

$$Y_{g}(t) = Y_{a} \operatorname{sen}(2\pi f_{r}t) \tag{5}$$

- Meshing Frequency: It is represented by senoidal excitation of frequency equal the meshing frequency, determined by product the rotation frequency and number of teeth of the gear, $f_e = Nf_r$. The signal that represents this component $Y_e(t)$ of amplitude Y_b is given for:

$$Y_e = Y_b sen(2\pi f_e t) \tag{6}$$

- Harmonics of the Meshing Frequency: The vibration signal that represents the relative deviations of the contour the tooth, can be given by $Y_h(t)$, where Y_{bn} is the amplitude of the n-th harmonic, n represents the harmonics number e ϕ_n the phase angle among signals:

$$Y_h(t) = \sum_{n=0}^{N} Y_{bn} \operatorname{sen}(2\pi n f_e t + \varphi_n)$$
(7)

It is know as the sum to normal components presents from vibration signal of geared systems, and they can to acentuate when to exist faults, as example, punctual faults in teeth. This can be modeled according with the expressions:

- Faults in gear teeth: These faults appear in the tooth reducing its stiffness. They provoke located modulation causing increase of lateral bands. When this defect is great, occur abrupt changes of the force on the tooth, and can to excite naturals frequencies from system, according McFadden (1992) apud Arato Jr. and Silva (2001). This phenomenon, can be modeled via a series pulses of equal amplitude, and the repetition period $T_r = 1/f_r$, that multiplied by pulses decreasing exponentially, presents a response defined for Imp(t). Such signal multiplied by a senoide that carry the resonance frequency from system Ω , is modulated by a frequency equal to rotation frequency the gear possessing faults, and can be represented:

$$Y_n(t) = Im \, p(t) \cdot \left[Y_o sen(2 \, \pi \Omega t) \right] \tag{8}$$

Considering the complex signal, containing all the components calculate previously, the vibration signal of pair geared can be expressed by sum of all the components showed, that are:

$$Y_{t}(t) = Y_{q}(t) + Y_{p}(t) + Y_{h}(t) + Y_{h}(t)$$
(9)

In order, to illustrate a simulated signal and to compare with experimental signal and finally to apply the join time-frequency methods, the Fig. 1 display a signal given by sum of several components showed above. In this case, the rotation frequency axis was assumed to 13. 33 Hz, similar the experimental rotation frequency of the system, the meshing frequency component f_c , equal is 413.33 Hz and its harmonics (826.6, 1240, 1653.33, 2066.66 e 2480 Hz) add a component that model a punctual fault on a tooth of the gear, exciting to resonant frequency from geared system, suche the frequency is 1280 Hz. The Fig. 1 illustrates the modeled signal contained the characteristics of each components determined above, in time and frequency domain, added its 40% of white noise. And can verified the combination of the several effects, obtained a vibration signal relatively complex and offer difficult in the interpretation via conventional methods, such as the visualization of periodic impacts provoked by punctual fault on the tooth, so verified the necessity to appeal to more robust analysis methods, such as, the join time-frequency methods (Silva and Irmão, 2004).

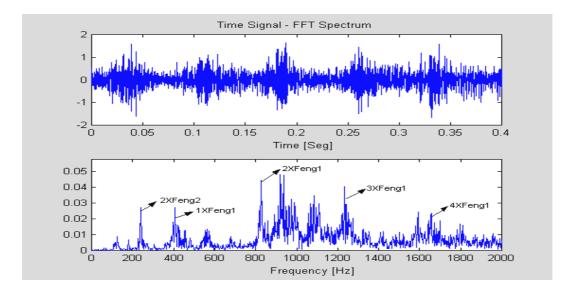


Figure 1. Signal in Time and Frequency Domain, modeling the normal components (Meshing Frequency and its Harmonics, Rotation Frequency) and punctual fault on the tooth of the gear, added 40% of white noise

5. Application of the time-frequency representations to simulated signals

The Figs. 2 e 3 exhibit the Spectrogram and Pseudo Wigner-Ville Distribution maps, respectively, showing the modeled signal, that receive the normal components, add the punctual fault on the tooth and 40% of white noise.

The figure are constituted of three graphs, distributed the following way: In horizontal, the signal in time domain, in the vertical position and left side of the figure be the spectrum signal, and in the center time-frequency map extracted in the superior view of the three-dimensional graph, the coordinates are the time axis, the frequency axis and the energy axis of the signal represented by the bar of colors (to right), which is related to energy signal.

Analyzing the maps of the Figs. 2 and 3, was observed on the frequency axis that the meshing frequency (413.33 Hz) and its harmonics (826.66, 1240, 1653.33, 2066.66 e 2480 Hz), as well the rotation frequency (13.33 Hz) are low visible, because the low frequency resolution in both maps.

On the times axis, was observed energy spots, equally spaced around the frequency of 1280 Hz, the frequency of resonance from system, allowing to visualize impacts equally spaced in the time of approximately 0.075 s, related to rotation frequency (13.33 Hz).

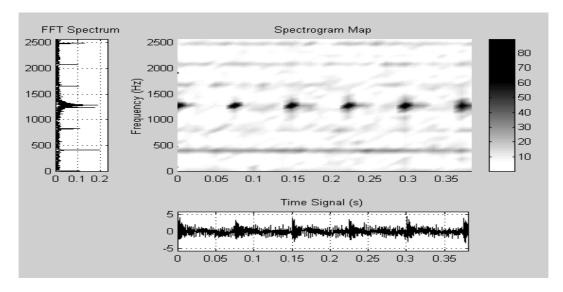


Figure 2. Spectrogram of the modeled signal (Punctual fault on tooth of the gear)

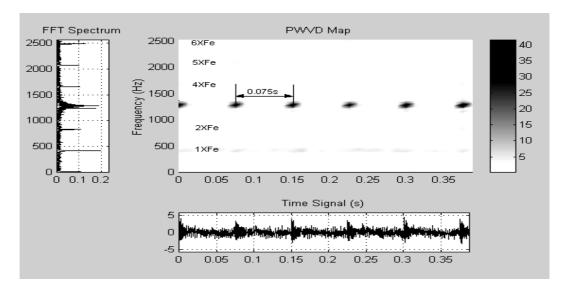


Figure 3. PWVD of the modeled signal (Punctual fault on tooth of the gear)

Establishing a comparative among the STFT and PWVD representations, was observed that neither notify the distanced of lateral bands similar multiples of the rotation frequency (13.33 Hz), due to adopted resolution. However, the maps PWVD were more cleaner in relation to maps STFT and possesses the content of energy, more accentuated in the fault frequency, allowing to analyze the signal with more precision. The PWVD method showed larger performance than STFT for this simulate signal.

6. Application of the time-frequency representations to experimental signals

The experimental setup is deliberate following, via its was realized to validate the models of faults in gears and obtained of a database produced in USP/LADIN. The data bench of tests was built with the objective of accomplishing acquisitions of vibration signals in gears subject to three types of conditions, normal, toothless and scratched, and starting these signals to evaluate the detection capacity, quantification and sensibility of the time-frequency techniques (Padovese, 2002).

The gearbox is constituted of two reductions, a relationship of reduction of 5.32. The gears for test are three different conditions: normal, with punctual fault and with extensive fault, these last represented by a gear lacking a tooth (gear toothless) and the other with 10 (ten) teeths consecutive scratched severely (scratched gear), respectively. All defects were artificially induced in the pinion gear z1.

The data acquisition was made through an accelerometer B&K 4393 coupled with a load amplifier B&K 2535, filtrated in 2 kHz and fragmented in 2048 points, by acquisition system linked to a computer for a sample frequency of 5.12 kHz. For each defect pattern was made the measure of the vibration signal in six different rotations: 400, 600, 800, 1000, 1200 and 1400 rpm. The signals were measured for the system without load and for load of 60% of the maximum nominal value, that is, 8.4 N.m.

Between the experimental signals acquired for the defect of scratched gear, those to rotations of 800 rpm were choose for application the time-frequency methods. They were choice due to have showed clearly the dynamics characteristics of the vibration signal for this gear condition. The Fig. 4 displays this signal in time domain and its spectrum.

The Table 1 show the frequencies used for the analysis of the signal and its respectivev abbreviations:

- FRE: Input Axis Rotation Frequency;
- FRI: Intermediate Axis Rotation Frequency;
- FRS: Output Axis Rotation Frequency;
- Feng1: First Reduction Meshing Frequency;
- Feng2: Second Reduction Meshing Frequency;

	Frequencies to 800 rpm	
	RPM	Hz
FRE	800	13.3
FRI	451	7.5
FRS	126.6	2.1
Feng1	24800	413.3
Feng2	7215	120.3

Table 1 – Meshing and Rotation Frequency for the Reducer to 800 rpm

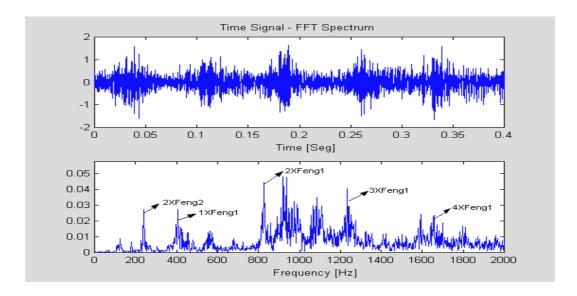


Figure 4. Experimental signal in the time and frequency domain, for scratched gear to rotation frequency of 800rpm

Observing the Fig. 4, verify that for a geared system relatively simple, the experimental signals can to generate a complex signal of difficult interpretation via conventional methods. By the analysis of the spectrum is possible to identify some meshing frequencies from gear damaged, however this analysis is difficult to identify some frequency that have relationship with the scratched frequency, for this is necessary an more robust method for analysis these signals. An more accurate analysis was obtained via time-frequency maps for the same signals.

The time-frequency maps were obtained using programming in Matlab environment. For the calculate of the Espectrogram, the toolbox of processing of signals was used. The number of points by window was of 64, and sampling frequency of 5120 Hz, corresponding a frequency resolution of 80 Hz for Espectrogram and 40 Hz for the distribution Pseudo Wigner-Ville.

The Figs. 5 and 6 shows Spectrograms and the Pseudo Wigner-Ville distribution obtained on the signal of pinion gear with defect of scratched in frequency rotation of 800 rpm.

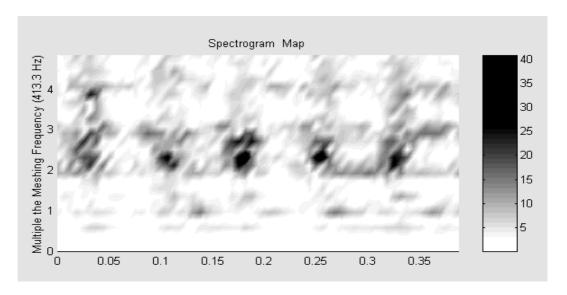


Figure 5. STFT map for the signal of the scratched gear (800 rpm)

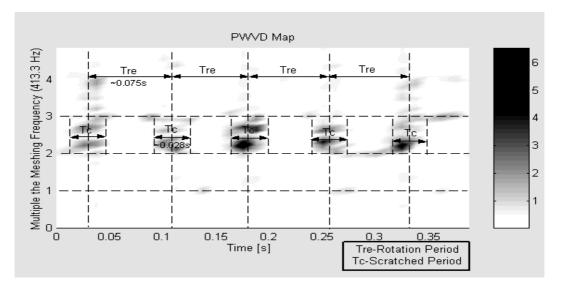


Figure 6. PWVD map for the signal of the scratched gear (800 rpm)

Observing the STFT representation map is verified that it show the principal components of frequency the system, its too show various stains which difficult the analysis, this certainly aggravated by the low resolution in axis frequency. However, an better definition is possible to see through the PWVD maps, corresponding show the Fig. 6, where is verified the superiority of the PWVD map, because evidences the frequencies related with the first and second harmonics of the meshing frequency the pinion gear.

Accomplishing the PWVD map analysis, was easily perceived that some periodic phenomenon exists in the signal, visualized through regular energy spots in time-frequency plan. It was observed that this periodicity corresponds to one rotation period of the input axis and therefore the rotation frequency of the damage gear. The meshing frequencies and other frequencies can be modulated predominantly by the input rotation frequency, this denounces that the defect exists on the frequencies components that operates in this rotation frequency, indicating which the scratched faults is located on the pinion gear. As the scratched defect consists of pronounced risks about ten teeth, appears in the signal an typical frequency of this defect, call of scratched frequency, correspond to inverse of the time that the gears meshing with ten defective teeth. In this case, the scratched frequency is 413.3/10, that is equal to 41.3 Hz. The scratched component appear in time-frequency map in form of vertical spots of almost constant thickness, and its thickness defines the scratched time, and therefore the scratched frequency for its rotation. When was measured this interval of time in the map resuted in approximately 0.028 s, which is related gives the frequency of approximately 36 Hz. Notice that this frequency is about 13% unless 41.3 Hz, but is important remember that this value is theoretical, and in practice, can occur small deviations, such as the instantaneous variation of the rotation, due the brake system coupled to gearbox (ex. attrition increase in the contact surface).

7. Conclusions

In this work, the distributions STFT and PWVD were applied for some types of simulate and experimental signals of geared systems could be verified that both allowed identifying the principal components characteristics of defects (rotation frequency, meshing frequency and its harmonics).

Comparing the time-frequency methods, was observed that the results obtained with experimental signals reproduce the behavior of the simulate signals illustrated from the models. The Espectrogram, as was verified present limitations due to its frequency resolution, complicated the identification of some components, and can confuse the analyst in the identification of other components.

By deliberated, the Pseudo Wigner-Ville Distribution (PWVD) present more cleaner maps and containing the basic components that interest the analysis. It presents a advantage obove the Espectrogram (STFT) in the resolution, which corresponds to the double of the STFT resolution, allowing to identify of form more precision some components and mainly transients, always presents in the signals from systems geared.

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