EFFICIENT PROCEDURE TO CALCULATE TRANSMISSION ERROR OF HELICAL GEAR PAIRS WITH GLOBAL TOOTH CONTACT ANALYSIS USING A COMBINED PSEUDO-INTERFERENCE AND LINEAR PROGRAMMING SOLUTION

Carlos Henrique Wink
Eaton Limited - Transmission Division
Rua Clark, 2061 - P.O. 304
Valinhos - São Paulo
13279-400 Brazil
Phone: 55 19 3881-9494
Fax: 55 19 3881-9183
e-mail: carloshwink@eaton.com

Alberto Luiz Serpa
State University of Campinas
Faculty of Mechanical Engineering
Department of Computational Mechanics
P.O. 6122 Campinas - São Paulo
13083-970 Brazil
Phone: 55 19 3788-3387
e-mail: serpa@fem.unicamp.br

Abstract. *In this paper an efficient and accurate computational procedure to determine load distribution of gear teeth regarding global tooth contact is presented. This procedure is applied to static transmission error calculation of helical gear pairs with intentional tooth profile modifications and manufacturing errors. The tooth contact zone is initially estimated using a pseudo-interference method and tooth elastic deflections are calculated employing the influence coefficient method. The tooth contact problem is reformulated based on maximum rigid body displacement so that it can be solved using standard optimization algorithms, such as those of linear programming. An interior-point method is applied to solve the contact problem and to determine the load distribution. Transmission error results of the loaded situation and computatonal effort required to solve the problem using this procedure are compared to an iterative-gradual load application method. In a numerical example of a helical gear pair, the computer time is reduced about 100 times when both pseudo-interference method and linear programming are applied. The significant gain in computational efficiency of the proposed procedure leads the global tooth contact analysis to its practical application as a design tool in the accurate transmission error calculation of helical gear pairs with modified tooth surfaces.*

Keywords*: gear transmission error, tooth contact analysis, pseudo-interference, interior-point method.*

1. Introduction

Noise has been one of the major areas of gear research over the years. Noise regulations, customer requirements and competitive pressures have increased the attention to sound quality in geared systems. The requirement of designing quiet gear pairs becomes even more crucial as other components of mechanical equipment are becoming less noisy.

The main source of noise and vibration of geared systems is the so-called transmission error (TE) (Houser, 1991). TE is the irregularity of motion transmitted by gear pairs due to deviations from the ideal tooth contact. Ideal involute gears transmit uniform motion since they are geometrically perfect and with infinite stiffness, which is not the case of real-life gears. Therefore, TE arises from topological modifications of teeth, manufacturing errors, shaft misalignments, and elastic deflections of the teeth and the support structure of the gear pair when it is under load (Welbourn, 1979).

Several mathematical models have been presented to calculate the TE under load, and different approaches have been proposed based on analytical, finite elements, empirical methods and hybrid methods (Özgüven and Houser, 1988). The TE modeling usually considers, for simplification, that each tooth pair is in contact exclusively on the plane of action. In this case the contact deviations from the plane of action are neglected. However, the present authors proposed a procedure to calculate the TE of helical gear pairs under load with global tooth contact analysis (Wink and Serpa, 2003), extending the work of Kurokawa *et al.* (1996) for spur gears. The present authors also carried out a study about the effects of the tooth contact deviations from the plane of action, and have shown that the global contact analysis procedure can be more precise in calculating the TE (Wink and Serpa, 2005). Some models based on finite element method can also consider the global tooth contact (Vijayakar, 1991).

In order to calculate the TE under load, it is necessary to determine the load distribution of the contacting teeth. The formulation of the solution of the load distribution in gears is equivalent to the formulation of the solution of the generalized elastic contact problem.

In the particular case of tooth contact analysis models that assume the existence of tooth contact exclusively on the plane of action, a few discrete points are evenly spread out on the lines of contact, and linear programming is successfully employed to solve the problem. A procedure based on a modified simplex-type algorithm firstly proposed by Conry and Seireg (1973) has been widely employed to solve the contact problem and to determine the load distribution in gear teeth (Beacham *et al.*, 1999; Norman, 1995; Park and Lee, 1993). The minimization problem stated by Conry and Seireg (1973) could be solved using a standard linear programming if it were not for a criterion of contact that requires a modification of the entry rules. This criterion of contact was stated to handle with the discrete points that can be out of contact for a certain rigid body displacement, and in which the acting load must be zero and not negative.

In the work of Wink and Serpa (2003), a method of gradual and incremental load application to determine load distribution in global tooth contact problems was proposed. The method consists of dividing the total load into small increments of loads, which are gradually applied to contacting points. In each iteration, part of the load is applied to the discrete points in contact, the deflections due to those applied loads are calculated, and the contact points are reevaluated in order to consider the effects on contact caused by the tooth deflections, according to the gear motion and kinematics. Thus, the actual contact of the teeth is considered in the analysis. Although those authors have shown good results on load distribution and TE calculation using that method, it is not computationally efficient and its accuracy is related to the degree of discretization of the total load (load increment size) and to the degree of discretization of the tooth surfaces (number of contacting points). By the other hand, other numerical solutions can be inefficient in solving the contact problem due to the huge compliance matrices required for global contact analysis.

Although global contact model spread discrete points out on the whole tooth surfaces, it is known that only a small region of the teeth is in contact at a certain position of mesh cycle and under a given torque. Therefore, if a method to identify the points pertaining to that contact region is employed, the analysis can be restricted to those discrete points so that a reduced contact problem, which properly represents the original problem, can be solved with less computational effort. Pimsarn and Kazerounian (2002) presented a method called pseudo-interference to estimate the mesh stiffness of spur gear. Using the pseudo-interference method the effective region of contact can be estimated and the number of discrete point for the analysis can be reduced. However, this reduced number of points for contact analysis can be too higher than those solved by the modified simplex-type algorithm in the models that assume the contact on the lines of action. By the other hand, interior-point methods have shown to be effective solving some large-scale linear problems (Vanderbei, 1997). Hence, in this work an interior-point method is evaluated to solve the reduced contact problem, which is obtained after applying the pseudo-interference approach.

The objective of this work is to propose a more efficient method in terms of computational effort and accuracy to determine the load distribution in gears and to calculate the static TE regarding global tooth contact analysis. The proposed procedure can be used as a design tool to precisely predict the TE in the design phase.

2. Gear contact model

The gear contact model regarding global tooth contact analysis initially proposed by Kurokawa *et al.* (1996) for spur gears, and later extended for helical gears by Wink and Serpa (2003) was adopted in this work. The geometrical model of a gear pair is shown in Fig. 1.

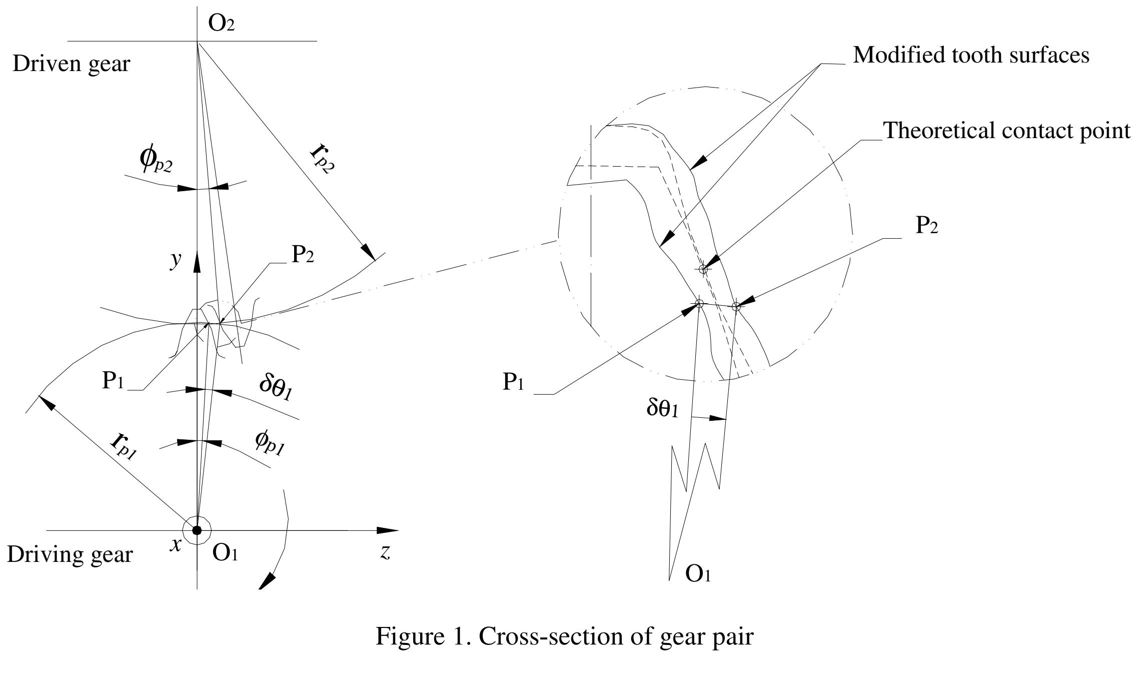

Figure 1. Cross-section of gear pair

The tooth surfaces of the driven gear are properly represented by discrete points, and respective contact points on driving gear teeth are calculated by geometry and kinematics theory of helical gears. Deviations from the ideal tooth surfaces caused by manufacturing errors such as adjacent pitch errors, profile errors, misalignment and lead errors, and intentional modification specified in the gear design such as tip profile relief and lead crowning can be considered in that model converting the longitudinal values to a convenient angular form, and adding them to the respective discrete points on tooth surfaces.

The angle $\delta\theta_1$ between mating points of the driven and driving gears is calculated for all discrete points that represent the tooth surfaces. Hence, the minimum angle $\delta\theta_{1min}$ can be determined and the longitudinal TE can be calculated by Eq. (1),

$$\delta t_2 = -\delta\theta_{1\,min} \cdot \frac{N_1}{N_2} \cdot R_{B2} \quad , \tag{1}$$

where, δt_2 is the longitudinal TE under no load for a particular position on the meshing cycle, $\delta\theta_{1min}$ is the minimum angular displacement of driving gear so that tooth surfaces can be in contact, N_1 / N_2 is the designed gear ratio and R_{B2} is the base radius of the driven gear.

This analysis may be repeated for several positions of mesh cycle in order to represent the gear motion, since contact path and number of tooth pairs in contact change along the mesh cycle.

In this work, the elastic deflections of teeth such as bending and local contact deflections are considered according to the work of Wink and Serpa (2003). Gear bodies, except teeth, and gear support structure are assumed to be rigid.

The deflection of the local contact zone is based on Hertz's compressive stress model, and it is calculated using the equation presented in Conry and Seireg (1973) with the function of free edge effect proposed by Umezawa and Ishikawa (1973), i.e.,

$$w_H = \frac{25 \cdot (1-\nu^2) \cdot p_N}{\pi \cdot E} \cdot c(\hat{\rho}) \cdot c(\hat{\varepsilon}) \quad , \tag{2}$$

where, w_H is the deflection of tooth contact surface at the point where load is applied and it is taken in the normal direction to the tooth surface, p_N is the normal load applied on discrete point, E is the Young's modulus of the material, ν is the Poisson's ratio, $c(\hat{\rho})$ and $c(\hat{\varepsilon})$ are the correction functions for the effect of the free edge shown in Umezawa and Ishikawa (1973), $\hat{\rho}$ is the distance between the loaded point and the free edge related to gear faces, and $\hat{\varepsilon}$ is the distance between the loaded point and the free edge related to the gear outside radius.

The bending deflection of the contacting teeth is calculated using the approximate formula proposed by Umezawa (1972) that is based on the deflection of a rack-shaped cantilever plate with finite width when a concentrated load is applied. This formula utilizes some functions to the plate bending deflections based on numerical solutions obtained by finite differences calculations. Park and Lee (1993) proposed new approximation functions based on the results of a finite element analysis taking into account the tooth foundation deflections. Umezawa's equation (1972) with the updated functions proposed by Park and Lee (1993) is employed in this work, i.e.,

$$w_B(x', y') = \frac{U \cdot f(\lambda \cdot \bar{x}') \cdot g(\bar{y}')}{MP} \cdot \sum_{k=1}^{N} p'_k \cdot \frac{v(\bar{r}_k)}{f\left(\lambda \cdot \left|\bar{x}'-\bar{\zeta}'_k\right|\right) \cdot g\left(\left|\bar{y}'-\bar{\eta}'_k\right|\right)} \cdot f(\lambda \cdot \bar{\zeta}'_k) \cdot g(\bar{\eta}'_k) \quad , \tag{3}$$

where,

$w_B(x',y')$ is the total deflection at point (x',y');

U is the absolute value of deflection of the rack-shaped cantilever plate at its origin when a concentrated load is applied there;

$g(\bar{y}') = \sqrt{G(\bar{y}')}$ is a function that determines the deflection caused by a concentrated load applied in the heightwise direction of the plate;

$f(\bar{x}') = \sqrt{F(\bar{x}')}$ is a function that determines the deflection caused by a concentrated load applied in the widthwise direction;

$v(\bar{r}_k)$ is a function that defines the deflection measured at a point distant r from the concentrated load point;

$\bar{r}_k$ is the distance from the concentrated load point $P(\zeta, \eta)$ to the point where the deflections are measured $P(x', y')$. It can be obtained using the Eq. (4);

$\bar{x}' = \frac{x'}{H}$, (variables with upper bar mean a division by plate height);

Variables with apostrophe such as x' and y' mean coordinates related to the rack-shaped cantilever plate.

k=1,2,...,N, where N is the number of loaded discrete points;

$$\bar{r}_k^2 = (\lambda \cdot \bar{x}' - \lambda \cdot \bar{\zeta}_k')^2 + (\bar{y}' - \bar{\eta}_k')^2 \quad , \tag{4}$$

and λ is a scale of coordinate applied on the coordinates x' e ζ'.

The influence coefficient method is employed in this work using Eq. (3), which makes it possible to calculate the bending deflection at any discrete point caused by a concentrated load applied to another point of tooth surface.

The total longitudinal deflection calculated by Eq. (2) and (3) at each discrete point can be transformed to angular displacement and can be added to the angles ϕ_{p2} and ϕ_{p1} shown in Fig. 1 related to driven gear and driving gear, respectively.

3. Contact problem and load distribution

In order to calculate the elastic deflections it is necessary to determine the intensity of load applied to each discrete point in contact over the tooth surfaces. The formulation of the solution of the load distribution in gears is equivalent to the formulation of the solution of the generalized elastic contact problem.

In this work, a combined pseudo-interference method and linear programming solution is proposed to solve the contact problem regarding the global tooth contact analysis.

3.1. Pseudo-interference method

The pseudo-interference method, proposed in the work of Pimsarn and Kazerounian (2002) to estimate the stiffness of spur gear tooth mesh, is applied in this work to the problem of global tooth contact analysis of helical gears in order to reduce the number of discrete points for analysis.

Using the pseudo-interference method, the region of contact is determined imposing a rigid body rotation on the driving gear, which would cause an interference with the driven gear teeth. This interference would be physically impossible assuming that there is no interpenetration of teeth. That is the reason it is called virtual or pseudo-interference. Figure 2 illustrates the pseudo-interference of a gear pair.

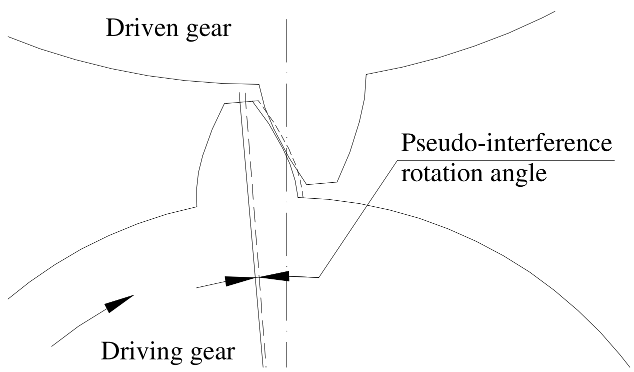

Figure 2. Illustration of the pseudo-interference of a pair of teeth

The rigid body rotation is estimated using a semi-empirical equation based on finite element model presented in the work of Pimsarn and Kazerounian (2002), i.e.,

$$K_{si}(r) = (A_0 + A_1 X) + (A_2 + A_3 X) \cdot \frac{r - R_p}{(1 + X)m} \quad , \tag{5}$$

where, $K_{si}(r)$ is the specific stiffness in N/μm at a loading point of radius r, X is the addendum modification coefficient, N is the number of teeth, R_p is the pitch radius, and the coefficients are:

$$\begin{aligned} A_0 &= 3.867 + 1.612N - 2.916 \cdot 10^{-2} N^2 + 1.553 \cdot 10^{-4} N^3 \quad , \\ A_1 &= 17.060 + 7.290 \cdot 10^{-1} N - 1.728 \cdot 10^{-2} N^2 + 9.993 \cdot 10^{-5} N^3 \quad , \\ A_2 &= 2.637 - 1.222N + 2.217 \cdot 10^{-2} N^2 - 1.179 \cdot 10^{-4} N^3 \quad , \\ A_3 &= -6.330 - 1.033N + 2.068 \cdot 10^{-2} N^2 - 1.130 \cdot 10^{-4} N^4 \quad . \end{aligned} \tag{6}$$

The stiffness of a pair of mating points is considered as two springs arranged in serial form where the specific stiffness of each point is calculated by Eq. (5), and the pairs of mating points are considered as parallel sets of springs. Therefore the equivalent mesh stiffness of the pairs of teeth in contact can be obtained from the specific stiffness.

Since the contact region is estimated by the pseudo-interference method, the discrete points pertaining to that region can be selected. Hence the compliance matrix of teeth can be built regarding only those discrete points. All the other points are neglected for the analysis.

3.2. Combined pseudo-interference and linear programming solution

An interior-point method is employed in this work to solve the reduced contact problem. The primal-dual algorithm available in a Matlab routine (Mathworks, 2005), which is a variant of the predictor-corrector algorithm proposed by Mehrotra, is applied to solve the contact problem in the TE calculation procedure.

In order to use the standard linear programming, the contact problem is reformulated based on maximum rigid body rotation, as follows:

Maximize: $\delta\theta$,
such that

$$\begin{aligned} & R_B \delta\theta \{e\} - [S]\{F\} - \{\varepsilon\} \leq 0 , \\ & \{e\}^t \{F\} R_B - T = 0 , \\ & F_k \geq 0 \ \ and \ \ \delta\theta \geq 0 \ , \end{aligned} \tag{7}$$

where, $\delta\theta$ is the rigid body rotation (or angular TE), R_B is the base radius, $\{e\}$ is a column vector of ones, $[S]$ is the compliance matrix with the influence coefficients, $\{F\}$ is the vector of loads, $\{\varepsilon\}$ is a vector of the initial separations, $\{e\}^t$ is the transpose of vector $\{e\}$, and T is the applied torque.

Two conditions must be satisfied to ensure a mathematical solution of the load distribution problem. The set of inequality constraints states a condition of compatibility, which means that at any point in the region of contact the sum of elastic deflection, given by $[S]\{F\}$, and the initial separations $\{\varepsilon\}$ must be greater than or equal to the rigid body displacement of the geared system $R_B \delta\theta \{e\}$. A condition of equilibrium of torque is stated by the equality constraint, in which the total moment of the loads acting at contacting points must be equal to the applied torque. Thus, under these conditions, the solution to maximize $\delta\theta$ is becoming the elastic deflections at the discrete points that present small initial separations as large as possible but limited by the equilibrium condition of torque.

In order to account the displacement of the contacting points due to the tooth deflections, a few iterations are required. The proposed method can be summarized in the following steps:

1. Input the gear geometric parameters and the load for analysis;
2. Perform the basic calculations and define the discrete points;
3. Calculate the angular distance between each pair of mating points (angles $\delta\theta_l$ in Fig. 1);
4. Estimate the rigid body rotation based on the pseudo-interference method - Eq. (5);
5. Find all discrete points pertaining to the contact region: $\delta\theta_l \leq$ (the estimated rigid body rotation + $\delta\theta_{lmin}$ under no load);
6. Build the matrix of influence coefficients (compliance matrix) regarding only the contact region points;
7. Solve the reduced contact problem using an interior-point method – Eq. (7);
8. Reevaluate the contact point taking into account the tooth deflections and the calculated rigid body rotation. If the mating points are not the same of those of the last iteration, then go back to step 3;
9. Calculate the longitudinal TE.

4. Numerical results

In order to evaluate the efficiency of the proposed procedure, some numerical examples were analyzed. A helical gear pair with design parameters defined in Tab. 1 was used as example. The tooth contact analysis and loaded TE calculation were carried out at 10 positions of the gear mesh cycle. The CPU time required to perform the analysis with the proposed procedure was compared to the time required to perform the same analysis with the method of iterative and gradual load application proposed by Wink and Serpa (2003). The error of both numerical methods was also used as a comparison.

The procedures implemented in Matlab (Mathworks, 2005) run under a personal computer with a Pentium Intel(R)Xeo processor of 2.8GHz, and 2.1GB of RAM.

Table 1. Basic design parameters of the gear pair used as example

Parameters	Driving gear	Driven gear
Number of teeth	27	43
Centre distance (mm)	70	
Normal module (mm)	1.93	
Normal pressure angle	20°	
Normal helix angle	15°	
Outside diameter (mm)	58	90
Root diameter (mm)	48	80
Addendum modification factor	0	0
Face width (mm)	15	15
Nominal torque for analysis (Nm)	30	
Young's modulus (GPa)	206	
Poisson's ratio	0.3	
Grid size in height (μm)	90	
Grid size in width (μm)	385	

The error of the numerical solutions related to the separation between contacting points were verified. It means that all mating points in contact and under load should present zero of separation as exact value. The deviations from the exact value are called in this work as contact errors, and they are illustrated in Fig. 3. The contact error affects the TE results but not in a direct relation. Thus, the effect of the contact error on TE results was also investigated.

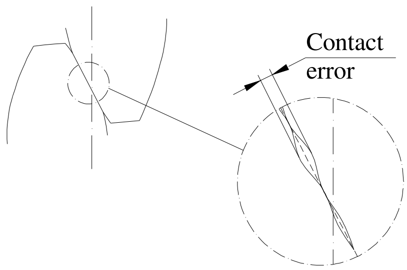

Figure 3. Illustration of the contact error

Figure 4 shows the results using the method of iterative and gradual load application proposed by Wink and Serpa (2003). The accuracy of that method is related to the degree of discretization of the load (load increment size). Thus, several load increment sizes (from 0.15 to 12N) were considered in this analysis. It can be observed in Fig. 4 that smaller load increments lead to lower contact errors but more computer time is required for the solution.

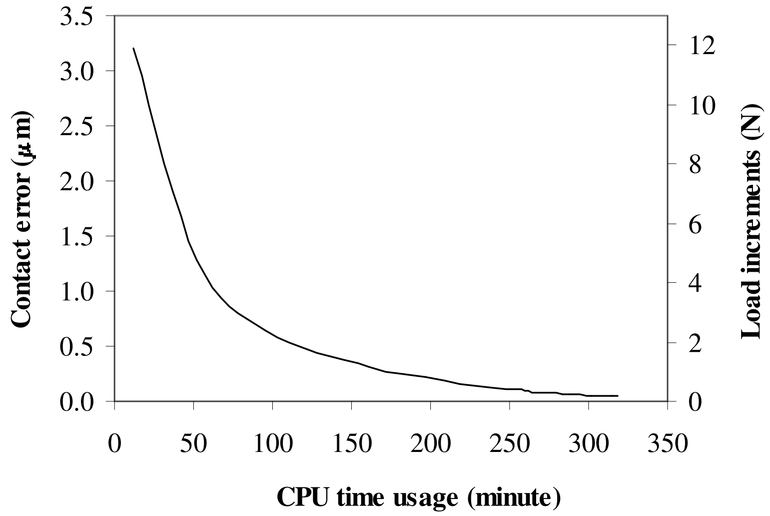

Figure 4. Results of the analysis using the method of iterative and gradual load application

Figure 5 shows the results using the combined pseudo-interference and interior-point method. For this analysis, different values were imposed to the convergence criterion of the algorithm and the contact errors were verified for the final solution. It can be observed in Fig. 5 that this method is faster and more accurate than the iterative-incremental method.

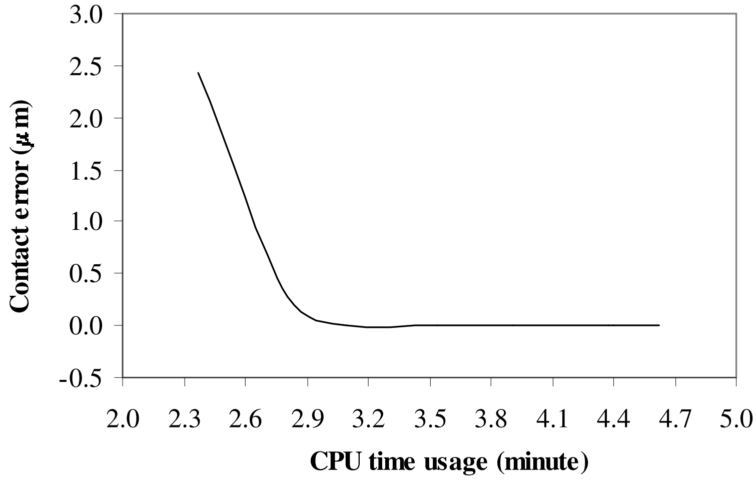

Figure 5. Results of the analysis using the combined pseudo-interference and interior-point method

The effect of the contact error on TE results is shown in Fig. 6. Those results were plotted for one position of the mesh cycle in which three pairs of teeth were in contact.

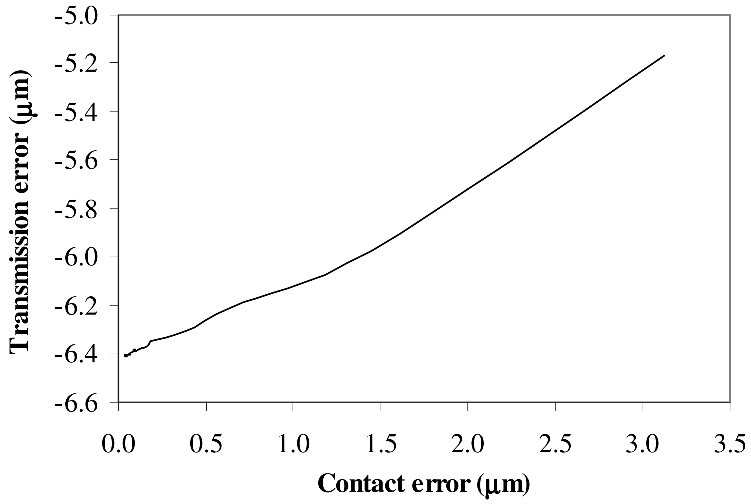

Figure 6. Effect of contact error on TE results for one position of the mesh cycle

For practical purposes of TE calculation, one could assume that an error of 1 per cent on TE result due to the numerical method would be acceptable. In this way, the performance of the both methods can be compared. The condition of the two methods that results in 1 per cent of error was selected. Table 2 shows the comparison of computer time to perform a 10-position calculation of loaded TE using both methods, as well as, the deviation to the exact value of TE due to the numerical method for the first position of mesh cycle. In that condition, the proposed method is about 100 times faster than the iterative-incremental method.

Table 2. Comparison to computer CPU time usage for the two procedures

Procedures	**CPU time (minute)**	**Error on TE result (%)**
Iterative and gradual load application method	318	1.3
Combined pseudo-interference and interior–point method	3	0.2

5. Conclusions

This work evaluated a combined pseudo-interference and linear programming solution for the contact problem of gear teeth under static load. This procedure was proposed to determine the load distribution and to calculate TE of helical gear pairs under load. The TE model performs a complete tooth contact analysis, not restricted to the plane of action. The proposed procedure was implemented into a computer code and its performance in terms of computational effort and accuracy was evaluated and compared to results of an iterative-gradual load application method. Numerical examples of a helical gear pair under static torque were carried out and showed that the proposed procedure is faster and more accurate than the other method evaluated, and that the interior-point method used was efficient in solving the large-scale contact problem. In the numerical simulations, the proposed procedure was about 100 times faster than the method of iterative and gradual load application for an assumed acceptable error around 1 per cent. In that condition of simulation the proposed procedure presented error in the TE results of 0.2 per cent against 1.3 per cent with the other method. These significant gain in computational efficiency obtained with the proposed procedure can lead the global tooth contact analysis to its practical application as a design tool in the accurate TE calculation of helical gear pairs under load.

6. Acknowledgements

The authors would like to thanks Eaton Limited, Light and Medium Transmission Division, Valinhos-SP, Brazil, for providing support for the development of this paper.

7. References

Beacham, M. R. *et al.*, 1999, "Development of Transmission Whine Prediction Tools", Journal of the Society of Automotive Engineers, pp. 887-897.

Conry, T. F. and Seireg, A., 1973, "A Mathematical Programming Technique for the Evaluation of Load Distribution and Optimal Modifications for Gear Systems", Journal of Engineering for Industry – Transactions of the ASME, Vol. 95, pp. 1115-1122.

Houser, D. R. "Gear Noise". In: Townsend, P. S., 1991*, Dudley's Gear Handbook*, McGraw-Hill, New York, pp. 14.1-14.32, Chap 14.

Kurokawa, S., Ariura, Y. and Ohtahara, M., 1996, "Transmission Errors of Cylindrical Gears Under Load – Influence of Tooth Profile Modification and Tooth Deflection Proceedings of the 7th International Power Transmission and Gearing Conference, ASME - Design Engineering Division, New York, Vol. 88, pp. 213-217.

Mathworks, Inc., 2005, "Matlab – the Mathematical Computer Language", http://www.mathworks.com.

Norman, M. E., 1995, "A New Design Tool for Optimising Gear Geometry for Low Noise", Proceedings of the Institution of Mechanical Engineers, Paper No. C492/034/95, pp. 231-244.

Özgüven, H. N. and Houser, D. R., 1988, "Mathematical Models Used in Gear Dynamics-A Review". J. of Sound Vibration, Vol.121(3), pp. 383-411.

Park, C. II and Lee, J. M., 1993, "The Load Transmission and Vibration Characteristics of Automobile Gear", J. of Society of Automotive Engineers, Paper No. 932917, pp. 1-11.

Pimsarn, M. and Kazerounian, K., 2002, "Efficient Evaluation of Spur Gear Tooth Mesh Load Using Pseudo-Interference Stiffness Estimation Method", J. of Mechanism and Machine Theory, Vol. 37, pp. 769-786.

Umezawa, K., 1972, "The Meshing Test on Helical Gears Under Load Transmission – 1st Report, The Approximate Formula for Deflections of Gear Tooth", Bulletin of the JSME, Vol. 15, No. 90, pp. 1632-1639.

Umezawa, K. and Ishikawa, J., 1973, "Deflection Due to Contact Between Gear Teeth with Finite Width", Bulletin of the JSME, Vol.16, No. 97, pp. 1085-1093.

Vanberbei, R. J., 1997, "Linear Programming: Foundations and Extensions (International Series in Operations Research and Management Science)", Kluwer Academic Publishers, 2nd ed., 440 p.

Vijayakar, S., 1991, "A Combined Surface Integral and Finite Element Solution For a Three-Dimensional Contact Problem", Int. J. Numerical Methods in Engineering, Vol. 31, pp. 525-545.

Welbourn, D. B., 1979, "Fundamental Knowledge of Gear Noise – A Survey", In IMechE Conference on Noise and Vibrations of Engines and Transmissions '79, Conference publications, Vol. 117, pp. 9-14.

Wink, C. H. and Serpa, A. L., 2003, "A New Approach to Calculate The Transmission Error of Helical Gear Pairs With Modified Tooth Surfaces", Proceedings of the 17th International Congress of Mechanical Engineering, São Paulo, Brazil, COB03-072.

Wink, C. H. and Serpa, A. L., 2005, "Investigation of Tooth Contact Deviations from the Plane of Action and Their Effects on Gear Transmission Error", Proceedings of The Institution of Mechanical Engineers, Part C – J. of Mechanical Engineering Science, Vol. 219(5), pp. 501-509.

8. Responsibility notice