

MULTIVARIABLE PREDICTIVE CONTROL WITH CONSTRAINS OF REFRIGERATION SYSTEMS

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Abstract Cooling machines are multi-input, multi-output, cross-coupled, time-varying, nonlinear systems whose inputs (manipulated variables) and outputs (controlled variables) present saturation and rate constraints. Classical control techniques such as PID controllers are not usually adequate to deal with the existing cross coupling among inputs and outputs. We implement a multivariable version of a predictive control algorithm (GPC – Generalized Predictive Control) taking in account some constraints. These constraints include output constraints like minimum and maximum superheat and input constraints like maximum compressor speed. The simulations showed an excellent control performance. The output variables (superheat and freezing-power) are kept at the set points while constraints are in activity. The proposed scheme provides an optimal solution for the compressor speed and the expansion valve opening, which minimizes the system energy consumption.

Keywords: Predictive Control with Constraints, Multivariable Control, Refrigeration Systems

1. Introduction

The world energetic consumption with refrigeration systems is second Imbabi (1990) is 50%. Beecham, (2001) have predicted an annual consumption of 10.6 billion gallons of gasoline just to power air conditioning in automotive applications in USA. Beecham, (2004)) have estimated that 66% of cars in Europe has air conditioning systems. In Brazil, refrigeration sector is responsible to 32% of residential electric consumption. This is 8% of Brazilian electric consumption (Achão, 2003). Face this figures, it is clear that energetic efficient improvement is very welcome to refrigeration systems.

Despite the low cost and its large application, the control of refrigerating machines based on-off strategy is inefficient from the viewpoint of energy consumption. That is because during the system start-up occurs energetic consumption peaks to reestablish the system equilibrium conditions.

Other aspect to be considered is the compromise between the temperature adjustment and life equipment, since in refrigeration systems with on-off control a precise temperature adjustment is obtaining with a high frequency on-off cycle, which cause components degradation.

From the viewpoint of control process, refrigeration systems are non-minimal phase multivariable process, with stronger cross coupling between the input-components output variables. Moreover, they are systems that have constraints in input and output variables and variable thermal load.

One way to deal with these challenges is to endow the refrigeration systems with variable speed compressors jointly electronic expansion devices. The standard on-off control is not capable to explore the whole capacity of these new actuators. In order to do this and to reach a better performance, it is necessary to use advanced control techniques like the Predictive Control.

The Predictive Control was first applied at complex industrial plants (Richalet *et al.*, 1976 and 1978). The high initial cost to computational implementation of these control algorithms has restricted the first applications at these complexes plants (chemical and petrochemical plants). Today, given the relative computers low cost, the application of Predictive Control to smaller plants like refrigeration system is economically viable.

This work presents the simulation results from application of a Multivariable Generalized Predictive Controller to a vapor compression refrigeration system. The considered component is a counter-flow double tube evaporator that is modeled by a 2 x 2 matrix transfer function. The objective is to keep the output variables (superheating and freezing power) at the set point values despite of load disturbances in the presence of variable constraints. The proposed solution using of a multivariable predictive controller and the inclusion of variable constraints in the control algorithm are significant improvements with respect to previous results (Silva and Galvez, 2001).

The results show that the proposed controller is an excellent solution for the simultaneous control of the compressor speed and the valve expansion aperture degree, which allows the system to have a good performance.

2. The Refrigeration System Model

Figure 1 shows a vapor compressor refrigeration system and its main components: the expansion valve, the evaporator, the compressor and the condenser.

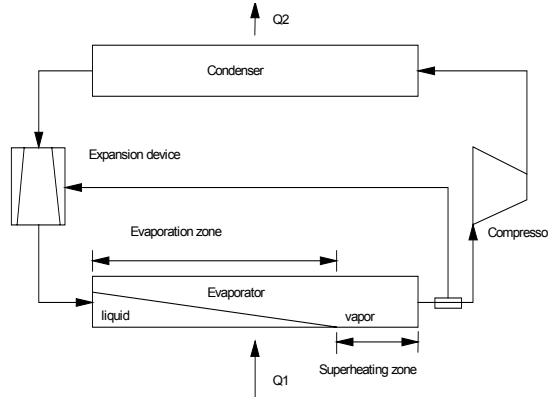


Figure 1. The Vapor Compression Based Refrigeration Cycle.

The system dynamic to be control (evaporator) is defined by the following matrix transfer function:

$$\begin{bmatrix} \Delta T(s) \\ Q_1(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} \dot{m}(s) \\ \dot{V}(s) \end{bmatrix} \quad (1)$$

The input variables are the mass flow rate $\dot{m}(s)$ and the volumetric flow rate $\dot{V}(s)$. The output variables are the superheating $\Delta T(s)$ and the cooling power $Q_1(s)$. The G terms are the functions that relate the input-output variables pairs. The cooling power is a variable of interest because it represents the heat absorbed by the evaporator. The superheating degree is considered other output variable in order to prevent liquid aspiration by the compressor and due to your role in whole cycle efficiency.

Figure 2 shows the interactions between input-output variables, that is, the cross coupling. The functions $G_{11}(s)$ and $G_{21}(s)$ establish the evaporator inlet mass flow rate influence on the superheating and cooling power, respectively. The functions $G_{12}(s)$ e $G_{22}(s)$ establish the evaporator outlet volumetric flow rate on the superheating and the cooling power, respectively.

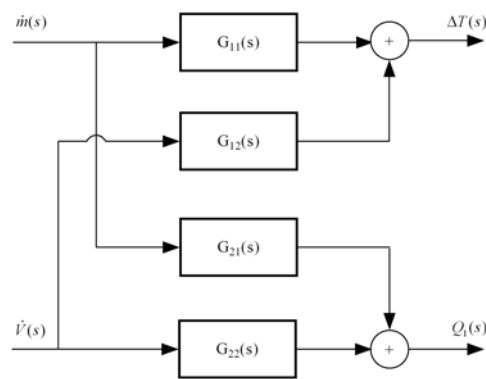


Figure 2. Evaporator Input-Output Variables and Cross Coupling.

Numerical values to the transfer functions coefficients are (Silva e Galvez, 2001):

$$\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} = \begin{bmatrix} \frac{-5.62}{(45 \ s + 1)} & \frac{-2.49 \ (-70 \ s + 1)}{(59.52 \ s + 1)} \\ \frac{33.89 \ (-36.37 \ s + 1)}{(25.65 \ s + 1) \ (67.79 \ s + 1)} & \frac{22.20 \ (630 \ s + 1)}{(80 \ s + 1) \ (90 \ s + 1)} \end{bmatrix} \quad (2)$$

In the decoupling case (ideal scene), the expansion valve aperture (mass flow rate) will be used to control the superheating and the compressor velocity (volumetric flow rate) will be used to control the cooling power. In this case $G_{12}(s)$ e $G_{21}(s)$ will have nulls gains. However, in order to have a good control performance, the cross coupling between systems input-output variables can not be reject.

3. The Generalized Predictive Controller

The Generalized Predictive Control (GPC) (Clarke *et al.*, 1987 a and b) has been implemented in many industrial and academic applications, presenting good performance and robustness. The GPC can be applied in many kinds of control problems, having been applied to several plants: mono and multivariable systems, non-minimal systems, systems with variable or unknown delay, systems with weakly damped poles and unstable open-loop plants.

The basic idea is the calculus of a future control signals sequence that minimizes a quadratic cost function defined over a time interval: the prediction horizon, N . The cost function measures the distance between the predicted system output and some predicted reference trajectory over the named control horizon plus a quadratic function that penalizes the control effort.

Considering the linear CARIMA (Controlled Auto-Regressive Integrated and Moving-Average) model that relates variations in the n plants outputs with the variations in the m plants inputs:

$$\mathbf{A}(q^{-1})y(t) = z^{-d}\mathbf{B}(q^{-1})u_j(t-1) + \mathbf{C}(q^{-1})\frac{\xi(t)}{\Delta(q^{-1})} \quad (3)$$

where :

$\mathbf{A}(q^{-1}) = I_{n \times n} + A_1 q^{-1} + \dots + A_{na} q^{-na}$, $\mathbf{B}(q^{-1}) = B_0 + B_1 q^{-1} + \dots + B_{nb} q^{-nb}$ and $\mathbf{C}(q^{-1}) = I_{n \times n} + C_1 q^{-1} + \dots + C_{nc} q^{-nc}$ are polynomial matrices.

The extended $\mathbf{B}(q^{-1})$ polynomial incorporates the process dead time. The variables $y(t)$, $u(t)$ e $\xi(t)$ are a $n \times 1$ output vector, a $m \times 1$ input vector and a white noise zero mean, respectively. The term $\Delta(q^{-1})$ is defined as $\Delta(q^{-1}) = 1 - q^{-1}$; na , nb e nc are the degree of input-output pair in $\mathbf{A}(q^{-1})$, $\mathbf{B}(q^{-1})$ e $\mathbf{C}(q^{-1})$.

It can be demonstrated (Camacho e Bordons, 1999) that the optimal outputs predictions j steps ahead are given by following equation:

$$\hat{y}(t+j|t) = \mathbf{G}_j(q^{-1})\Delta\mathbf{u}(t+j-1) + \mathbf{G}_{jp}(q^{-1})\Delta\mathbf{u}(t-1) + \mathbf{F}_j(q^{-1})y(t) \quad (4)$$

If the future control signals increments are zero, the predicted outputs in Eq. (4) depend only on past values of the process output and input variables. This is known as the process free response and corresponds to the last two terms in Eq. (4). The first term depends on future values of the control signal and is called forced response.

Equation (4) can be rewritten as:

$$\hat{y}(t+j|t) = \mathbf{G}\tilde{\mathbf{u}} + \mathbf{f} \quad (5)$$

where $\mathbf{G} = \mathbf{G}_j$, $\tilde{\mathbf{u}} = \Delta\mathbf{u}(t+j-1)$ and $\mathbf{f} = \mathbf{G}_{jp}(q^{-1})\Delta\mathbf{u}(t-1) + \mathbf{F}_j(q^{-1})y(t)$.

In Eq. (4) and Eq. (5), $j = 1 \dots N$ and the polynomial matrices \mathbf{G}_j , \mathbf{G}_{jp} e \mathbf{F}_j satisfy the following Diophantine equations:

$$\mathbf{I} = \mathbf{E}_j(q^{-1})\mathbf{A}(q^{-1})\Delta + q^{-1}\mathbf{F}_j(q^{-1}) \quad (6)$$

$$\mathbf{E}_j(q^{-1})\mathbf{B}_j(q^{-1}) = \mathbf{G}_j(q^{-1}) + q^{-1}\mathbf{G}_{jp}(q^{-1}) \quad (7)$$

The GPC algorithm consists in finding the control sequence increments $\Delta\mathbf{u}(t+j)$ that minimize the following cost function:

$$J_{GPC} = E \left\{ \sum_{j=N_1}^{N_2} \|\hat{y}(t+j|t) - w(t+j)\|_R^2 + \sum_{j=1}^{N_u} \|\Delta\mathbf{u}(t+j-1)\|_Q^2 \right\} \quad (8)$$

Or in the matrix form:

$$J_{GPC} = (\mathbf{G}\tilde{\mathbf{u}} + \mathbf{f} - \mathbf{w})^T R(\mathbf{G}\tilde{\mathbf{u}} + \mathbf{f} - \mathbf{w}) + \tilde{\mathbf{u}}^T \mathbf{Q} \tilde{\mathbf{u}} \quad (9)$$

where $\hat{y}(t+j|t)$ is the output prediction at time t to output for the time $(t+j)$ (Eq. 4); $w(t+j)$ is the future reference trajectory; N_1 is the minimum cost horizon; N_2 is the maximum cost horizon (prediction horizon); N_u is the control horizon; $\lambda(j)$ is a weight control sequence; $\Delta u(t)$ is future signals control increments in the control horizon; \bar{R} and \bar{Q} are diagonal matrices with weighting terms, the parameters α and λ .

If there is no constraints, the minimization of Eq. (8) or Eq. (9) gives the optimum:

$$\tilde{\mathbf{u}} = (\mathbf{G}_{N_{12}}^T \bar{R} \mathbf{G}_{N_{12}} + \bar{Q})^{-1} \mathbf{G}_{N_{12}}^T R(\mathbf{w} - \mathbf{f}_{N_{12}}) \quad (10)$$

Note that control signal effectively sent to the system at time t is given by:

$$\mathbf{u}(t) = \mathbf{u}(t-1) + \tilde{\mathbf{u}}(t) \quad (11)$$

that is, the control signal sent to the system at the present time is given by the sum of the past control signal and the control signal increment calculated at the present time.

3.1. The Generalized Predictive Controller with Constrains

The use of the predictive controllers in industry is due to its capacity to deal with constrains in a direct way as well the possibility to control multivariable process easily.

Usually constrains are classified in two categories: the soft constrains and the hard constrains. The first type are constrains which violation is permitted during some time period or constrains that represents actuators limits, for example, the maximum superheating degree can be greater than a minimum value, despite of the fact this implicate in a system efficiency reduction. On the other hand, the superheating value is not allowed to be lower than a minimum value because in this case the compressor will aspirate liquid. This is a hard constrain type.

We can see that hard and soft constrains have different implications. Hard constrains are related to safety limitations, output product quality and technological requirements. Soft constrains normally are related to actuators limitations or physical restrictions. If the constrains are not directly considered in the GPC control law formulation, the control signal is calculated by the Eq. (11) and it will physically constrained anyway. If $u(t)$ violates its constrains, the control signal that will applied to the plant is u_{\max} , instead the optimum control, u_c , Fig. (3a). If $u(t)$ does not violate its constrains, the control signal that will applied to the plant is u , instead the optimum control, u_c . Figure (3a) and (3b) are based on Camacho and Bordons (1999). This practical operation way will result in an inadequate closed-loop performance and may lead to even an unstable system behavior.

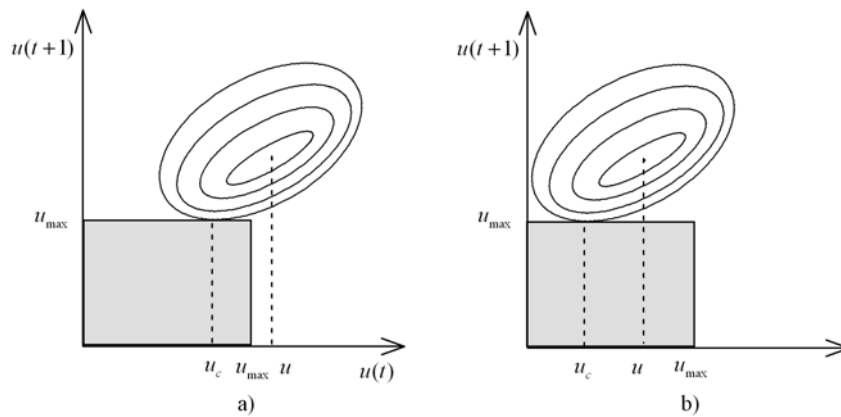


Figure 3. Constrains on the Control Signal.

When the constrains are enclosed in GPC control law we have the following quadratic programming problem:

$$\text{Minimize } J(\tilde{\mathbf{u}}) = \tilde{\mathbf{u}}^T \mathbf{H} \tilde{\mathbf{u}} + \mathbf{b}^T \tilde{\mathbf{u}} \quad (12)$$

$$\text{Subject to } \mathbf{P} \tilde{\mathbf{u}} \leq \mathbf{r} \quad (13)$$

where the time dependence of was omitted for clearer notation and \mathbf{P} and \mathbf{r} are the matrix and vector constrains respectively and theirs structures depended on which constrains are considered.

$$\mathbf{H} = (\mathbf{G}^T \mathbf{G} + \lambda \mathbf{I}) \quad (14)$$

$$\mathbf{b}^T = 2(\mathbf{f} - \mathbf{w})^T \mathbf{G} \quad (15)$$

Since the GPC uses a linear model plant, when constrains are present, any future control signal or output signal or state can be given by a linear combination of the future control increments. This fact is what allows us to write the inequalities given by the Eq. (13).

In this work, we are considering three constrains type: constrains on control increments Eq. (16), constrains on control amplitude Eq. (17) and constrains on output signals Eq. (18). Others constrains type can be founded in (Kuznetsov e Clark, 1994) and (Camacho e Bordons, 1999).

$$du_{\min} \leq u(t) - u(t-1) \leq du_{\max} \quad \forall t \quad (16)$$

$$u_{\min} \leq u(t) \leq u_{\max} \quad \forall t \quad (17)$$

$$y_{\min} \leq y(t) \leq y_{\max} \quad \forall t \quad (18)$$

From the Eq. (16), Eq (17) and Eq (18), we can write \mathbf{P} and \mathbf{r} Eq. (13) as:

$$\mathbf{P} = \begin{bmatrix} \mathbf{I}_{N \times N} \\ \mathbf{T} \\ -\mathbf{T} \\ \mathbf{G} \\ -\mathbf{G} \end{bmatrix} \quad \text{e} \quad \mathbf{r} = \begin{bmatrix} \mathbf{I}(du_{\max} - du_{\min}) \\ \mathbf{l}u_{\max} - \mathbf{T}\mathbf{l}du_{\min} - u(t-1)\mathbf{l} \\ -\mathbf{l}u_{\min} + \mathbf{T}\mathbf{l}du_{\min} + u(t-1)\mathbf{l} \\ \mathbf{l}y_{\max} - \mathbf{f} - \mathbf{G}\mathbf{l}du_{\min} \\ -\mathbf{l}y_{\min} + \mathbf{f} + \mathbf{G}\mathbf{l}du_{\min} \end{bmatrix} \quad (19)$$

where:

\mathbf{l} is a vector of dimension $\sum_i^n NU_i$ having 1 as element and n is the number of outputs;

\mathbf{T} is lower triangular matrix of dimension $\sum_i^n NU_i \times \sum_i^n NU_i$ having 1 as element.

Note that without constrains the quadratic problem given by Eq. (12) is solve making the gradient of J equal to zero, which leads to:

$$\tilde{\mathbf{u}} = -\mathbf{H}^{-1}\mathbf{b} \quad (20)$$

Several methods can be used to solve the problem with constrains Eq. (12) and Eq. (13): the Gradient Projection Method of Rosen, the Dual Primal Interior Point Method and the Predictor Corrector Interior Point Method (Pereira, 1997).

In this work, we use the approach proposed by Camacho and Bordons (1993), that consists in transforming the a GPC Quadratic Problem into a Linear Complementary Problem (Cottle *et. al.*, 1992). The Linear Complementary Problem (LCP) is solved by Lemke's algorithm (Bazaara e Shetty, 1979), which is essentially a pivoting method. We do not present this solution algorithm due to space constrains, but it can be founded in details in the references.

4. Results and Discussion

In this section, we present the simulation results for the Generalized Predictive Control applied to the plant model describe by Eq. (2). The results show significant improvements with respect to a previous work (Silva and Galvez, 2001) in which a multi-loop control strategy was adopted with two independents GPC SISO controllers used to control the superheating and the refrigeration power. In that work it was not considered any kind of constrains either in the control or in the manipulated variables, as it was done in this work. Figure 4 shows the schematic representation of the multiloop strategy and the multivariable strategy.

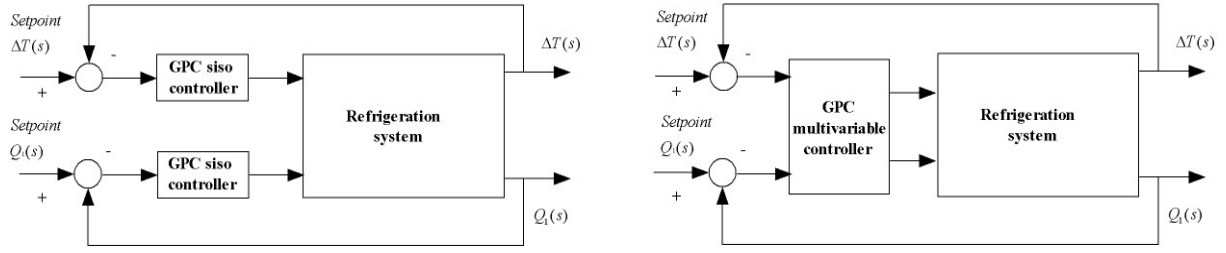


Figure 4. Multi Loop and Multivariable Controllers.

4.1. The Unconstrained Case

To implement the GPC, we use a sample period $T_s = 1$ s. So the discrete version of the evaporator matrix transfer Function, Eq. (2) is given by:

$$\begin{bmatrix} G_{11}(q^{-1}) & G_{12}(q^{-1}) \\ G_{21}(q^{-1}) & G_{22}(q^{-1}) \end{bmatrix} = \begin{bmatrix} \frac{-0.1235 \ q^{-1}}{1-0.9780 \ q^{-1}} & \frac{2.9242-2.9656 \ q^{-1}}{1-0.9833 \ q^{-1}} \\ \frac{-0.0006891 \ q^{-1}+0.0006910 \ q^{-2}}{1-1.9471 \ q^{-1}+0.9477 \ q^{-2}} & \frac{1.9212 \ q^{-1}-1.9182q^{-2}}{1-1.9765 \ q^{-1}+0.9766 \ q^{-2}} \end{bmatrix} \quad (21)$$

The multivariable GPC performance without constraints is presented in the Fig. (5). The superheating set-point is zero. The set point to the cooling power is a square wave with amplitude 0.5 and a frequency of 0.005 Hertz. The adjustment parameters are in Tab. 1.

At the beginning, the cooling power set point is -0.5 . To satisfy the superheating and cooling power set-points, the controller initially reduces the refrigerant volumetric flow rate at evaporator outlet (reducing the compressor speed) and increase the mass flow rate at evaporator inlet (increasing the expansion valve aperture). Initially, these actions combined increase the evaporation pressure and the superheating. After this, a reduction of expansion valve aperture and an increase of compressor speed take the system to a new equilibrium point maintaining the superheating degree and the cooling power at the set point. We can observe that the cooling power reaches the set-point without oscillations.

The system equilibrium is maintained until the time $t = 100$ s, when the cooling power set-point change to $+0.5$ (an variation of 200%). As reply to this new change, there is a fast increase of the compressor speed. As the compressor speed increase, the pressure reduces quickly at the evaporator outlet, leading to a reduction of the evaporation temperature and, therefore, the superheating degree decrease quickly (Machado, 1995). To compensate, the superheating degree reduction, the expansion valve aperture is reduced, this action reduces the mass flow rate at the evaporator inlet and, consequently the superheating degree increase and the system reach a new equilibrium state.

We can observe that cooling power behavior is excellent. Excluding the variations at the cooling power set-point change, we can see that the superheating degree is kept in its desired set point.

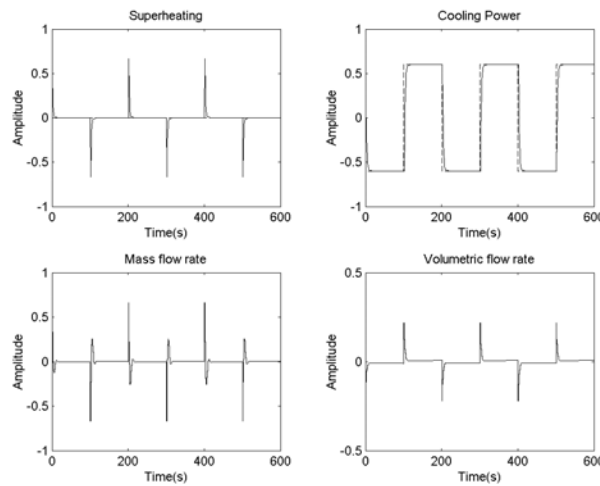


Figure 5. The Closed-loop Unconstrained Case Response.

Table 1. GPC parameters unconstrained case. U_1 - inlet mass flow rate, U_2 – outlet volumetric flow rate, Y_1 - superheating, Y_2 – cooling power

Controller Parameters	Variable			
	U_1	U_2	Y_1	Y_2
N_2 – Prediction horizon	-	-	5	4
N_I – Minimum cost horizon	-	-	1	1
N_u – Control horizon			3	3
λ - Control signal weight	0.001	0.001		
α - Set-point weight	-	-	0.9	0.9

4.2. The Constrained Case

The oscillations of the superheating are undesired for two reasons: the first one, a very small superheating degree represents a risk, with the liquid aspiration by the compressor; and the second one, there is a system efficiency reduction, since a big superheating degree implies in a smaller evaporation area. There are two ways to try to diminish the amplitude of the oscillations: modifying the controller parameters Tab. 1 or including constrains in the GPC control law. We are considering just the second case.

We have included constrains to maximum and minimum superheating values, that are the output amplitude constrains in Eq.(18). Others constrains were included, as following: the maximum and minimum expansion valve aperture and the maximum and minimum compressor speed. They are the input amplitude constrains in Eq. (17). The last constrains type included were the maximum and minimum input variables increments. They are the slew rate constrains in Eq.(16). Fig. 6 shows the simulation results. The constrains numerical values are in the Tab. 2. The controller adjustment parameters are equal to the case without constrains (Tab. 1).

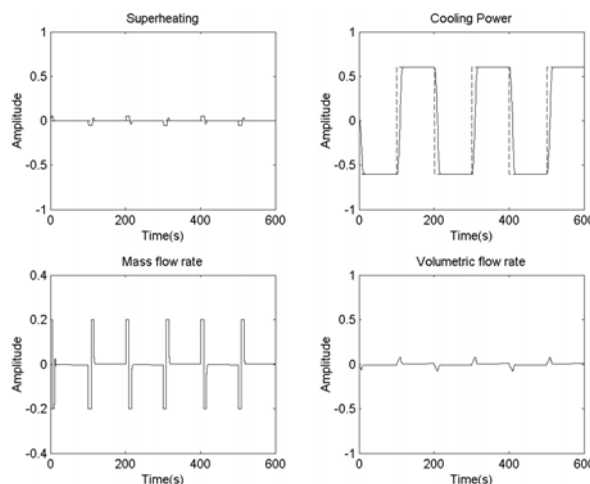


Figure 6. The Closed Loop Constrained Case Response.

Table 2. Constrains Values.

Variable	Limits
Superheating	$-0.05 \leq \Delta T_s \leq 0.05$
Cooling power	$-0.6 \leq \dot{Q} \leq 0.6$
Mass flow rate	$-0.2 \leq \dot{m} \leq 0.2$
Volumetric flow rate	$-0.2 \leq \dot{V} \leq 0.2$
Mass flow rate increment	$-0.4 \leq \Delta \dot{m} \leq 0.4$
Volumetric flow rate increment	$-0.4 \leq \Delta \dot{V} \leq 0.3$

Fig. 7 shows details of the control variables evolution. We can note that constrains are integrally respected. It is clear that system performance is better than the performance of the case without constrains. The superheating degree overshoot is just 5 % when compared to the step-point value. In the no constrains case, this overshoot is equal to 67%. The expansion valve aperture has a maximum and minimum values equals to 0.2 and -0.2 , respectively. In the no constrains case, these values were 0.65 and -0.65 , respectively. The cooling power answer was modified: the rise time was reduced to 18s to 14s and there are not overshoot. Finally, we can observe that the compressor speed presents the

maximum and minimum values of the 0.08 e -0.085 , respectively. In the unconstrained case, these values were 0.218 and 0.221, respectively.

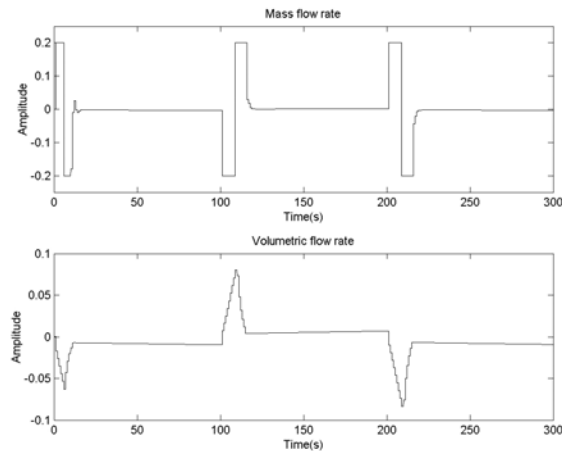


Figure 7. The Control Signals.

The results presented show that proposed controller is an excellent solution for the simultaneous control of the compressor speed and the valve expansion aperture degree. This allows the system have a good performance, leading to a smaller energy consumption and a smaller equipment degradation.

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