EVALUATION OF AN ANALYTICAL MODEL FOR ADHESIVELY BONDED REPAIR IN COMPOSITE PLATES

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Abstract. *The increasing applications of composites in aeronautical structures and the resulting need for joining and repairing them impose the necessity of developing accurate and practical design tools. This work aims at evaluating the applicability of a one-dimensional analytical model for adhesively bonded scarf repair in composites plain plates under in-plane tension. The elastic part of this model is thoroughly described. In order to prevent numerical problems, modifications in its mathematical formulation are introduced. Then, the algorithm is implemented using MATLAB™. This computational code is validated by means of a comparison of its output to the results of a simple example solved by hand. The code outcome concerning the analysis of adhesive shear strain distribution seems reasonable, except for negative values obtained for certain configurations. Next, the same problem is approximated by a set of different two-dimension finite element models, computed using the MSC.NASTRAN™ solver. This set comprises stepped-lap joint models, with different degrees of mesh refinement. Based on their results, the difficulty in meshing such joints is highlighted, the advantages of the utilization of locally refined meshes are emphasized, and singularities are identified. Finally, the comparison of the analytical to the numerical results leads to the conclusion that the analytical model is applicable only to qualitative studies of joints between (or repairs in) laminates with a relatively high number of plies.*

Keywords*: composite materials, advanced joints, adhesively bonded joints*

1. Introduction

The current growth of the usage of composites not only in military but also in civil aircraft is based on several advantages related to fiber reinforced materials, in particular to the high strength and stiffness to weight ratios, which may provide weight reduction and consequently performance enhancement. Nonetheless, airplanes operators require nowadays much more than purely lofty technical efficiency. High dispatchability, and low acquisition and, specially, operational costs are characteristics that are becoming more and more decisive to ensure aircraft competitiveness. Moreover, certification authorities constantly update the airworthiness requirements to fulfill the society demand for safer transportation.

These requirements are translated by structural development engineers into several design guidelines. Among them is the need of conceiving easily reparable, highly efficient airframes. In the case of composite structures, this structural efficiency is closely related to its joints, which represent interruptions of geometry and materials, causing thus stress concentration. Besides, they are almost unavoidable due to the complexity intrinsically associated to aeronautical structures (Hart-Smith, 1987).

Therefore, since at the initial design phases, aspects related to structural assembly and maintenance, which comprises structural joints and repairs, must be carefully taken into account in order to ensure a successful design. At this stage of the product development cycle, on account of the number of unknowns and design possibilities, simple reliable models for structural pre-sizing and quick analysis are notably useful.

Joints can be mainly bolted or bonded. However, thanks to the advantages associated with the later technique, researches aiming at enlarging its applicability in aerospace structures have been constantly receiving more investments.

The first relatively mature analytical models for adhesively bonded joints and repairs - which are identical from the structural point of view - between composite adherends were developed by Hart-Smith in several works presented in the beginning of the seventies. Since then, different models, based essentially on improvement of his works, have been proposed.

One of these further upgraded works is the analytical method for adhesively bonded scarf repair in fiber reinforced laminated plain plates under in-plane tension, proposed by Springer and Ahn (1998), which is adaptation of the Hart-Smith´s (1973) model that takes into account the anisotropy of each ply in the laminate and repair patch separately.

This work aims at investigating the applicability of this model, considering further improvements herein included, by comparing the outputs of the corresponding computationally implemented algorithm with the results of related finite element models.

2. Analytical Model

The analytical method implemented herein is based on the Springer and Ahn (1998) model for calculating the failure in-plane tensile loads per unit width of the scarf repair, considering some adjustments. For that, the following assumptions and limitations are considered:

- The laminate is assumed to behave according to the classical lamination theory, i.e. each lamina is considered quasihomogeneous, orthotropic, linear elastic, to be in a state of plane stress and thickness-wise transverse shear strain are neglected;
- The interlayer is assumed homogeneous, isotropic and linear elastic;
- Viscoelasticity is neglected;
- A state of plane strain is assumed, i.e. transverse deformations are disregarded ;
- Since only relatively small scarf angles are commonly employed, the adhesive is analyzed as an one-dimensional shear strain model, i.e. its normal strength is disregarded;
- The small eccentricity in the load path is ignored, i.e. out-of-plane stress components are neglected;
- The repaired plate is subjected only to in-plane tensile loads;
- The joint is assumed plain, i.e. the effect of curvatures are not accounted for;
- The maximum strain criterion is considered as the ply failure criterion. It allows the identification of the mode of failure, which is important information for joints design;
- The first ply failure criterion, which is conservative, is assumed as laminate failure criterion for the adherends;
- Manufacturing imperfections such as variations in the adhesive thickness and voids are ignored.

In order to compute the failure in-plane tensile load, the following failure modes considered for the repaired laminate are:

- The parent laminate may fail in tension;
- The repair patch may fail in tension;
- The interlayer may fail in shear;
- Part either of the parent laminate or of the repair patch may fail in tension and part of the interlayer may fail in shear (Springer and Ahn, 1998).

To find the repair failure load, firstly a relative small value of the applied load per unit width is assumed. This load is then slowly increased. At each iteration step, parent laminate, repair patch and interlayer failures are analyzed separately.

2.1. Interlayer Failure Load in Shear

Thus, in order to compute the interlayer failure load in shear, initially the interlayer shear strain distribution shall be determined. For that, the scarf joint is modeled as a stepped-lap joint with one-ply thick steps, as presented in the Fig. 1.

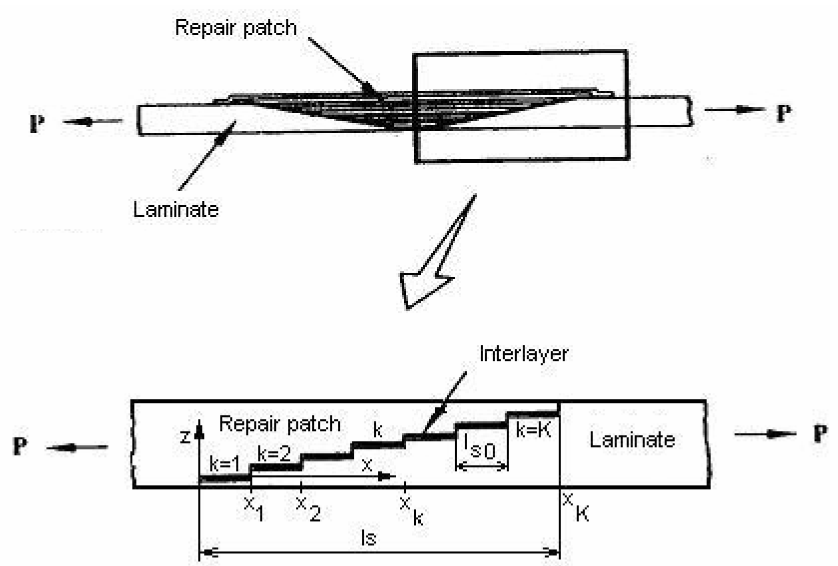

Figure 1 - Stepped-lap approach for scarf repairs (Based on Springer and Ahn, 1998)

Considering the equilibrium of forces for the laminate and repair patch of an infinitesimal long slice of the joint, yields Eq. (1):

$$\frac{d\left({}^{k}N^{L}\right)}{d\xi} - {}^{k}\tau l_{s} = 0$$
$$\frac{d\left({}^{k}N^{R}\right)}{d\xi} + {}^{k}\tau l_{s} = 0 \qquad (1)$$

where ${}^{k}N^{L}$ and ${}^{k}N^{R}$ are the in-plane loads per unit width inside the parent laminate and the repair patch, respectively; ls is the total scarf length, ξ is the ratio of x over ls, ${}^{k}\tau$ is the shear stress in the interlayer; and k refers to the k-th overlap segment.

By neglecting shear deformations of the parent laminate and the repair patch, compatibility of an infinitesimal long slice of the joint leads to the Eq. (2):

$$\frac{d\left({}^{k}\gamma\right)}{d\xi} = \frac{l_{s}\left({}^{k}\varepsilon^{L} - {}^{k}\varepsilon^{R}\right)}{h^{I}} \qquad (2)$$

where ${}^{k}\varepsilon^{L}$ and ${}^{k}\varepsilon^{R}$ are the off-axis in-plane strains in the parent laminate and in the repair patch, respectively; h_I is the interlayer thickness and ${}^{k}\gamma$ is the interlayer shear strain.

The constitutive relations for the adherends and adhesive of the k-th step are expressed by Eq. (3) and Eq. (4), respectively.

$${}^{k}\varepsilon^{L} = \left({}^{k}a_{11}{}^{L}\right)\left({}^{k}N^{L}\right)$$
$${}^{k}\varepsilon^{R} = \left({}^{k}a_{11}{}^{R}\right)\left({}^{k}N^{R}\right) \qquad (3)$$

$${}^{k}\tau = {}^{k}\gamma G^{I} \qquad (4)$$

where ${}^{k}a_{ii}{}^{L}$ and ${}^{k}a_{ii}{}^{R}$ are the membrane component of the elastic flexibility matrix of the parent laminate and repair patch, respectively, and G^{I} is the interlayer shear modulus.

Substituting Eq. (3), which are derived from the constitutive relations, in the adherends compatibility Eq. (2), and then combining it with Eq. (1), which is originated from the equilibrium of forces in the repair patch and parent laminate, and with Eq. (4), that results from the adhesive constitutive relations, the differential equation governing the interlayer behavior is obtained:

$$\frac{d^{2}\left({}^{k}\gamma\right)}{d\xi^{2}} = \gamma\left({}^{k}\lambda\right)^{2} \qquad (5)$$

where ${}^{k}\lambda$ is defined as in Eq. (6):

$${}^{k}\lambda = \left(\frac{G^{I} l_{s}^{2}\left({}^{k}a_{11}{}^{L} + {}^{k}a_{11}{}^{R}\right)}{h^{I}}\right)^{1/2} \qquad (6)$$

Thus, the general solution for the interlayer elastic shear strain shall be expressed as in Eq. (7).

$${}^{k}\gamma = {}^{k}C e^{{}^{k}\lambda\left(\xi - {}^{k}\xi\right)} + {}^{k}D e^{-{}^{k}\lambda\left(\xi - {}^{k-1}\xi\right)} \qquad (7)$$

where ${}^{k}C$ and ${}^{k}D$ are constants related to the boundary and continuity conditions; ${}^{k-1}\xi$ is the value of the left-hand end of the step, ${}^{k}\xi$ is the value at the right-hand end of the step and ${}^{k-1}\xi \leq \xi \leq {}^{k}\xi$.

It can be observed that modifications in the formulation proposed by Springer and Ahn (1998) are herein included. Since a set of algebraic equations are solved to compute the constants ${}^{k}C$ and ${}^{k}D$, ill-conditioning of the system and consequent numerical problems must be avoided. For that, the general solution for the interlayer elastic shear strain is defined as a function of a non-dimensional joint length. Moreover, instead of the difference of hyperbolic sinusoidal functions, Eq. (7) is expressed as a sum of exponential functions. Furthermore, the argument of the function is re-written using the relative step length, so that to prevent poor-conditioned system, especially when dealing with long steps (ESDU 79016, 1998).

Hence, in order to solve a general repair with K-th steps, 2.K constants shall be computed based on boundary and continuity conditions.

As boundary conditions it is assumed that at the inside edge of the repair patch (ξ=0), the axial load per unit width is zero in the laminate and is equal to the applied load per unit width (P) in the repair patch. At the outside edge of the repair (ξ=1) the axial load per unit width is zero in the repair patch and is equal to P in the laminate (Springer and Ahn, 1998). In practice, external plies may be included in the repair. This model allows the inclusion of up to two external plies. By including Eq. (3) into Eq. (2) and applying these boundary conditions, yields Eq. (8):

$$\frac{d\left({}^{1}\gamma\right)}{d\xi}=\frac{-Pl_s\left({}^{1}a_{11}^{R}\right)}{h^{I}}\ at\ \xi=0$$
$$\frac{d\left({}^{k}\gamma\right)}{d\xi}=\frac{Pl_s\left({}^{k}a_{11}^{L}\right)}{h^{I}}\ at\ \xi=1 \qquad (8)$$

Moreover, it is considered that at the edge of each overlap segment ($\xi={}^{k}\xi$), the shear strain in the interlayer is continuous and the in-plane loads are equal and opposite on the left and the right sides of the laminate parent and repair patch. It is assumed that the load is transmitted only by continuous layers. By substituting again Eq. (3) into Eq. (2) and applying these continuity conditions, Eq. (9) is obtained:

$$\frac{d\left({}^{k+1}\gamma\right)}{d\xi}=\frac{d\left({}^{k}\gamma\right)}{d\xi}{}^{k}\Omega+{}^{k}\Psi \quad at\ \xi={}^{k}\xi \qquad (9)$$

where constants ${}^{k}\Omega$ and ${}^{k}\Psi$ are defined as:

$${}^{k}\Omega=\left[\frac{\left({}^{k+1}a_{11}{}^{R}\right)+\left({}^{k+1}a_{11}{}^{L}\right)}{\left({}^{k}a_{11}{}^{R}\right)+\left({}^{k}a_{11}{}^{L}\right)}\right]$$
$${}^{k}\Psi=\frac{Pl_s}{h^{I}}\left[\frac{\left({}^{k}a_{11}{}^{R}\right)\left({}^{k+1}a_{11}{}^{L}\right)-\left({}^{k}a_{11}{}^{L}\right)\left({}^{k+1}a_{11}{}^{R}\right)}{\left({}^{k}a_{11}{}^{R}\right)+\left({}^{k}a_{11}{}^{L}\right)}\right] \qquad (10)$$

Finally, with the general solution expressed by Eq. (7) and boundary and continuity conditions written in form of the differential Eq. (8) and Eq. (9), an algebraic equation system is formed, from which the remaining constants are computed. As a result, interlayer shear strain distribution at the entire joint region for a given load is determined. The interlayer failure load is evaluated by progressively increasing the load per unit width until the maximum shear strain at any point of the interlayer is reached.

2.2. Parent Laminate/Repair Patch Failure Load in Tension

To determine the applied tensile load under which either the parent laminate or the repair patch fails, the in-plane load in the laminate and repair patch are computed as function of longitudinal position ξ. These relations stem from the integration of the differential equations derived from the equilibrium conditions, Eq. (1), and later substitution of the interlayer constitutive relation, Eq. (4), and the general solution for the interlayer elastic shear strain, Eq. (7), resulting in Eq. (11):

$${}^{k}N^{L}(\xi)=\frac{G^{I}l_s}{{}^{k}\lambda}\left[{}^{k}Ce^{-{}^{k}\lambda{}^{k}\xi}\left(e^{{}^{k}\lambda\xi}-e^{{}^{k}\lambda{}^{k-1}\xi}\right)-{}^{k}De^{{}^{k}\lambda{}^{k-1}\xi}\left(e^{-{}^{k}\lambda\xi}-e^{-{}^{k}\lambda{}^{k-1}\xi}\right)\right]+N^{L}\Big|_{0}^{{}^{k-1}\xi} \qquad (11)$$

Equation (11) allows the computation of the axial load distribution along the joint length. With that, the laminate failure of each step of the parent laminate and repair patch at each load step can be checked.

2.3. Computationally Implemented Code

The previously presented analytical model is computationally implemented employing the software MATLAB™. With the purpose of verifying the computational implementation, a simple example is solved by hand. It consists of a three-layer cross-ply scarf joint with identical adherends made up of unidirectional graphite-epoxy tape and without external plies. The results agreed with each other.

3. Finite Element Model

The use of finite element modeling techniques in the analysis of joints may be indispensable in some situations, where closed form solutions are not available or when details about the joint under load are required.

On the other hand, finite element modeling analyses are generally costly and time-consuming tasks. Moreover, changes in finite elements models may be cumbersome.

As a consequence of that and keeping in mind that the aim of this work is to discuss preliminary design techniques for joints/repairs, the finite element method is used herein aiming at verifying the results obtained with the analytical code previously implemented and assess its applicability.

For that, the simple example previously introduced on item 2.3 is simulated with finite element models. Since plain strain state is assumed, the longitudinal in-plane and the out-of-plane joint axes are modeled with quadrilateral, whenever possible, and triangular isoparametric shell elements. Symmetry is considered and only half of the joint needs to be modeled. In-plane tensile load is applied indirectly in form of prescribed displacement of edge nodes. MSC.NASTRAN™ solver is used (MSC.NASTRAN, 2002).

The proposed example is approximated by stepped-lap models, as shown in Fig. 2.

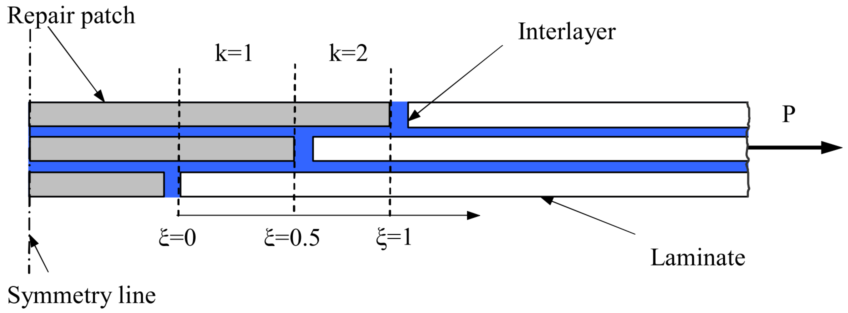

Figure 2 - Stepped-lap finite element model sketch

Two models without local refinement are employed to assess the required overall mesh refinement: Model I with one element through the interlayer thickness and 32,330 degrees of freedom, and Model II with three elements through the interlayer thickness and 56,738 degrees of freedom. Three additional models are used to investigate the necessary local refinement at the steps ends: Model III with 33,005 degrees of freedom, Model IV with 39,980 degrees of freedom and Model V with 43,980 degrees of freedom.

4. Results

4.1. Analytical Model

In order to investigate some characteristics of the analytical model, different joints are modeled using the implemented code.

Firstly, a laminated scarf joint tested by Mayer (2003) is modeled. This six-ply specimen with external plies is made up of graphite-epoxy fabric with the following typical properties: parent laminate E_1^L=66,600 MPa, E_2^L=66,600 MPa, G_{12}^L= 4,600 MPa, υ_{12}^L=0.06 and h^L=0.35 mm; repair parent E_1^R=58,800 MPa, E_2^R=58,800 MPa, G_{12}^R=2,500 MPa, υ_{12}^R=0.02, and h^R=0.35 mm; and interlayer G^I=241 MPa, h^I=0.18mm.

Though the value of the failure load is not expected to agree with experimental results obtained by Mayer (2003), once the analytical model implemented herein considered only elastic interlayer behavior, this joint presents adherends with stiffness mismatch, which allows the evaluation of important joints qualitative behaviors explained by theory and observed in practice. For instance, it can be noticed in Fig. 3 that the highest peaks of the interlayer shear strain are located at the joint ends; and also that the load transfer is concentrated at the end of the stiffest adherend.

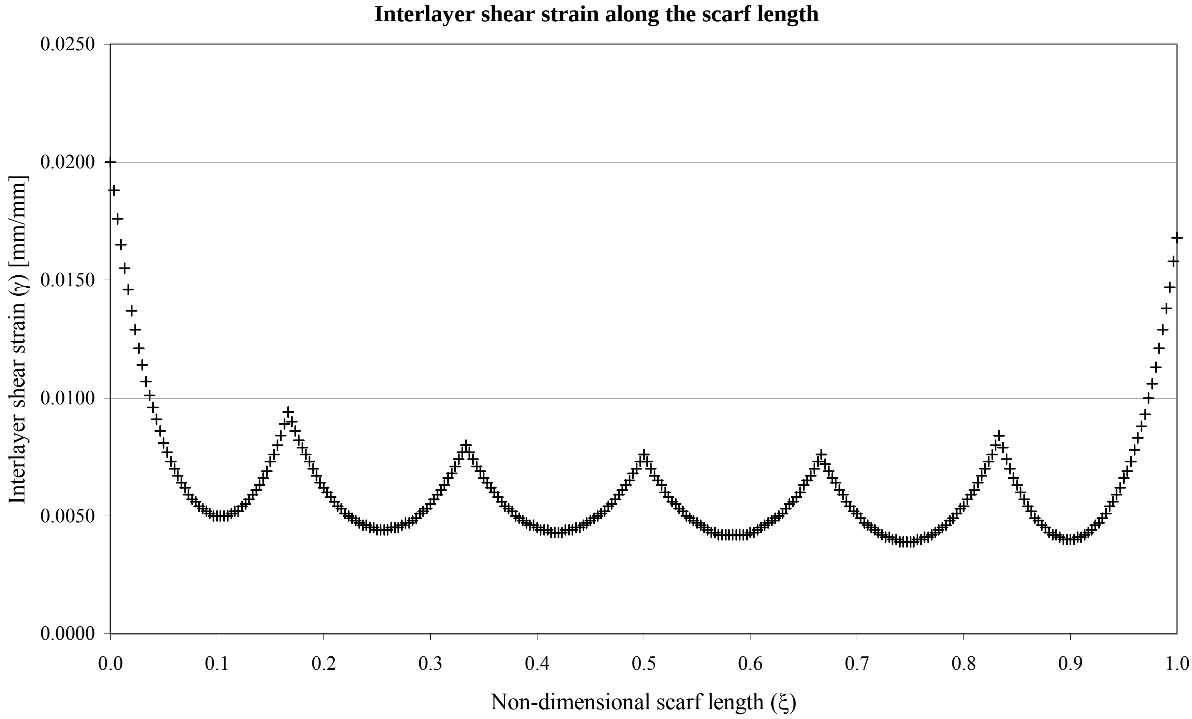

Figure 3 - Interlayer shear strain distribution: adherends with stiffness mismatch based on Mayer (2003)

Next, with the purpose of studying the influence of the lay-ups at the interlayer shear strain distribution, a three-layer laminated plate scarf repaired without external plies is modeled using the code. This simple example was also modeled using finite elements, as previously introduced on item 3. It can be observed in Fig. 4 that when the stiffness of adjacent plies differs considerably, the analytical model yields negative shear strain in some regions of the joint.

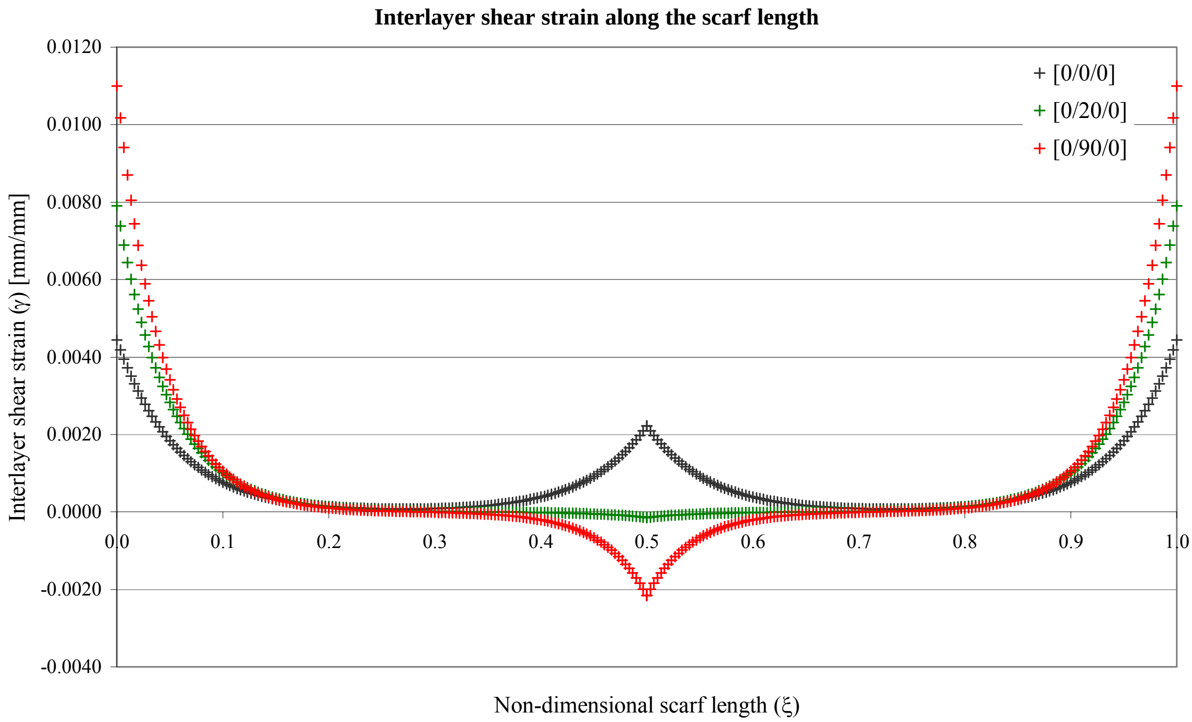

Figure 4 - Interlayer shear strain distribution: various lay-ups for a three-layer joint sketched in Fig. 2

The source of this unexpected result is not clearly identified. It might be related to numerical problems, as suggested by. Hart-Smith (1973 p.27): "One small computational problem was that […] the computer would predict physically unrealizable large negative strains in the adhesive". It might also be associated with the violation of simplifying hypotheses, such as the assumption of state of plane strain, for instance. Mortensen (1998) discussed the limitations of two-dimensional models when presenting a more complex adhesively bonded analytical model, which considers transverse deformations.

Finally, the influence of modeling a scarf joint with this stepped-lap model over the interlayer shear strain distribution is discussed. The stepped-lap approach is a discretization of the scarf technique, as illustrates Fig.1. For Hart-Smith (1973), mathematically speaking, the stepped-lap family of joints represents perturbations about the scarf joint solution. Then, aiming at investigating this approximation, the previously introduced, three-layer laminated plate scarf repaired is modeled using the code. The effect of discretization is introduced by an artificial increase in the number of steps, which consists of the hypothetical reduction of the overlap step length per ply and the ply thickness, whilst the number of these imaginary lamina are proportionally increased, ensuring thus that the scarf angle and length remain the same.

The results of this analysis show that by increasing the number of steps, the interlayer strain distribution tends to assume a homogeneous form, as expected, once that, for this proposed example, as previously stated, the adherends have identical elastic properties, i.e. no stiffness mismatch exists.

4.2. Finite Element Models

From the comparison between the models with one and three elements through the interlayer thickness, it is observed that similar results concerning the interlayer shear distribution throughout the joint are achieved, except for differences at the lap ends.

Thus, the comparison of the outputs of models with different degree of local refinement at step ends is performed. Figure 5 shows that despite the extremely refined mesh at step ends adopted, no convergence seems to be reached. These results lead to the conclusion that singularities might occur at the far ends of each step lap. This conclusion is reinforced considering that similar result, reported as "[…] mathematical stress infinity […]", was also found at analysis of double-lap joints using the finite element approach (Military, 1997). Moreover, Mortensen (1998) modeled a single-lap joint using finite element approach and mentioned also singularities present at the corners of the adherend facing the adhesive layer at the ends of the overlap.

In order to attempt to achieve convergence also at the step ends, all elements of the models with highest degree of local refinement, are replaced by 8-nodes isoparametric quadrilateral and 6-nodes isoparametric triangular elements. Though the higher order of the interpolation function within the elements utilized, the convergence is also not reached.

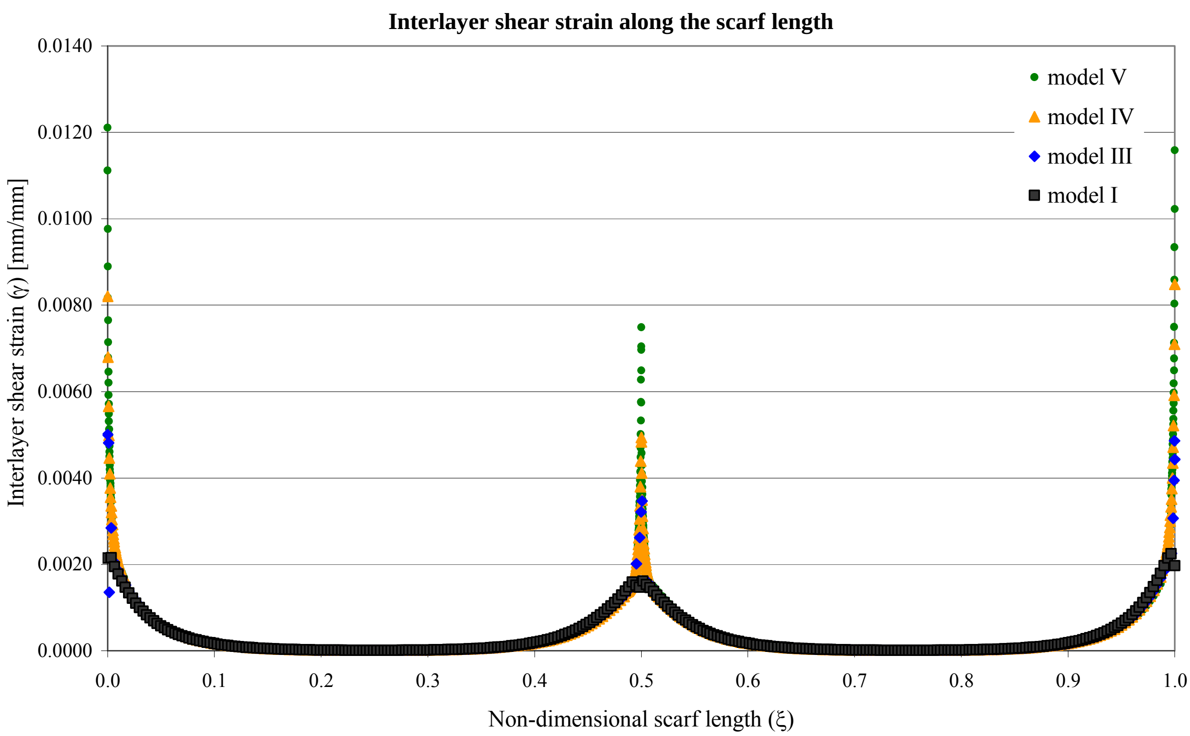

Figure 5 - Interlayer shear strain: models with local refinement at step ends

4.3. Comparison between the Analytical and the Finite Element Model

From the comparison between the results of the stepped-lap finite element model, with the mostly refined steps ends, and the analytical method for stepped-lap repairs, it can be observed that the result obtained by the finite element model agrees relatively well with the output of the implemented code, except for the step ends.

5. Conclusion and Comments

Based on the formerly presented study it can be concluded that the utilization of this one-dimensional analytical model for the analysis as well as the design of scarf repair/joints between composites adherends is relatively limited.

Concerning the prediction of the failure in-plane tensile load, which is one of the main outputs desired by design and stress engineers, the results of the analytical model herein implemented are not expected to agree with experimental values, mainly due to the complex state of stress originated at the joint ends. However, Springer and Ahn (2000) showed that, after estimating the properties of the interlayer by matching the model to one experimental data point, the so-called “backcalculation”, trends for further predictions were well predicted.

Regarding the number of plies of the laminated plates to be jointed with the scarf technique, the analysis using this analytical model is restricted to adherends with many plies, but no universal rules exist (Hart-Smith, 1981). This limitation seems to represent no severe practical implication since only for applications with a large number of plies, considering well designed joints, the adhesive shall be the weakest link. In stepped-lap or scarf joints with few steps, the potential bond shear is normally far in excess of the adherends strength (Hart-Smith, 1973).

Concerning the lay-up configuration, the analyses lead to the conclusion that the analytical model shall not be used when excessive rigidity mismatch between adjacent plies exists. When investigating single-lap joints, Mortensen (1998) recommended using 0^{o} layers facing the adhesive, so that to improve load transfer from adhesive to adherend. This design guideline could also, regarding some aspects, be extended to joint ends of advanced joints.

In relation to the finite element approach, it seems not adequate for the initial design phases. This is due not only to the high computational costs and time consuming tasks involved but also mainly to the fact that design changes might be cumbersome. Moreover, non-commonly used elements and / or approaches shall to ensure convergence.

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7. Responsibility notice