INDICIAL RESPONSE OF THIN WINGS IN A COMPRESSIBLE SUBSONIC FLOW

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Abstract. *A numerical method to compute the indicial response of a wing in a compressible subsonic flow is developed. The indicial response computed is the variation of the aerodynamic coefficients after a sudden change in the angle of attack of the wing. The method is a vortex lattice method, where the unsteady effects are computed by the intermittent emission of free vortex in the vortex sheet of the wing. The obtained results are restricted to thin wings and small angles of attack, because a linearized mathematic model is used. The effects of the Mach number, aspect ratio and sweep angle in the indicial response are studied. The results have good agreement with similar ones found in previous works. The method can be also applied to study arbitrary movements of the wing, using the superposition principle and the integral of Duhamel, due to the linearity of the system.*

Keywords*: unsteady aerodynamics, gusts, indicial response, vortex lattice, compressible potential flow*

1. Introduction

The aeroelastic phenomena have great effects in the design of modern aircrafts. They can be divided in two principal groups: the static phenomena and the dynamic ones. In the analysis of the static aeroelastic phenomena, transient effects are negligible. Thus, it covers only the interaction between the elasticity of the structure and steady aerodynamics effects. However, structural vibrations occurs in the dynamic aeroelastic phenomena, and, thus, its analysis require a reasonable dynamic modeling of the structural, inertial and unsteady aerodynamic forces. The phenomena of flutter, dynamic response to gusts and blade vortex interaction in helicopter rotors are some examples of important dynamic aeroelastic phenomena that have great influence in the structural design of aircraft wings. As shown, accurate models of the unsteady aerodynamics are necessary to a good analysis of these problems. An experimental dealing with unsteady aerodynamics modeling is very complex and expensive, being almost impracticable in the study of aerodynamic loads due to arbitrary vibrations of lifting surfaces. Thus theoretical analysis is required.

Theoretical computation of unsteady aerodynamic loads due to arbitrary vibrations of a lifting surface can be done through the composition of the response to more simple movements. This kind of dealing is possible only if the principle of superposition is valid, as in linear systems, for example. If the mathematical model is linear, according to Bisplinghoff, Ashley and Halfman (1955), it is possible to compute the aerodynamic response to arbitrary movements, using the response for harmonic oscillations of the wing and the Fourier integral. This approach is useful, because the problem of harmonic oscillations is simpler to solve. Another alternative, which also uses the superposition principle, is to use the obtained results for the indicial response and the Duhamel integral. The indicial response is the transient of the aerodynamic loads after an abrupt step change in the angle of attack of the lifting surface. The indicial methodology is more suitable than the harmonic one in the modeling of more abrupt movements, such as the response to vertical gusts or rapid deflections of control surfaces. This occurs because in these cases the harmonic methodology has a slow convergence.

The present work presents a numerical method to compute the indicial response of plane wings submitted to a compressible subsonic flow. The scheme is a vortex lattice method, where the unsteady effects are computed by the intermittent emission of free vortex, generating the vortex sheet of the wing. In the development of the mathematical model used, the assumptions of inviscid, irrotational and isentropic flow are done. Besides that, the equations are linearized. Thus the model is valid only for small perturbations (low angles of attack and low thickness of the wing profiles). These assumptions are reasonable for the kind of practical application the method is proposed.

The first theoretical studies in unsteady aerodynamics appeared during the 20s and 30s decades. Wagner (1925), Kussner (1936) and Theodorsen (1935) developed the classical analytical results for the indicial response, sharp edge gust penetration and harmonic response, respectively, of airfoils in incompressible flow. Since those years, many analytical and numeric solutions, harmonic or indicial, were developed. Analytical solutions, however, are too complex and restricted for a few cases, in general. Thus, the search for numeric solutions, more fast and general, becomes attractive. Heaslet and Lomax (1949) and Beddoes (1984) presented results for the indicial response of airfoils in supersonic and subsonic flows, respectively. Jones (1940) and Lomax *et al.* (1952) presented results for the indicial response of finite wings in incompressible and supersonic flows, respectively. There are very few works dealing with finite wings in compressible subsonic flows, as the present one. Vepa (1977) presents results for this condition, using a finite state modeling method. This work was used to validate the results obtained using the present methodology. Singh and Baeder (1997) and Sitaraman and Baeder (2004) compute the same thing using CFD codes.

The generalized vortex lattice method was developed to harmonic oscillations of finite wings in subsonic (Soviero and Bortolus, 1992), supersonic (Soviero and Resende, 1997) and transonic (Soviero and César, 2001) flows. It was also used in the computation of the indicial response of airfoils in subsonic and supersonic flows (Hernandes and Soviero, 2003) and finite wings in incompressible flow (Miranda and Soviero, 2004). In the present work, it is extended to the computation of the indicial response of finite wings in compressible subsonic flow.

2. Problem description

The problem consists in a plane finite wing (no thickness) submitted to a compressible subsonic flow of freestream velocity U. Initially, this wing is with zero angle of attack. In a given instant, it suffers an abrupt step change in the angle of attack, which becomes a non-zero finite value (in the present work, as said, it is considered that the angle of attack variation is small enough to maintain the wing C_L, lift coefficient, in the linear region). This problem, named indicial response, or response to a step change of angle of attack, consists in determine the time evolution of the aerodynamic coefficients of the wing (in the present work just the lifting coefficient is computed), from the perturbation instant to the development of the steady flow. Figure 1, shown below, illustrates the problem.

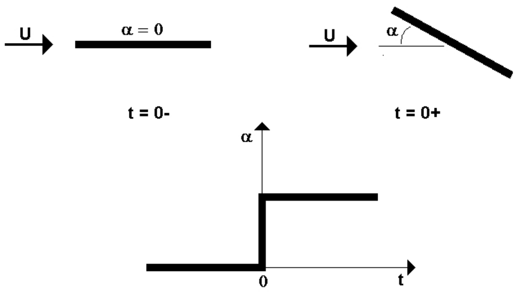

Figure 1. Illustration of a step change in the angle of attack of a plane wing.

3. Mathematical model

As specified in the introduction, the present work considers an inviscid, irrotational and isentropic flow. Besides that, it considers small perturbations of the flow (low angles of attack and low thickness of the wing profiles). Thus, the linear theory and the superposition principle can be used. With these assumptions, the classical linearized equation of the compressible and unsteady velocity potential can be used to compute the velocity field over the wing.

$$(1-M^2)\phi_{xx}+\phi_{yy}+\phi_{zz}-\frac{2M}{a}\phi_{xt}-\frac{1}{a^2}\phi_{tt}=0 \tag{1}$$

The problem can be modeled numerically using the singularities of the classical aerodynamics (in the present work are used the doublet and vortex singularities). The boundary condition is the tangency of the flow over the wing on every instant of time. That is, the composition of the non-disturbed flow with the induced flow by the potential jump generated at the wing after the alteration of the angle of attack must produce a velocity field such that the velocity component normal to the wing is null in every point of the wing and every instant of time. Thus, calling of W_P, the normal component of the induced velocity in a point P of the wing by the potential jump generated (positive direction is downward) and considering sin $\alpha \cong \alpha$ (small angle of attack), it follows that:

$$U\alpha = W_P \tag{2}$$

Equation 2 must be satisfied in every instant of time and every point P over the wing surface. The following equation is valid in the linear compressible theory to compute the ΔC_P (difference of the pressure coefficient between the lower and the upper sides of the wing):

$$\Delta C_P = -\frac{2}{U^2}\left(\phi_t + U\,\phi_x\right) \tag{3}$$

The ϕ_t term is the non-circulatory term and the ϕ_x term is the circulatory one. Another concept that is one of the pillars of the numeric model here used is the Kelvin Theorem, which guarantees that any potential jump, $\delta\phi$, generated over the wing, remains constant when it is dragged with the freestream velocity to the vortex sheet of the wing.

4. Numerical model

The numerical method proposed is based on three well known concepts of the theoretical aerodynamics: the impulsive generation of circulation in a perfect fluid, the correspondence between a superficial distribution of normal doublets and a linear distribution of vortex and the emission of free vortex, creating the vortex sheet of the wing. The wingspan and chord are discretized in rectangular panels. Each panel has a control point in its centre, where the boundary condition given in Eq. 2 must be satisfied in every instant of time. In the initial instant (t=0), the step change in the angle of attack occurs. In this moment, it occurs an impulsive generation of potential jump, normal to the wing, of intensity $\delta\phi_n^0$ (the superscript 0 indicates that the superficial distribution of normal doublets was created in the instant t=0 and the subscript n indicates that it is related to the panel n). Figure 2 shows the doublets distribution, in the initial instant, for a rectangular and a swept wing. Both wings are discretized with four panels in the semi-span and two panels in the chord. The symmetry between the two semi-spans is taken in account, to reduce the computational cost.

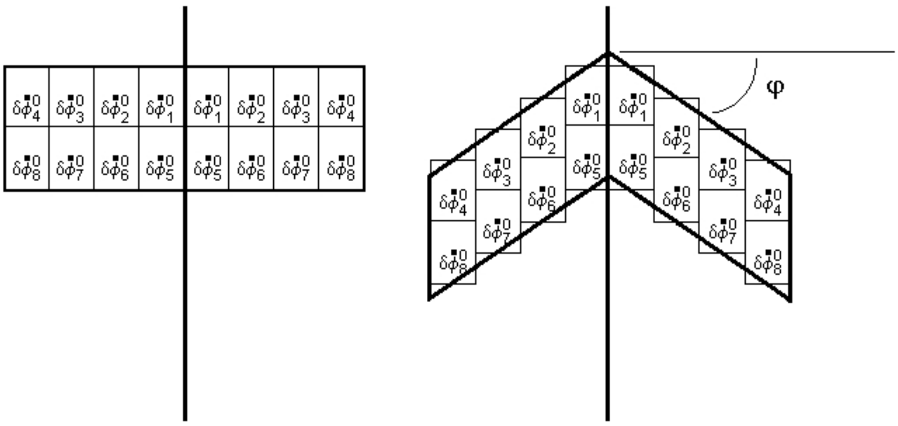

Figure 2. Rectangular and swept wings with discretization in two panels of chord and four panels of semi span.

Just after the impulsive generation of the normal doublets distribution, it is substituted by a linear distribution of vortex in the boundaries of each panel, in the instant t=0+, of intensity $\delta\Gamma_n^0$. As long as the superficial distribution of normal doublets is constant in each panel, the vortex linear distribution is also constant in each panel boundaries. As already shown, this is a well-known concept in theoretical aerodynamics. This change is possible because the induced flow of both distributions is identical physically. This concept is used in the present work because it is mathematically easier to deal with linear distributions, than area ones. Figure 3, below, illustrates this concept.

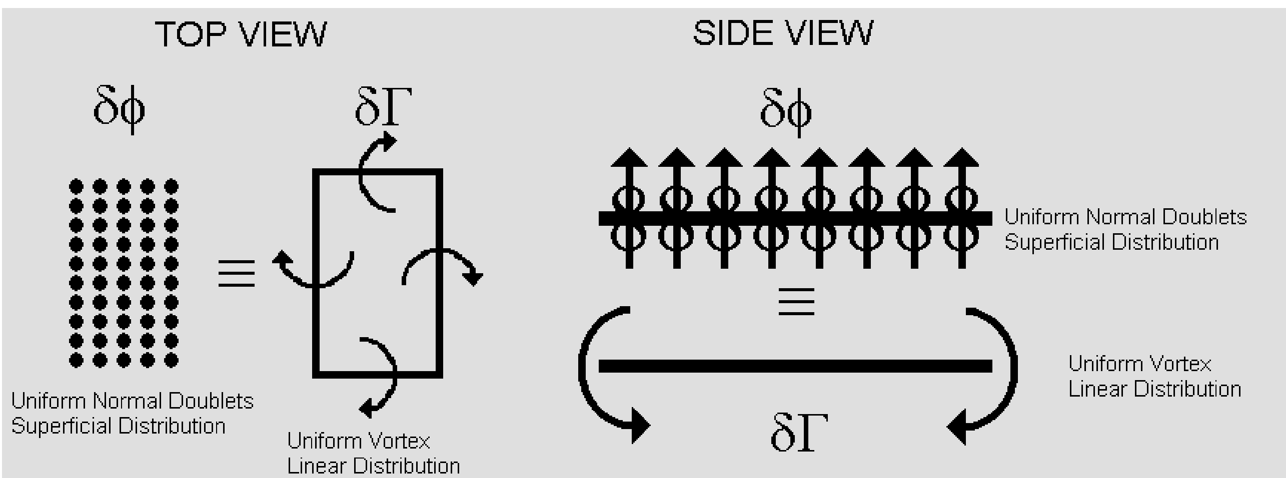

Figure 3. Equivalence between normal doublets area distributions and vortex linear distributions.

At the same instant (t=0+), the trailing vortex of the most downstream row of panels are emitted in the vortex sheet and are dragged by the flow. In the next step of time (t=1), occurs the same process: a new doublets superficial distribution is generated; this distribution is replaced by a distribution of vortex in the boundaries of the panels, in the instant t=1+, and the free vortex are emitted into the vortex sheet. The process continues in the subsequent instants until the steady flow is reached. The discretization of the time (dt) used is such that the distance traveled by the free vortex, which are dragged with the freestream velocity (U), during a time step is equal to the length of one panel in the chord direction. For a discretization of the chord in m panels and a chord length c, it implies that dt = c / (U m). This relation between time step and chord discretization is suitable to show well the time evolution of the lift over the wing. Figure 4, presents the process for a swept wing, in a top view.

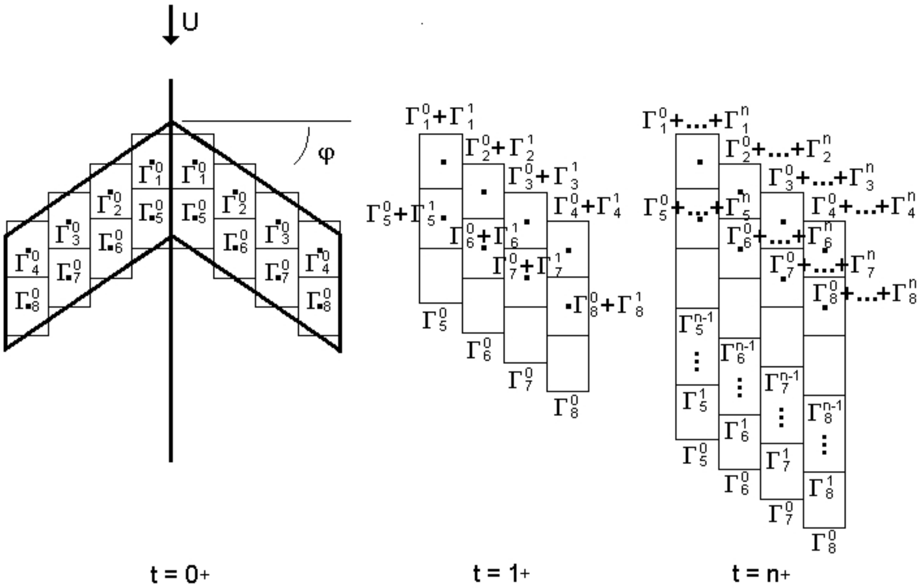

Figure 4. Numerical process: appearing of the vortex and emissions in the vortex sheet in three different instants.

The intensity of the potential jumps in each panel and instant of time is computed through the application of the boundary condition (Eq. 2), at the control points. This application is done in the half of the time step. So, if w_{ij}^{k} is the velocity induced in the control point i, in the instant t+dt/2, by an unitary potential jump created in panel j, in the instant k, it follows, from the Eq. 2, that:

$$\begin{pmatrix} w_{11}^{t} + a_{11}^{t} & w_{12}^{t} & \cdots & w_{1n}^{t} \\ w_{21}^{t} & w_{22}^{t} + a_{22}^{t} & \cdots & w_{2n}^{t} \\ \vdots & \vdots & \vdots & \vdots \\ w_{n1}^{t} & w_{n2}^{t} & \cdots & w_{nn}^{t} + a_{nn}^{t} \end{pmatrix} x \begin{pmatrix} \partial\phi_{1}^{t} \\ \partial\phi_{2}^{t} \\ \vdots \\ \partial\phi_{n}^{t} \end{pmatrix} = \begin{pmatrix} U\alpha - \sum_{j=1}^{n}\sum_{k=0}^{t-1} w_{1j}^{k} \\ U\alpha - \sum_{j=1}^{n}\sum_{k=0}^{t-1} w_{2j}^{k} \\ \vdots \\ U\alpha - \sum_{j=1}^{n}\sum_{k=0}^{t-1} w_{nj}^{k} \end{pmatrix} \qquad (4)$$

The only unknowns of the Eq. 4 are the components of the vector of the potential jumps. Applying Eq. 4 in each instant of time, it is possible to compute the variation of the potential jumps with time. The matrix that multiplies the vector of the potential jumps is called the influence coefficients matrix. It needs to be computed just once. The vector of the right side of the Eq. 4 needs to be computed in each step of time, because the induced velocities vary with the time. Inside the influence coefficients matrix, there are the terms a_{ii}, which are the impulsive velocities in each panel, created with the impulsive generation of potential. These terms appears only at the principal diagonal of the matrix, because its value in a given panel depends only on the intensity of the potential jump over the panel itself. They can be computed, using the linear piston theory (Bisplinghoff, Ashley and Halfman, 1955) and applying the boundary condition in the instant t=0. In this instant, all the terms w_{ij}^{k} are null (in a compressible flow, the perturbation created in the flow by the appearing of a potential jump is not instantaneous). Thus, at instant t=0, Eq. 4 reduces to:

$$a_{ii} \; \partial\phi_{i} = U\,\alpha \qquad (5)$$

The linear piston theory states that, in the initial instant, the pressure differential over the wing is constant and equal to:

$$\Delta P = -2\rho_\infty a_\infty U\alpha \Rightarrow \Delta C_P = \frac{4\alpha}{M_\infty} ; \Delta C_P = \frac{\Delta P}{\frac{1}{2}\rho_\infty U^2} \tag{6}$$

In the above equation, a_∞ is the free stream sound velocity and ρ_∞, the free stream density. From Eq. 3 and 6, as ϕ_x is null, because the potential distribution is constant over the wing in the initial instant:

$$\Delta P = -2\rho_\infty a_\infty U\alpha \Rightarrow \partial\phi = -\frac{2\alpha U^2}{M_\infty} dt \tag{7}$$

Thus, from Eq. 5 and 7:

$$a_{ii} = \frac{1}{2 a_\infty dt} \tag{8}$$

As shown in the Eq. 7, the values of the potential jumps in the instant t=0 are given directly from the piston theory. Thus, the Eq. 4 is applied only from the instant t=1. The terms w_{ij}^k, of the induced velocities by the vortex distributions in each instant of time are computed through the expression for the induced velocity by a vortex filament in a compressible flow. In the incompressible flow, where the perturbation created by the vortex is felt instantaneously in all the points of the flow, the expression for the induced velocity by a straight vortex filament, of intensity Γ and extremities A, (X_A,Y_A), and B, (X_B,Y_B), in a point P, (X_P,Y_P), of the flow is given directly by the integration of the Biot-Savart Law along the filament (see Fig. 5). That is:

$$W_{P\overline{AB}} = -\frac{\Gamma}{4\pi(x_P - x_A)}\left[\frac{(y_P - y_A)}{\sqrt{(x_P - x_A)^2 + (y_P - y_A)^2}} + \frac{(y_B - y_P)}{\sqrt{(x_P - x_A)^2 + (y_P - y_B)^2}}\right] \tag{9}$$

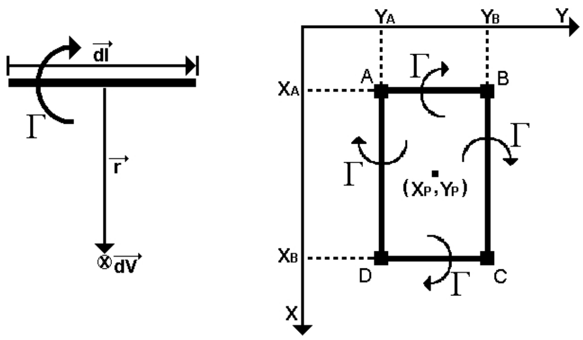

Figure 5. Illustration of the Biot-Savart Law (left side) and vortex panel with extremities A, B, C, D.

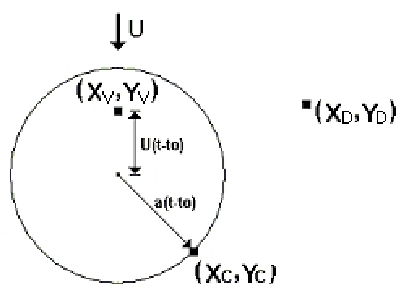

Figure 6. Influence region, at instant t, of a perturbation (X_V, Y_V) appearing in the flow at the instant t_0.

This computation in a compressible flow is analogous, but a little more complicated, because when the vortex appears in the flow, the perturbation induced by it propagates with a finite velocity. In this case, when the vortex is created, it appears a perturbation wave in each point of the filament. This perturbation propagates in all directions with the sound velocity (U / M_∞, where M_∞ is the freestream Mach number) and is convected with the flow velocity. In the plane, the perturbation wave is a circumference. Points out of this circumference did not "feel" the perturbation created by the vortex appearing yet. This concept is shown in Fig. 6. A perturbation appears in the flow at the point V, (X_V,Y_V), at the instant $t=t_0$. In the instant t, the perturbation wave has not reached the point D, (X_D,Y_D), yet and, thus, the induced velocity by the vortex in the point D is still null at the instant t. On the other side, at the same instant, the perturbation wave have just reached the point C, (X_C,Y_C), and from this time it will "feel" the perturbation induced by the vortex. It is possible to compute the value of $\tau_{VC}=(t-t_0)$, which is the time necessary to the perturbation created in the point V to reach the point C. From Fig. 6:

$$\tau_{VC} = \frac{-U\,(x_C - x_V) + \sqrt{-U^2\,(y_C - y_V)^2 + a^2\left[(x_C - x_V)^2 + (y_C - y_V)^2\right]}}{(a^2 - U^2)} \qquad (10)$$

The expression for the vortex filament in the compressible flow depends on the time the perturbation reaches the point. In fact, the expression for the incompressible flow, given in the Eq. 9 is a special case of the compressible equation, which occurs when $\tau=0$ (in Eq. 10, when $a\rightarrow\infty$, which is the incompressible condition, $\tau\rightarrow 0$). For the vortex filaments of Fig. 6, the following equations should be applied:

$$W_{P\overline{AB}} = -\frac{\Gamma}{4\pi(x_P - x_A)}\left[\frac{(y_P - y_A)}{\sqrt{\left[(x_P - x_A) - U\,\tau_{AP}\right]^2 + (y_P - y_A)^2}} + \frac{(y_B - y_P)}{\sqrt{\left[(x_P - x_A) - U\,\tau_{BP}\right]^2 + (y_P - y_B)^2}}\right] \qquad (11)$$

$$W_{P\overline{BC}} = -\frac{\Gamma}{4\pi(y_B - y_P)}\left[\frac{(x_P - x_A - U\,\tau_{BP})}{\sqrt{\left[(x_P - x_A) - U\,\tau_{BP}\right]^2 + (y_P - y_B)^2}} + \frac{(x_P - x_B - U\,\tau_{CP})}{\sqrt{\left[(x_P - x_B) - U\,\tau_{CP}\right]^2 + (y_P - y_B)^2}}\right] \qquad (12)$$

The equations for the other filaments, CD and DA, are analogous to Eqs. 11 and 12, respectively. In the above equations, if τ is negative for the extremities of the filament, there are two possibilities. The first is the perturbation has not reached any point of the filament. In this case, the value of the induced velocity is null. The second one is the perturbation has reached the filament, but not its extremities. In this case, the equations above are applied, just changing the extremities coordinates for the effective ones at that instant. Equation 11 is valid only for a fixed vortex. The equation for a free vortex, which is convected with the freestream velocity, in a compressible flow is the same of the equation for a fixed vortex filament in an incompressible flow, that is, Eq. 9. The only difference is that the perturbation last a finite time to reach the vortex, in the compressible flow, and this is taken in account. The lifting coefficient for unit of angle of attack (in radians), can be computed integrating the Eq. 3 over the wing. Thus:

$$\Delta C_{L\alpha} = \frac{1}{\alpha}\sum_{i=1}^{n}\Delta C_{Pi}\Delta x_i \Delta y_i \Rightarrow \Delta C_{L\alpha}(t) = -\frac{2}{\alpha U}\left(\frac{1}{U dt}\sum_{i=1}^{n}\partial\phi_i^t + \sum_{i=1}^{n}\sum_{k=0}^{t}\Gamma_i^k\right) \qquad (13)$$

In Eq. 13, the circulatory term is already transformed in the integration of the circulation, through the Kutta-Joukouski Theorem.

5. Results and conclusions

Figure 7 shows comparisons of the results of the present method with results of similar previous works, for the $C_{L\alpha}$ of the wing (in radians units) as function of time, after a step change in angle of attack. The time, in the graphs, is the non-dimensional time (s=U t/c, where c is the chord of the wing). Figure 7a shows the results for a wing of infinite span, obtained with the present method (discretization of 20 panels in chord) and the analytical method of Lomax *et al* (1952), for Mach numbers of 0.5 and 0.8. Figure 7b shows the results for a rectangular wing of aspect ratio 6 (AR 6), for Mach numbers of 0.3, 0.5 and 0.7, obtained by Vepa (1977), with his finite state modeling, and the results of the present method (discretization of 5 panels in chord and 20 in the span, that is, 5x20). Both the comparisons had good agreement. If a higher discretization in chord (and consequently in time) were used, the agreement would be still better. Qualitatively, the response of all the methods begins, at the initial instant, near the value predicted by the piston theory (see Eq. 6). After that, the $C_{L\alpha}$ decays rapidly, but begins to increase again, now more slowly, until reach the steady flow value.

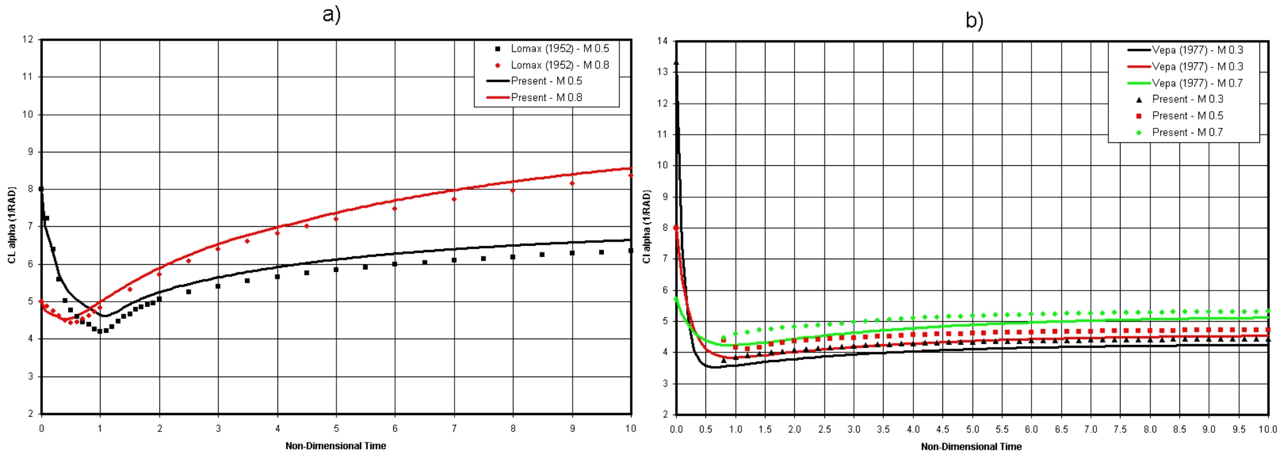

Figure 7. Comparisons: a) 2D results of Lomax *et. al* (1952) for Mach 0.5 and 0.8 and results of the present method; b) results of Vepa (1977) for a rectangular wing of AR 6 and results of the present method.

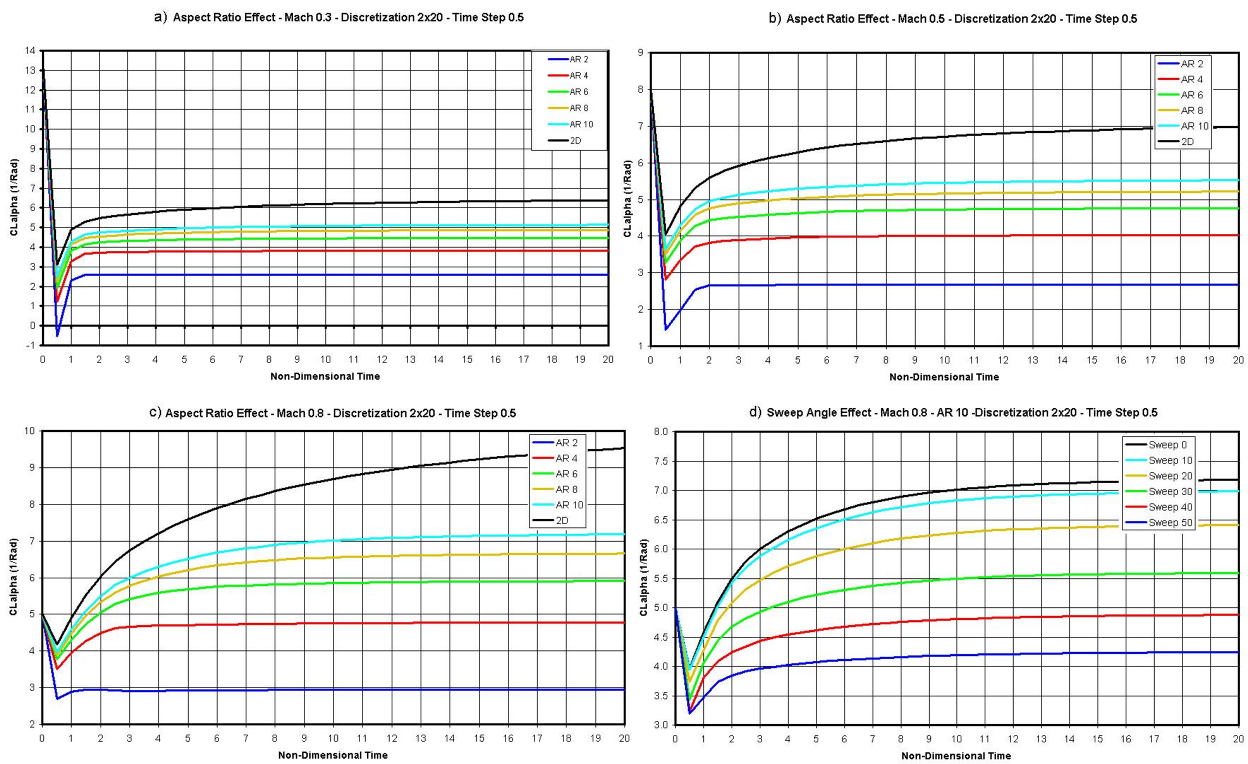

Figure 8. Influence of the aspect ratio for rectangular wings at a) Mach 0.3, b) Mach 0.5, c) Mach 0.8 and d) influence of the sweep angle in a wing of AR 10 for Mach 0.8.

Figure 8 shows results of the present method for several Mach numbers, aspect ratios and sweep angles. The goal is to check the influence of these parameters in the indicial response. The discretization used in this figures was 2x20, which is small, principally in chord, but is enough to address the qualitative influence of the parameters of the wing. Figure 8a shows the indicial response in $C_{L\alpha}$ for a rectangular wing at Mach 0.3 and several aspect ratios. Figures 8b and 8c show the same, for Machs 0.5 and 0.8. Figure 8d shows the response of a wing of aspect ratio 10 (taper ratio 1), at Mach number 0.8, with several sweep angles. The results show conclusively that the steady state is reached more slowly with the increase of the aspect ratio and Mach number. This is expected, because the propagation of the perturbations becomes slower with the increase of the Mach number. The effect of the aspect ratio can be explained by the effect of the wing tip vortex, which has more effect in smaller aspect ratios. At the other side, the sweep angle has no significant influence in the time to steady state. The only influence, of course, is the value of the $C_{L\alpha}$ in the steady state.

The method showed to be able of obtaining good results for the indicial response of wings in compressible flow. Its principal advantage is to be applicable to different situations, while analytical results are restricted. Besides that, the code is very simple and does not demand much computational resources, different of CFD codes, which can be used for the same purposes (Singh and Baeder, 1997; Sitaraman and Baeder, 2004), but are much more time and money demanding. For obtaining better quantitative results, principally in the beginning of the response, it is necessary a great chord discretization (50 or 100 panels), which demands some hours of computation, but not more than CFD demands. As already shown, the indicial responses for several values of step changes of angle of attack can be superposed, through the Duhamel Integral, to study the response to arbitrary oscillations. Thus, the present method can be very useful in preliminary projects of aircrafts.

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