ESTIMATING THE FRETTING FATIGUE LIMIT BY FINITE ELEMENT ANALYSIS

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Abstract. *This paper is an attempt to make a finite element (FE) analysis of fretting fatigue by searching an appropriate mesh refinement level able to consider the experimentally observed contact size effect into the calculation of fatigue strength. The basic idea underlying such methodology is based on the fact that there is an intrinsic approximation of the stress state along a finite element due to the integral formulation of the method. The usual FE procedure proposed by other authors to incorporate the stress gradient in the fatigue strength analysis requires the determination of the stress state, as accuratelly as possible, in a sufficiently large number of points (elements) within a process volume and then calculate the average volume stress. The approach developed in this work considers the finite element itself as the process volume.*

Keywords*: fretting fatigue, contact size effect, stress gradient, finite element analysis, multiaxial fatigue.*

1. Introduction

Fretting fatigue is the cause of premature failure in a number of mechanical assemblies subjected to vibrations. Examples include riveted or bolted lap joints, spline couplings and fan blade/disc fixings (Shaffer and Glaeser, 1994). To gain a better understanding of this phenomenon, various experimental apparatus and methodologies have been developed (Kinyon et al., 2002) considering a number of different contact configurations (Harish and Farris, 1998; Shaffer et al., 1994; Wittkowsky et al., 1999)

Bramhall (1973) first verified the existence of a critical contact size, dividing infinite and finite fretting fatigue regimes. This work revealed that for contact sizes smaller than the critical one lives were very long ($>10^8$ cycles), while for larger contacts lives were finite. Since then, new and better controlled experimental data have been produced by other researchers confirming the effect of the contact size on fretting life (Araújo, 1998; Nowell, 1988). For instance, Nowell conducted experimental series using both flat *dog bone* specimens and cylindrical pads made of Al4%Cu alloy. The results provided by his work have been extensively considered to validate methodologies for prediction of fretting fatigue strength (Castro, 2003).

On the fretted surfaces, there may be a strong stress concentration that creates favorable conditions to crack nucleation. However, as the crack grows away from the surface, a high stress gradient will slow down such crack, so that it may eventually arrest (Araújo and Nowell., 1999). Since not only the localized stress state, but also the stress gradient should be considered to model such problem, the fretting fatigue life prediction of structural elements becomes a hard task (Hills and Nowell, 1994). Moreover, previous works (Araújo and Nowell, 2002; Castro, 2003; Fouvry et al., 2002) showed that the application of multiaxial fatigue criteria upon the fretting surface provide very conservative life estimates, although they may be useful to estimate both, the crack nucleation site (Fouvry et al., 2002) and the crack growth orientation (Swalla and Neu, 2003). In order to incorporate the effect of the stress gradient in the analysis many authors have considered a process volume approach (Araújo and Nowell, 2002; Foubry et al., 2002; Naboulsi, and Mall, 2003; Namjoshi et al., 2002; Swalla and Neu, 2003), which consists in the application of the fatigue criteria upon the stress history approximated onto a volume rather than in a single point.

This paper is an attempt to make a finite element (FE) analysis of fretting fatigue by searching an appropriate mesh refinement level able to consider the stress gradient effect into the calculation of fatigue strength. The basic idea underlying such methodology is based on the fact that there is an intrinsic approximation of the stress state along a finite element due to the integral formulation of the method.

2. Experimental data

The FE analysis presented in this paper will be validated considering the work conducted by Nowell (1988). A schematic representation of the configuration tested is showed in Fig. 1 where R is the radius of cylindrical fretting pad (Figure 3), P is the normal load, σ_B is the bulk stress and Q denotes the tangential load induced by the spring A. The contact loads P and Q and the bulk stress σ_B were applied as shown in Fig. 2, i.e., P is a static load and Q and σ_B are in phase sinusoidal functions of time. Moreover the pressure p_0 can be computed by the equation

$$p_0 = \frac{2P}{\pi C}, \tag{1}$$

where C is the contact area between the specimen and the cylindrical pad. Therefore p_0 is proportional to P.

Table 1. Experimental parameters and critical contact size range.

Series	p_0 (MPa)	Q_{max}/P	σ_{Bmax} (MPa)	a_{crit} (mm)
1	157	0.45	92.7	0.28–0.38
2	143	0.24	92.7	0.54–0.72
3	143	0.45	92.7	0.18–0.27
4	143	0.45	77.2	0.36–0.54
5	120	0.45	61.8	0.57–0.71

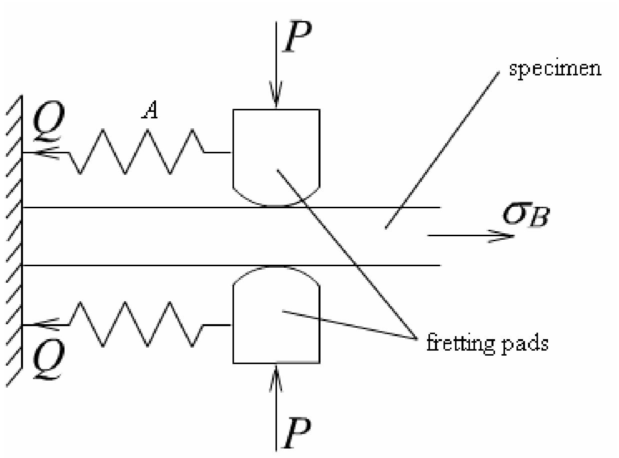

Figure 1. Experimental scheme used by Nowell.

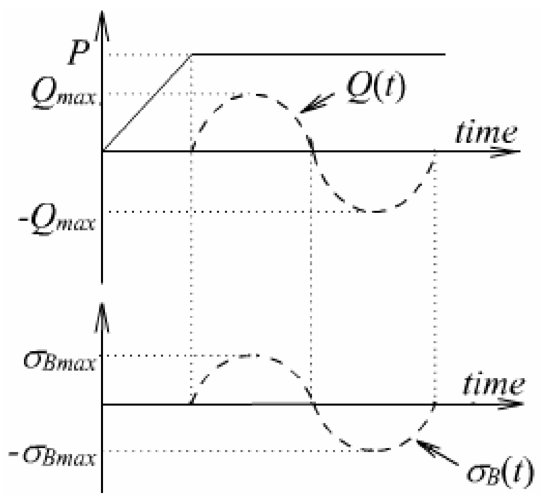

Figure 2. Load modes of the fretting experiments conducted by Nowell.

Five experimental series were conducted. In each one of this data series, the parameters p_0 (the peak pressure), Q_{max}/P and σ_{Bmax}, were kept constant, while the pad radius was varied from 12.5 to 150mm. Here the subscript *max* denotes the maximum value reached by Q and σ_B over time (Figure 2). These parameters for each series are presented in Tab. 1. The importance of varying R keeping p_0 constant is that it is possible to produce a data series where all specimens are submitted to the same superficial stress state although they experience different stress decays along the depth.

The tests carried out by Nowell revealed an effect of the pad radius (or contact size, a) on fretting life. It was observed that for small contact sizes fretting tests last infinitely while for large contacts specimens broke within a finite number of cycles. The range defined by the largest contact to show infinite life and the smallest contact to provide finite life was termed (Bramhall, 1973) the critical contact size range, a_{crit}. Table 1 reports such range for each data series.

3. FE model

To simulate the experiments, the FE code *ef++*, developed by the Mechanics of Materials Research Group of the University of Brasília, was used. As a graphical interface this code uses the GiD platform (Ribó, 2000), which carries out both, pre – geometry creation, meshing and boundary conditions assignment – and post processing – stress, strain and displacement field visualization. A contact element was recently implemented (Bernardo, 2003) into the FE code, which allows the calculation of the stress field under fretting situations (Bernardo et al., 2003). The Crossland (Crossland, 1956), Dang Van (Dang Van and Papadopoulos, 1987) and Prismatic Hull (PRH) (Gonçalves et al., 2003; Mamiya and Araújo, 2000) multiaxial fatigue criteria have been implemented to the code (Dantas et al., 2003) so that a direct fatigue strength analysis can be carried out. The program is able to apply these criteria on the nodal stress history.

In order to perform the FE analysis of the configuration depicted in Fig. 1 and considering its symmetry, the model showed in Fig. 3 was adopted. The fundamental parameters (Bramhall, 1973) necessary to carry out the stress analysis are: the Young's modulus (E = 74GPa) and the coefficient of friction (f = 0.75), while for the fatigue strength computation data concerning the fatigue limit under alternate bending (f_{-1} = 124MPa at 5×10^7 cycles) and under alternate torsion – estimated for Al4%Cu as t_{-1} = 72MPa at 5×10^7 cycles (Forrest, 1962) – are necessary.

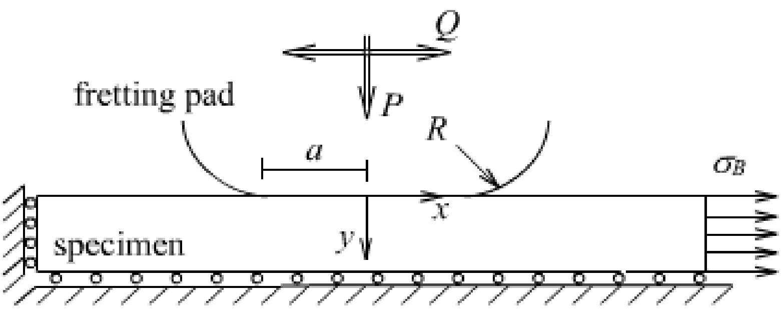

Figure 3. Scheme of the adopted model.

4. FE mesh

An example of a characteristic mesh generated for both fretting pad (150mm radius) and specimen is depicted in Fig. 4. Plane strain linear elastic triangular elements are used on the pad and specimen domains. To simulate the contact problem bi-dimensional two-node interface elements were considered, as shown in Fig. 5. In the same figure it can be noted that a structured mesh region was defined under the contact surface. Within such region the mesh refinement level and the finite element size are characterized by three parameters, *viz*, the contact mesh width, l_c, and the element width and depth, l_e and h_e, respectively.

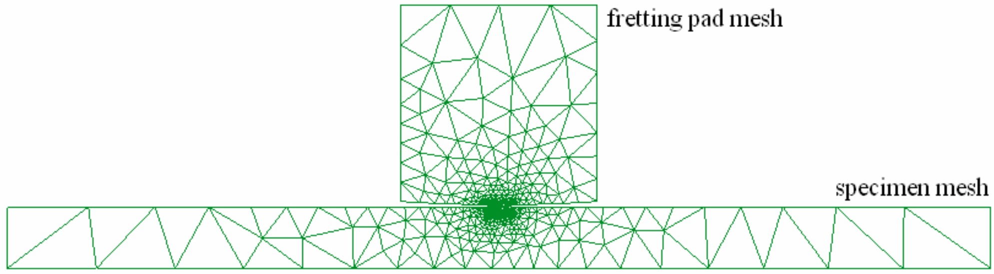

Figure 4. Mesh of both 150mm radius pad and specimen.

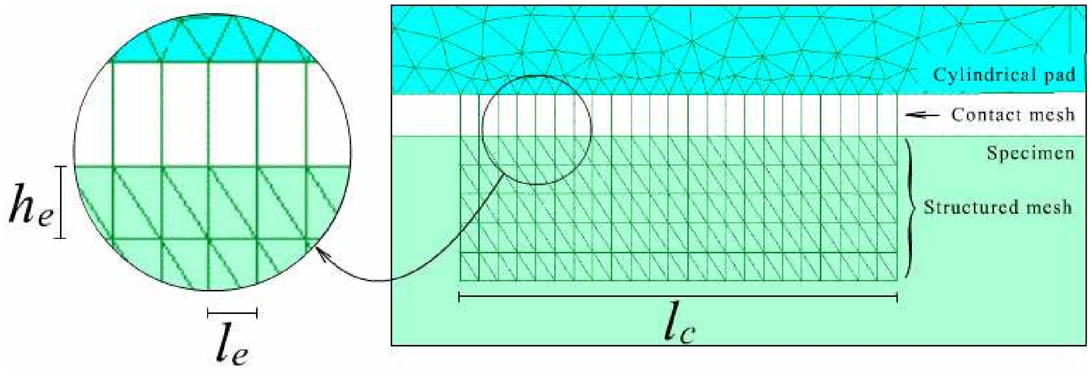

Figure 5. Mesh parameters.

In a recent work conducted by one of the authors (Bernardo, 2004) it was shown that the numerically computed stress field for Nowell's contact configuration approximates well from the analytical solution for the adimensionalized parameters $l_c/a > 2.10$, $l_e/a < 0.0726$ and $h_e/a < 0.0146$. Furthermore, the authors of that work also noticed that for l_e less than 20 μm and/or l_e/h_e ratio far from the unit, the numerical simulation becomes unstable.

As mentioned before there is a number of authors, including ourselves, who have tried to associate the contact size effect in fretting fatigue life with the presence of a stress gradient in the vicinity of the contact region. In this paper it is claimed that such an effect can be incorporated to the calculation of the fatigue strength by choosing an appropriate mesh refinement level. The basic idea underlying such methodology is based on the fact that there is an intrinsic approximation of the stress state along a finite element due to the integral formulation of the method. Hence, the methodology proposed here is to vary the element size, which will provide different average stress states, until the numerical fatigue strength predictions match the experimental data. Therefore, as can be seen, the aim of this work is not to generate a well refined mesh capable to capture the analytical stress field.

The reader should notice that the usual FE procedure proposed by other authors (Swalla and Neu, 2003; Namjoshi et al., 2002) to incorporate the stress gradient in the fatigue strength analysis needs to determine the stress state, as accurately as possible, in a sufficiently large number of points (elements) within the process volume and then calculate the average volume stress. The approach developed in this work considers the finite element itself as the process volume. Thus the computational cost of the analysis is substantially reduced. Furthermore, it will be assessed in this paper whether the element size may be associated with material parameters such as the grain size. In order to do so, all fretting tests were simulated considering structured meshes having the same l_c and l_e and varying the element depth h_e.

5. Results

Numerical simulations of fatigue strength have been conducted by considering the Crossland, Dang Van and PRH multiaxial models. Only space limits us to present here a detailed description of these criteria, which have been well documented and discussed in the literature (Crossland, 1956; Dang Van and Papadopoulos, 1987; Gonçalves et al., 2003 Mamiya and Araújo, 2000; Papadopoulos et al., 1997).

The results of the numerical analysis are then compared with Nowell's experimental data. Following these results are presented and evaluated, but before a fatigue strength index (*FSI*) is defined. This parameter, which is obtained for each multiaxial fatigue criteria studied, is used to quantify the fatigue strength of a component.

5.1 Fatigue strength index

A generic multiaxial fatigue criterion based on the stress invariant approach can be expressed by the inequality

$$g_1(\tau) + \kappa g_2(\sigma) \leq \lambda, \tag{1}$$

where κ and λ are material constants, $g_1(\tau)$ is a function of the deviatoric stress history and $g_2(\sigma)$ denotes a function of the normal stress history. In this setting, the *FSI* to compute the fatigue strength can be defined as

$$FSI = \frac{g_1(\tau) + \kappa g_2(\sigma) - \lambda}{\lambda} \times 100\% . \tag{2}$$

If this index assumes positive values, inequality (1) does not hold, it essentially means that the criterion predicts failure of the mechanical structure. On the other hand, if *FSI* assumes values less than or equal to zero the component is safe, according to the model.

5.2 Fretting fatigue strength computation using FE model

The numerical simulation was conducted aiming to determine the size of the finite element necessary to provide the correct fretting fatigue strength prediction within each data series considered. Two set of structured meshes each having l_e equal to 83 and 104μm were adopted to model the experimental data, both maintaining the parameter l_c = 2.4mm. For each set four different meshes were generated considering h_e values varying from 160 to 400μm as reported in Tab. 2. In this table the predicted critical contact size range a_{crit} is also reported for each value of h_e (these results concern the structured mesh where l_c = 2.4mm and l_e = 83μm and was varied h_e). The smallest value of this range corresponds to the greatest contact semi-width, a, that presented non-positive values of *FSI* in the numerical simulation of the configuration tested, while the other value of the range is the smallest a where positive I values were computed. Notice that the fatigue strength index is computed at every node of the mesh. The predicted range was obtained considering the node where this index was maximum on the specimen domain. Bolded data in Tab. 2 correspond to the predicted critical contact size range which matched the experimental data.

An alternative manner to present such results is depicted in figures 5 and 6. These graphs plot the fatigue strength index against the contact semi-width for l_e = 83μm. The two dashed vertical lines are used to define the experimental critical contact size range. It can be seen that for series 1 data the h_c value which predicts the correct contact size range is h_e = 267μm for the Crossland and PRH criteria and h_e = 400μm for the Dang Van model.

Table 3 reports the values of h_e necessary to predict the correct contact size range according to the multiaxial fatigue criteria considered in this work for l_c = 2.40mm and l_e = 83μm, respectively. All values obtained are between 160 and 400μm. To illustrate some of the results reported in Tab. 3, figures 8 and 9 are used. They contain graphs of the PRH fatigue strength index versus element depth (h_e) for the critical contact size ranges of series 1 (Fig. 8) and series 3 (Fig. 9) data. It can be observed in Fig 6 that the PRH model predicts that the correct a_{crit} for series 1 data can be obtained for h_e = 267μm while for series 3 values of h_e between 160 and 200μm are required. Other graphs for different mesh characteristics and considering the other models assessed are not shown due a space limitation.

Table 2. Critical contact sizes in series 1 with l_c = 2.40mm and l_e = 83μm.

	Critical contact size a_{crit} (mm)		
h_e (μm)	Crossland	Dang Van	PRH
160	0.10-0.19	0.10-0.19	0.10-0.19
200	0.19-0.28	0.10-0.19	0.19-0.28
267	**0.28-0.38**	0.19-0.28	**0.28-0.38**
400	0.38-0.57	**0.28-0.38**	0.38-0.57

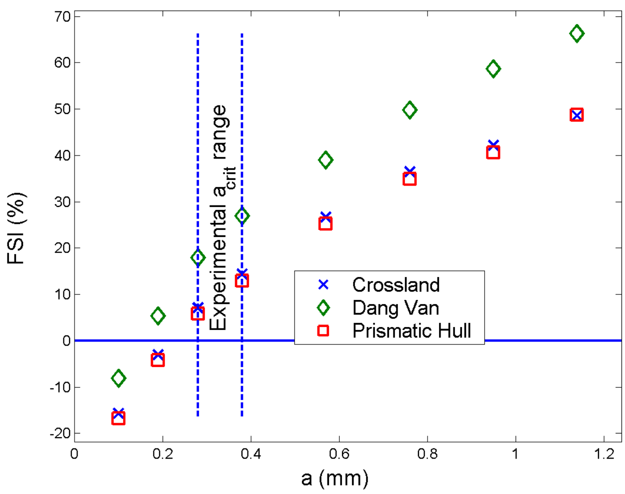

Figure 6. *FSI* for data series 1 with mesh parameters l_c = 2.40mm, l_e = 83μm and h_e = 200μm.

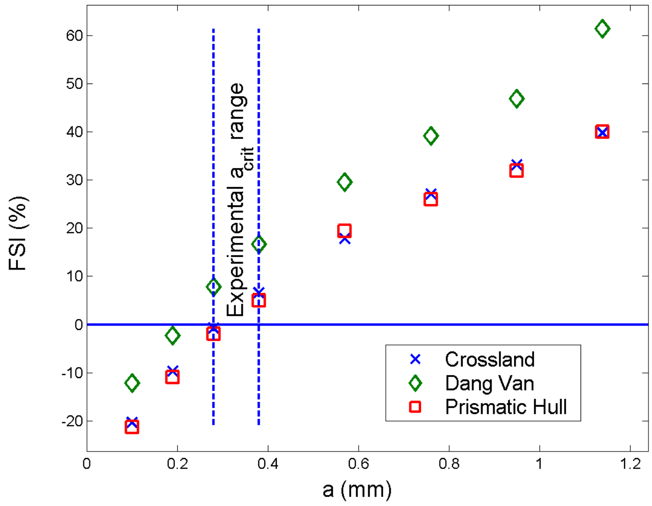

Figure 7. *FSI* for data series 1 with mesh parameters l_c = 2.40mm, l_e = 83µm and h_e = 267µm.

Table 3. Values of h_e that provide the same fatigue strength results using l_c = 2.40mm and l_e = 83µm compared to experimental data.

	h_e (µm)		
Series	Crossland	Dang Van	PRH
1	267	400	267
2	267	400	267
3	160-200	267	160-200
4	200-267	267	200-267
5	160	200	160

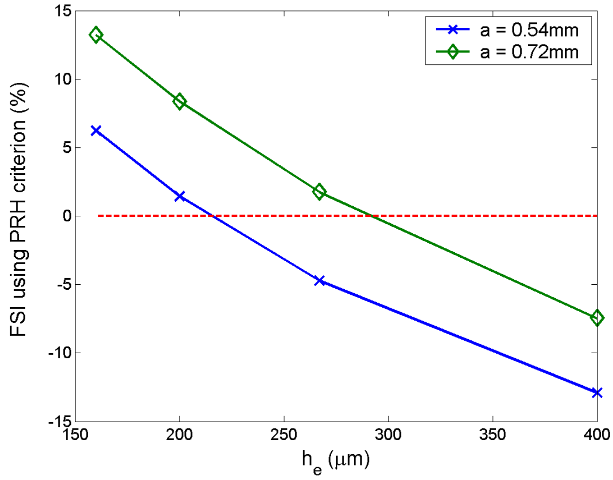

Figure 8. *FSI* (using PRH criterion) versus h_e with mesh parameters l_c = 2.40mm and l_e = 83µm for data series 1 considering the contact semi-widths that limit the a_{crit} for this series.

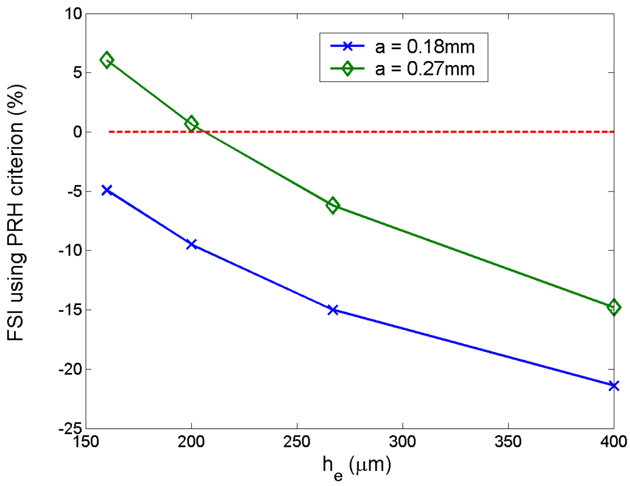

Figure 9. *FSI* (using PRH criterion) versus h_e with mesh parameters l_c = 2.40mm and l_e = 83µm for data series 3 considering the contact semi-widths that limit the a_{crit} for this series.

6. Discussion and conclusion

The analysis carried out in this work showed an alternative procedure to numerically compute the effect of the stress gradient in the prediction of fretting fatigue strength. It was shown that the choice of an appropriate mesh refinement level and element size is capable to successfully predict the critical contact size effect observed in Nowell's fretting experiments.

More specifically, it was found that the Crossland and the PRH criteria required the same element size to predict the correct a_{crit} (Table 3), while Dang Van model presented a larger *FSI* for the same element dimension.

In the used set of structured mesh neither of the criteria studied found a unique range of element size (h_e range) capable to predict a_{crit} for all data considered (Tab. 3). However, one should remember that: (i) the fatigue limit in torsion was estimated and this introduce inaccuracies to the analysis; (ii) the size of the element is determined based on the prediction provided by the multiaxial fatigue criteria, hence if such criteria are not working well for the tests considered they will provide a poor analysis. Notice that these models do not take in account the effect of the relative slip between the contacting surfaces on fretting life. Although this influence may not be very strong, since in the partial slip regime the amount of wear is small, it will certainly introduce some level of inaccuracy into the calculation.

It is worthy of notice that recent approaches (Swalla and Neu, 2003; Naboulsi and Mall, 2003) used to carry out a FE fretting fatigue strength analysis rely on the utilization of very fine meshes, which usually contain elements whose size is considerably smaller than a characteristic material grain size. For instance, the Al4%Cu alloy grain size is around 100µm (Nowell, 1988). It was observed that, in order to compute a stress field which closely agrees with the analytical solution for series 1 data and pad radius R = 12.5 mm an element with h_e = 1.46µm and l_e = 7.24µm was required. Besides being computationally very expensive such an approach seems to violate the continuum assumption.

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