HYDRODYNAMIC LUBRICATION APPLIED TO BEARINGS WITH OSCILLATING MOTION

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Abstract. *The purpose of this work is to develop a mathematical model for the hydrodynamic lubrication problem applied to bearings with oscillating motion. In contrast with common bearings, this type of bearing does not perform a complete rotation, belonging to a newly defined class: the bearings with oscillatory movement. For the analysis of the lubrication problem, we start from simple solutions for oscillating plates, parallel and inclined to each other, resulting in Couette and Poiseuille flows, respectively, which were solved by using the same approach as Stokes' 2nd problem. The lubrication or friction reduction between the two surfaces in relative movement is caused by the induced movement of a viscous fluid inside the narrow and variable gap between them. The same basic assumptions of the classical Reynolds' lubrication theory were assumed. In the flat oscillating plates problem, the lower plate is assumed to have an oscillating movement and creates a Couette-Poiseuille combined flow inside the gap. After simplification, the momentum equation is solved to determine the velocity distributions. From integration of the mass conservation equation, we obtain a second order differential equation for the pressure distribution. It is shown that, at the limit of zero rotational velocity, this expression becomes the classical Reynolds' equation of hydrodynamic lubrication. As result we show graphcally the pressure distribution as function of angle θ and time t. We also solved the classical Reynolds' equation to compare with the results obtained from this model.*

Keywords*: Hydrodynamic Lubrication,Oscillating Motion, Bearings.*

1. Introduction

Since the beginning of century XX, due to great industrial demand, the research in the tribology area had evolved very much. Development in the quality and service characteristics of the lubricants had also collaborated in the solution of the tribological problems. However, the rhythm of our industrial society demands higher velocities, load and precision, many times in very hostile environments, requiring additional developments in this subject.

The economy gotten in particular cases improvements are very small and these tribological improvements aren't always sufficiently understood. Due the great amount of tribological contacts in the machines, a small particular improvement in each one of these contacts allows to reach a great economy.

In the energetic point of view, we can remember that almost half of the energy produced in the world is used to win the friction, what allows to deduce that better tribological projects have great importance in the future of the humanity. Factors like the international crisis of the oil in the end of 70's and the increasing ecological concern from 90's had also stimulated the research in tribology searching for more efficient systems, mainly in the direction of reduction of pollutants emission, noise and lubricants consumption, which must be discarded after use. (Stoterau, R. L.; 2004)

The analysis of the mechanical projects that must support loads and allow relative displacement between parts, always takes us to the question: Which is the best solution for the problem to support load through the interface with acceptable friction and wear?

In this work we take the viscous fluid lubrication as subject of study, applied to bearings with oscillating motion. The surfaces in contact are separated by the fluid film and the sustentation pressure is generated from the relative movement of the surfaces.

According Stoterau, R. L. (2004) historically the first scientific studies on friction and lubrication were done by Leonardo da Vinci in 16th century. In the following century Isaac Newton in 1687 introduced the viscosity concept. The 19th century, stimulated for the necessities of the industrial revolution, was fertile in researchers on the subject. This century presents the figures of Navier in 1823 and Stokes in 1845 which develop the equations for viscous fluids. Petrov in 1883 was the first person to research on the nature hydrodynamics of the viscous friction. He obtained the expression for determination of the viscous friction coefficient, also called Petrov's law. Beauchamp Tower in 1883 during his experiments to observe the best form of a shaft lubrication validated experimentally the Reynolds equation. Osborne Reynolds in 1886 presented additional solutions for the Beauchamps Tower bearings, giving solutions for situations that involve the problem of squeeze film. The notion that a radial bearing have gaps, was developed concerning to the problem of cavitation in bearings and demonstrating the viscosity dependence with temperature. From these considerations Reynolds establishes the bases of the fluid film lubrication. Reynolds mathematically explained and formulated the problem of hydrodynamic lubrication.

2. Fluid Mechanics Applied to Hydrodynamic Lubrication

For the analysis of the lubrication problem, we take as starting point classical problems of fluid mechanics, as the Couette and Poiseuille flows and the 2nd Stoke's problem (White, F. M., 1974) in order to get enough fundamentals of the involved concepts, the physical properties and the possible simplifications in the mathematical model.

The lubrication or friction reduction between two bodies is followed by the movement of a viscous fluid through a narrow gap with inclination between the two bodies. At least one of the two bodies is in movement. This theory was developed by Reynolds (White, F. M., 1974).

The problem is the journal bearing shown in Figure 1. The low plate moves with a velocity U and creates inside the gap a combined Couette and Poiseuille flow.

We assume that the a superior plate is fixed, the gap is very narrow, or, $h(x) << L$, and it decreases from h_0 in the entrance to h_L in the exit. The problem then is to determine the pressure and velocity distributions.

Assuming in Fig. 1 that the flow is two-dimensional, it means, $\frac{\partial}{\partial z} = 0$ in z direction. We also add the hypothesis of Stoke's Flow that is worthless inertia. For this consideration we have: $\rho u \frac{\partial u}{\partial x} << \mu \frac{\partial^2 u}{\partial y^2}$.

Finally we also assume an incompressible fluid and steady state. Thus we obtain for an analysis of the hypotheses above and for order of magnitude, the simplified equations of mass continuity and the momentum conservation in x and y.

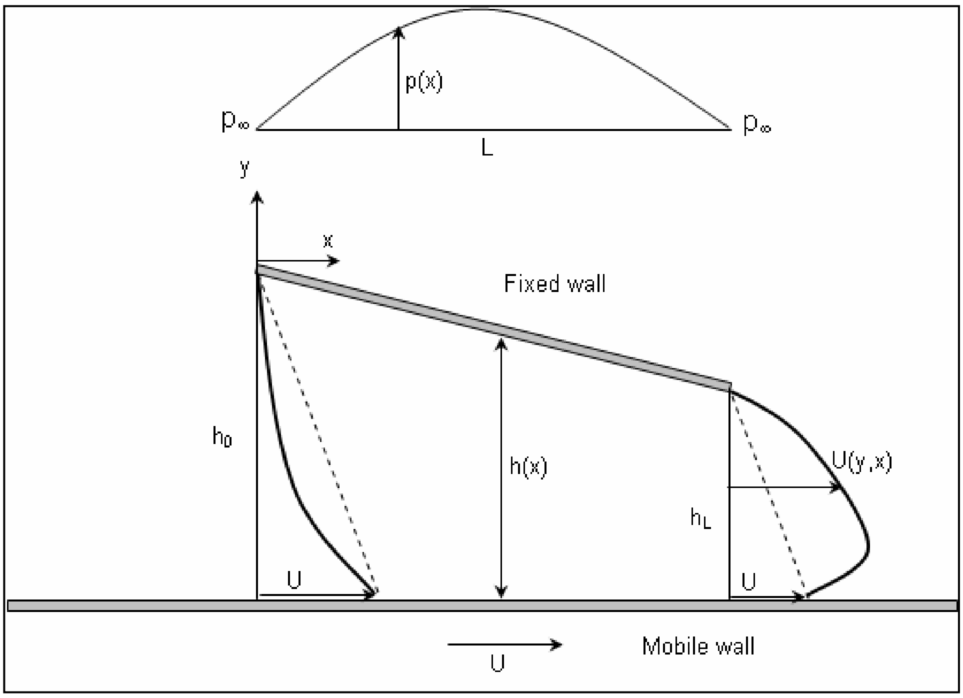

Figure 1. Flow in a narrow gap

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{Mass Continuity} \qquad (1)$$

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = \upsilon \frac{\partial^2 u}{\partial y^2} \qquad \text{Momentum Conservation in x direction} \tag{2}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} = 0 \qquad \text{Momentum Conservation in y direction} \tag{3}$$

In any section of length L, the velocitie profile is a combination of Couette and Poiseuille flow and can be obtained integrating two times the Eq.(2) in y.

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial P}{\partial x} y + C_1 \qquad \text{and} \qquad u = \frac{1}{2\mu}\frac{\partial p}{\partial x} y^2 + C_1 y + C_2$$

Considering the boundary conditions: y = 0, u = U and y = h, u = 0, we have:

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x} y(y-h) + U\left(1 - \frac{y}{h}\right) \tag{4}$$

Coming back to the system of differential equations, the integration of the Eq. (1) gives

$$\int_0^h \frac{\partial u}{\partial x} dy = -\int_0^h \frac{\partial v}{\partial y} dy = -v(h) + v(0) = 0 \tag{5}$$

Derivating U(y) with respect to x and integrating in y, a second order differential equation for the pressure distribution is finally obtained:

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) = 6\mu U \frac{\partial h}{\partial x} \tag{6}$$

Here we assume that U is constant and the variation of the gap h(x) is known. Then we can find p(x) to conditions p(0) and p(L) = p_∞. The equation is a simplified form of the Reynolds equation for the hydrodynamic lubrication.

2.1. Solution for a linear inclined gap

The Eq.(6) can numerically be integrated for any variation of h(x). A closed solution can be obtained for a linear inclined gap as the Fig. 1.

$$h = h_0 + \left(h_L - h_0\right)\frac{x}{L} \tag{7}$$

Substituting the expression of h in the Eq.(6) and integrating twice we have:

$$\frac{p - p_\infty}{\mu U L / h_0^2} = \frac{6\left(x/L\right)\left(1 - x/L\right)\left(1 - h_L/h_0\right)}{\left(1 + h_L/h_0\right)\left[1 - \left(1 - h_L/h_0\right)\left(x/L\right)\right]^2} \tag{8}$$

This expression is plotted in Fig.2 for some contraction ratios of the gap.

When the inclination is small and the gap is approximately plain, the pressure distribution is also approximately symmetrical with $p_{máx}$ located in x/L = 0,5.

As the inclination increases, $p_{máx}$ increases in direction to the exit plan and in the order of $\mu UL / h_0^2$, being able to reach very high values.

This provides a great force of sustentation and the sliding block supports a great load without contact between the plates.

If we revert the movement of the wall in Fig. 2 to move in the left direction where U < 0, the pressure in the Eq.(8) changes to negative. The fluid simply will not develop a great negative pressure, but certainly it will cavitate and form a vacuum in the gap.

Thus, the flow inside a narrow and inclined gap can also not support much load or exactly not to provide good lubrication.

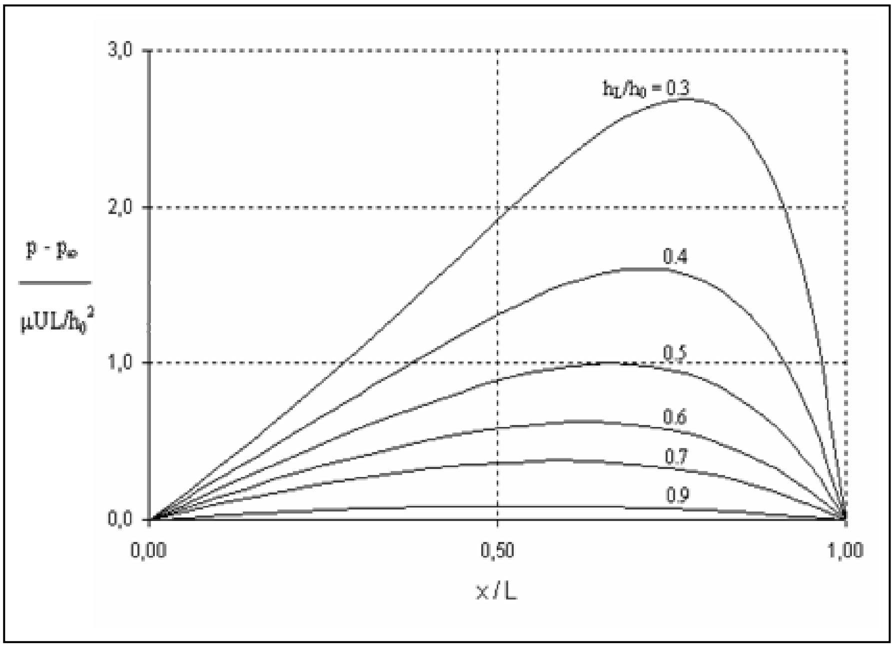

Figure 2. Pressure distribution in a two-dimensional linear and inclined gap

This effect is inevitable in a rotating bearing where the gap contracts and then expands; a partial cavitation always occurs causing wear of the bearing.

2.2. Solution for a linear inclined gap with oscillating motion

After developing a series of classical cases of the fluid mechanics, with the purpose of getting a theoretical basics to develop the model of bearings with oscillatory motion, we finally reached the formulation of the hydrodynamic lubrication problem of such type of bearings.

Considering the problem of Fig.3, and the following hypotheses:

i. $h << L$;

ii. $\dfrac{\rho UL}{\mu}\left(\dfrac{h}{L}\right)^2 << 1$;

iii. Two-dimensional flow.;

iv. Incompressible fluid.

Now we have the problem is finding the components of the velocity u(x, y, t) and v(x, y, t) field, as well as the pressure p(x, y, t) field that satisfies the following system of differential equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{Mass Continuity} \qquad (9)$$

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\frac{\partial^2 u}{\partial y^2} \qquad \text{Momentum Conservation in x direction} \qquad (10)$$

$$\frac{\partial p}{\partial y} = 0 \quad \text{Momentum Conservation in y direction} \tag{11}$$

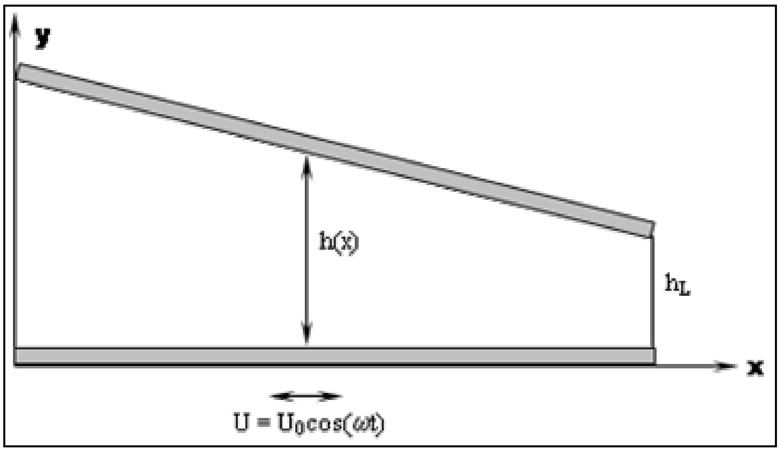

Figure 3. Problem of Boundaries in Movement

The boundary conditions are:

Lower wall: u(x,0,t) = U_0 cos(ωt) e v(x,0,t) = 0
Upper wall: u(x,h,t) = v(x,h,t) = 0
Entrance and exit of the gap: p(0,t) = p(L,t) = p_∞

Considering a solution in the complex form:

$$u(x,y,t) = U(x,y)\, e^{i\omega t} \tag{12}$$

$$p(x,t) = P(x)\, e^{i\omega t} \tag{13}$$

$$v(x,y,t) = V(x,y)\, e^{i\omega t} \tag{14}$$

We have the following system:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \tag{15}$$

$$U\, i\omega = -\frac{1}{\rho}\frac{d\,P}{d\,x} + \upsilon \frac{\partial^2 U}{\partial y^2} \tag{16}$$

Now submitting the equations (15) and (16) to the following boundary conditions:

Lower wall: U(x,0,t) = U_0 and V(x,0,t) = 0
Upper wall: U(x,h,t) = V(x,h,t) = 0
Entrance and exit of the gap: P(0,t) = P(L,t) = P_∞

This problem can be considered as an extension of the parallel plates problem where one of them oscillates, that now it includes an additional term $F(x) = -\frac{1}{\rho}\frac{dP}{dx}$ that makes the Eq.(16) non-homogeneous. Due to the linear character of this equation, the general solution will be the addition of the general solution of the homogeneous equation with a particular solution:

$$U(x,y)=U_0\frac{senh\left[(h-y)\sqrt{\frac{i\omega}{\upsilon}}\right]}{senh\left(h\sqrt{\frac{i\omega}{\upsilon}}\right)}+\frac{1}{2\mu}\frac{dP}{dx}y(y-h) \tag{17}$$

Thus:

$$u(x,y,t)=\left\{U_0\frac{senh\left[(h-y)\sqrt{\frac{i\omega}{\upsilon}}\right]}{senh\left(h\sqrt{\frac{i\omega}{\upsilon}}\right)}+\frac{1}{2\mu}\frac{dP}{dx}y(y-h)\right\}e^{i\omega t} \tag{18}$$

Coming back to the system of differential equations, the integration of the Eq. (15) it gives:

$$\int_0^h\frac{\partial U}{\partial x}dy=-\int_0^h\frac{\partial V}{\partial y}dy=-V(h)+V(0)=0 \tag{19}$$

Derivating U(x, y) with respect to x and integrating in y, we finally obtain the differential equation for the pressure distribution:

$$\frac{d}{dx}\left(h^3\frac{dP}{dx}\right)=\frac{6\mu U_0}{cosh^2\left(\frac{h}{2}\sqrt{\frac{i\omega}{\upsilon}}\right)}\frac{dh}{dx} \tag{23}$$

At this moment it is very interesting to show that, at the limit when rotational oscilatory velocity tends to zero, this expression become the classical Reynolds equation for hydrodynamic lubrication in Eq.(6).

Integrating this equation and using the known boundary conditions P(0,t) = P(L,t) = P_∞,we obtain an analitycal expression for the pressure distribution.

$$P(x)=P_\infty+12\mu U_0\sqrt{\frac{\upsilon}{i\omega}}\left[\int_0^x\frac{\tanh\left(\frac{h}{2}\sqrt{\frac{i\omega}{\upsilon}}\right)dx}{h^3}-\left(\int_0^L\frac{\tanh\left(\frac{h}{2}\sqrt{\frac{i\omega}{\upsilon}}\right)dx}{h^3}\right)\frac{\int_0^x\frac{dx}{h^3}}{\int_0^L\frac{dx}{h^3}}\right] \tag{24}$$

Once the presure is known, the load components can be evaluated. It is convinient to determine the components of the resultant load along and perpendicular to the center line, the line drawn through the shaft center and the bearing center. For this purpose, we make a change of coordinates and now we describe the Eq.(23) and Eq.(24) as function of angle θ. $\varepsilon=\frac{e}{c_r}$, where ***e*** is the eccentricity between bearing and journal ans c_r is the radial clearence. Figure 4 describes the coordinate system adopted and the expression for h becomes:

$$h(\theta)=c_r(1+\varepsilon\cos\theta) \tag{25}$$

And the differential equation to be solved for the pression distribution becomes:

$$\frac{d}{d\theta}\left(h^3\frac{dP}{d\theta}\right)=\frac{6\mu RU_0}{\cosh^2\left(\frac{h}{2}\sqrt{\frac{i\omega}{\upsilon}}\right)}\frac{dh}{d\theta} \tag{26}$$

According Cataruzzi, E. F., (1998) the forces can be determined by :

$$F_x=-2.Z.\int_0^{\pi}P.r_b\cos(\theta)d\theta \tag{27}$$

$$F_y=2.Z.\int_0^{\pi}P.r_b\sin(\theta)d\theta \tag{28}$$

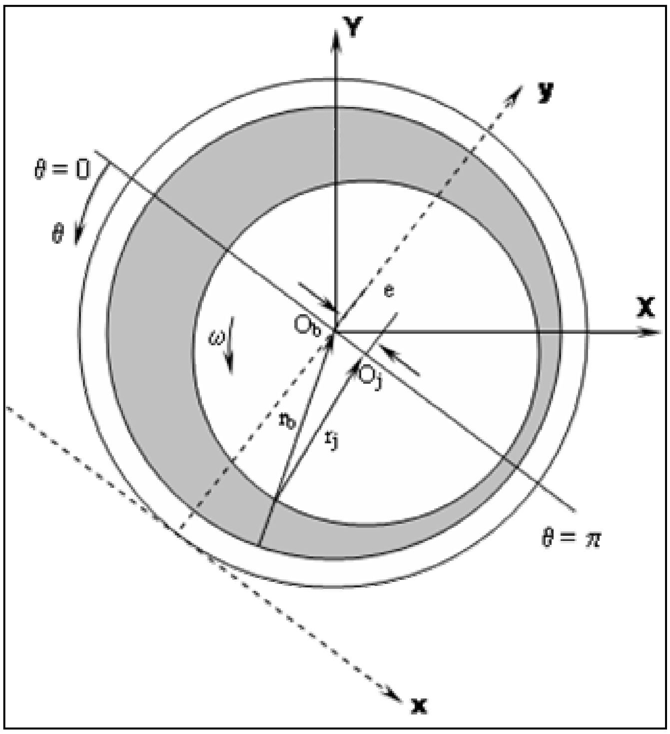

Figure 4. Coordinate system and force in a journal bearing

Z is the length of the bearing in the axial direction.
At this moment we can found the hydrodynamic forces in this kind of bearings with oscillating motion.

3. Results

First of all, to determine the integration of the Eq.(26) for the pressure distribution in θ direction , we used a numerical procedure found in MATLAB® that is the adaptive quadrature. Adaptive quadrature is a numerical integration procedure in which the interval of integration is recursively subdivided until an error tolerance is reached for the approximate integral on each subinterval. (Gander, W., Gautschi, W., 2000)

Due to the oscillating movement of the journal, we can expect an oscillating distribution of the pressure in θ direction as function of time t, because when the journal rotate with ω in the same direction that θ increase, we have the pressure distribution as shown in the Fig. 6a. When the journal rotate with ω in the other direction, we have the pressure distribution as shown in the Fig. 6b.

Assuming some values for bearing and journal dimensions, radial clearece,viscosity, rotational velocity, we tried to find conditions that can assure that we have hydrodynamic lubrication and the oil film thickness is more than 4 times the RMS rugosity of bearing's surface where here we consider 2μm. (Norton, R. L., 1996)

Figure 7a shows the pressure distribution in θ direction plotted in the interval from θ =0 to θ =π, with the journal rotating with ω in the same direction of θ. Fig. 7b shows the pressure distribution as function of angle θ and the time t. We can also note the similarity of the Fig.7a with Fig. 2. The difference is shown in Fig. 6b due to proposed solution in complex form for P(x,t) in the Eq.(13).

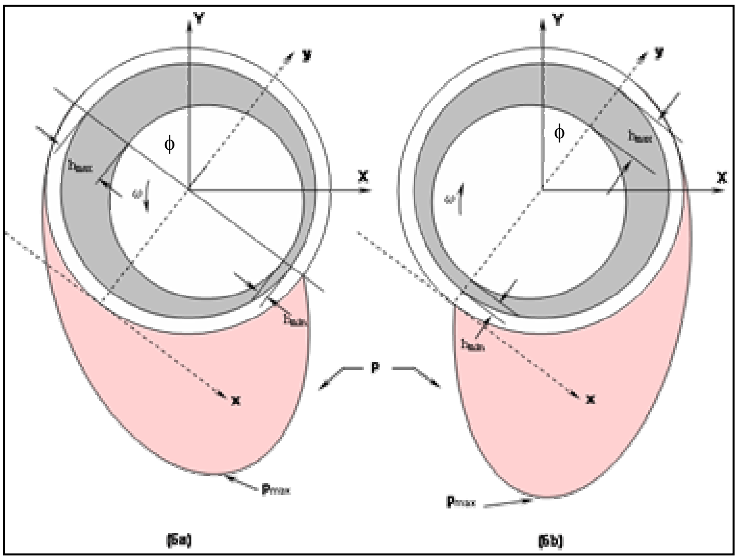

Figure 6. Expected pressure distribution according to the journal movement.

Table 1. Constant values used to solve numerically the problem of hydrodynamic lubrication with oscillating oil film bearings

Properties and Units		Symbol	Value
Journal Radius	[m]	R	$20 \cdot 10^{-3}$
Bearing Length	[m]	Z	$80 \cdot 10^{-3}$
Eccentricity ratio		ε	0.52
Radial clearance	[m]	c_r	$40 \cdot 10^{-6}$
Rotational Velocity	[rad/s]	ω	20.94
Cinematic Viscosity	[m^2 / s]	ν	$1.38 \cdot 10^{-5}$
Absolute Viscosity	[Pa.s]	μ	$1.17 \cdot 10^{-2}$
Shaft Weight	[N]	W	1500

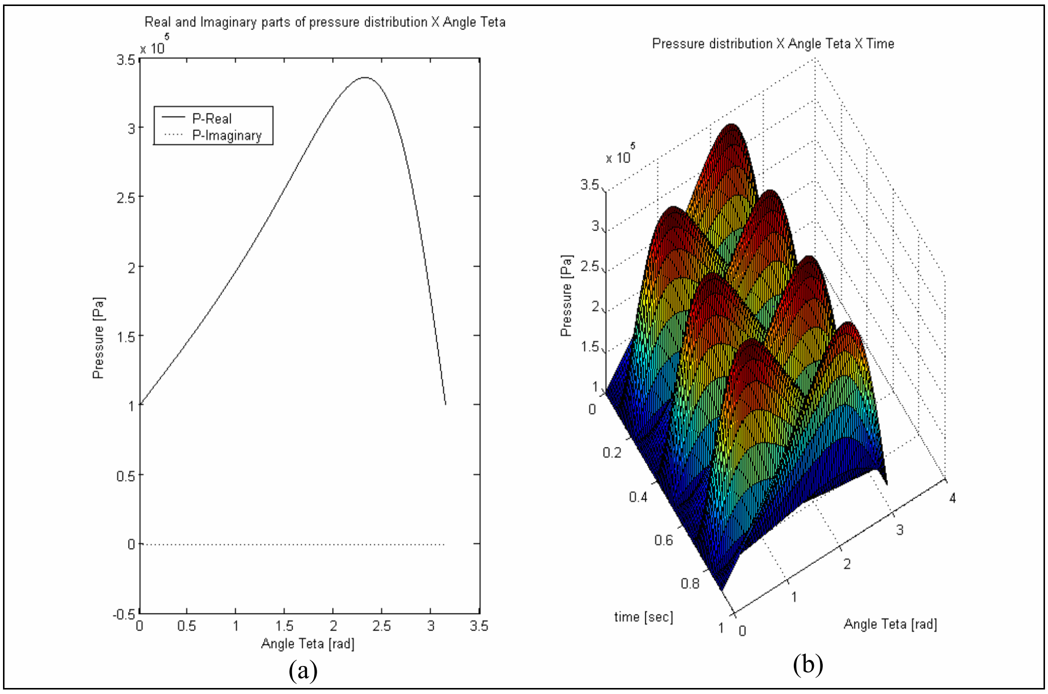

Figure 7. Pressure distribution X Angle θ X Time in a two-dimensional linear and inclined gap.

Figure 8 shows a comparison between the pressure distribution in θ direction for the two cases analysed, with and without oscillating motion. It is possible to note not only the equality between the two curves obtained, as well, to verify numerically that when the oscillating rotational velocity ω tends to zero, we fell into the classic solution of the Reynolds' equation for a linear inclined gap assuming the same input data.

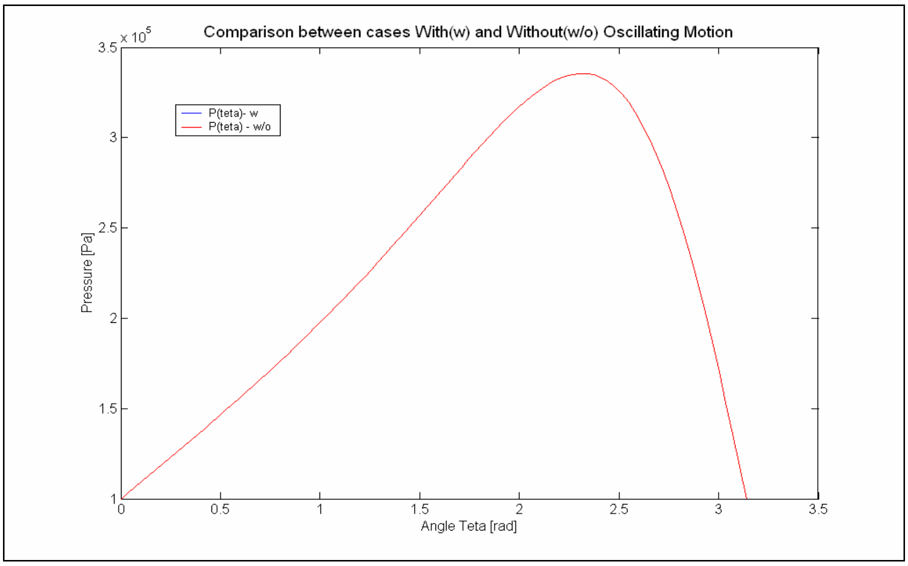

Figure 8. Comparison between the pressure distribution in θ direction for the two cases analysed.

Another interesting result is showed at Fig. 9, that allow us to understand the behaviour of the loading carrying forces according to the shaft position and the oscillating rotational velocity. The minimum force found must be the same as half of the shaft weight, because we are considering the shaft carried by two bearings. For this eccentricity ratio of 0,52 we found this minimum force equal to half shaft weight and we verified that the minimum force is found when the angle of equilibrium ϕ on the Fig.6 is zero and the velocity is maximum $U = U_{0.}$ The maximum force is obtained where absolute value of ϕ is maximum.

Using the Eq.(25), we can find the minimum oil film thickness, that is 19.2μm or more than 9 times the RMS rugosity of bearing's surface.

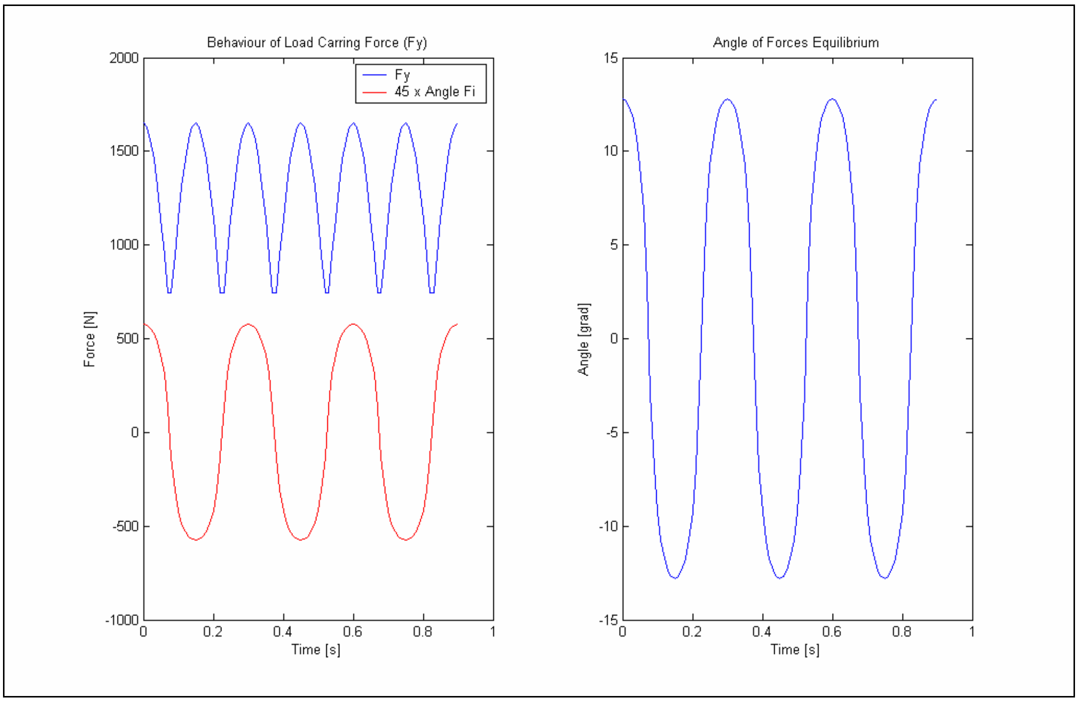

Figure 9. Behaviour of Load Carrying Forces

4. Conclusion

For the analysis of the lubrication problem, we start from simple solutions for oscillating plates, parallel and inclined to each other, resulting in Couette and Poiseuille flows, respectively, which were solved by using the same approach as Stokes' 2^{nd} problem. The same basic assumptions of the classical Reynolds' lubrication theory were assumed. Thus we had to verify the hypothesis of Stokes' flow or worthless inertia. After check this important information that assures the linearity of the Eqs. (9), (10) and (11), we continued solving this model.

Figure 7 can clearly demonstrate the behaviour of the pressure distribution as function of angle θ and time t as expected, according Fig. 6. To validate this model we plotted in the same figure the two solutions. One assuming this model for oscillating motion bearings and other assuming the Reynolds' equation. As results we obtained the Fig. 8 that can assure the conformity of this model with the model proposed by Reynolds.

Verifying the hydrodynamic lubrication of this system assuming 2μm as rugosity of finishing surface of the bearing, we can assure that the minimum oil film thickness found for this rotational velocity and lubricant, allows a complete lubrication.

The next step of our work is integrate this equations in a rotordynamic system for example the Jeffcot Rotor and analyse its behaviour.

5. Acknowledgements

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7. Responsibility notice