AN APPLICATION OF ARMAV MODEL TO THE IDENTIFICATION OF MODAL PARAMETERS

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Abstract. *The dynamical characteristics of a five-degrees-of-freedom mechanical system, excited by uncorrelated random forces, were identified through the ARMAV model. The least-square method was used to determine the matrices coefficients of the auto-regressive and moving average parameters from simulated data in time domain. Through the representation of the ARMAV model in the state space, the modal parameters identification was obtained with good accuracy. To represent a more realist situation, numerical simulations were performed considering the response data corrupted by noise and taking only some of the outputs. The proper order of the model was obtained using the stability diagrams. It was observed the efficiency of the least-square method by means of simulated tests, particularly when using the ARMAV model in the modal parameters identification of a system with close modes and light damping ratios.*

Keywords: *modal analysis, ARMAV, least-square method.*

1. Introduction

The latest developments of experimental modal analysis (EMA) have been an essential subject to the solution of engineering problems related to structural vibrations.

One of the fundamental areas of EMA is known system identification, what corresponds to the identification of the dynamical properties of a structure from input and output data obtained on experimental tests. The dynamical properties are described by a group of parameters that characterize the natural frequencies, damping ratios and mode shapes of a system in analysis, called modal parameters. These identified parameters can be used as a tool to adjust or to validate analytic models, to detect the presence of a fault and to perform structural modifications.

The experimental identification of the system modal parameters can be performed with frequency or time domain data. The frequency domain methods are widely used in identification problems. However, they present some limitations mentioned by Gontier (1993), mainly when the system has special characteristics such as close natural frequencies, light damping ratios and measurements with high noise level. Besides, Maia (1997) says that the leakage phenomenon becomes critical when the amount of data is small, what turns impossible a good resolution in frequency domain. In recent decades, the time domain methods have been used in the analysis of mechanical vibrations as an alternative to solve these problems. The time domain methods are characterized by an evaluation of modal parameters from excitation and response data obtained directly from time domain. Then it is not necessary to evaluate the Inverse Fourier Transform with its inherent numerical errors.

The identification methods can also be distinguished by the amount of system inputs and outputs used. The MIMO (Multi-Input/Multi-Output) techniques consider multiple system excitations and responses obtained simultaneously. The use of these techniques is increasing, when compared with SISO (Single-Input/Single-Output) techniques that work only with two signals data at each evaluation. It is due to the fact that, besides the reducing of spent time on data acquisition and analysis, it allows to estimate the modal parameters in a more accurate way even when the structures have strongly coupled modes and light damping ratios (Ewins, 1984 and Papakos, 2003).

Since the 70's, many works on systems identification were developed, mainly due to technological progresses. It was done emphasis on the developments of time domain methods such as Ibrahim Time Domain Method (ITD), the Least Squares Complex Exponential Method (LSCE), Polyreference Complex exponential Method (PRCE) and the Eygensystem Realization Algorithm (ERA) (Maia, 1997 and Yang, 1994).

A remarkable class of time domain methods is based on the parametric Auto-Regressive Moving Average (ARMA) models. In these methods the dynamical properties are described through linear difference equations, which auto-regressive and moving average coefficients set up the input-output relationship of the system. The main characteristic of those models is that the system output can be calculated as linear combination of the present inputs, past inputs and present outputs. The application of these models in the vibrations analysis was consolidated from the works developed by Gersh (1975), Wu (1977) and Pandit (1977). Variations of those models, as its extension to a vectorial form, called auto-regressive moving average vector models (ARMAV), have been applied with success in the identification of systems with complex characteristics. Among several papers founded in literature we named the followings: (Boudex, 2003), (Garibaldi, 1998), (Giorcelli 1994) and (Piombo, 1993).

In this paper, the ARMAV model was used to identify the modal parameters of a simulated system of five degrees-of-freedom (dof) with non-proportional damping, excited by random white noise forces.

The matrices coefficients of the auto-regressive and moving average parts were determined through the least-square approach. From those coefficients a state space model of the system was built in order to estimate the natural frequencies, damping ratios and mode shapes.

In actual situations, it is necessary to know the model order to identify the modal parameters. The problem of estimation the order of a model has been the main task of a great number of researchers. Among several existent techniques, we found the Akaike's Final Prediction Error technique (FPE) (Akaike, 1969), the Akaike's Information Criterion (AIC) (Akaike, 1974) and the Minimum Description Length technique (MDL) (Shwarz, 1978).

The approach used in this paper allows the estimate of the model order through stability diagram, which have been widely applied because its easy application.

2. State Space System Model

The objective of structural identification is the search of a model that represents the dynamical characteristics of a system. The physical systems are continuous structures that can be represented in a convenient model by spatially discrete sized parameters, which equations form a system of second order ordinary differential. The dynamical characteristics of a linear time-invariant *n* dof mechanical system can be described by the following matrix differential equation, called equation of motion:

$$\mathbf{M}\,\ddot{\mathbf{q}}(t) + \mathbf{C}\,\dot{\mathbf{q}}(t) + \mathbf{K}\,\mathbf{q}(t) = \mathbf{f}(\mathbf{q},t) \tag{1}$$

where **M, C** and **K** are the $n \times n$ system mass, damping and stiffness matrices, respectively. The $n \times 1$ time dependent vectors $\mathbf{q}(t)$, $\dot{\mathbf{q}}(t)$ and $\ddot{\mathbf{q}}(t)$, represent the masses displacement, velocity and acceleration through a generalized coordinates. The $n \times 1$ vector $\mathbf{f}(\mathbf{q},t)$ corresponds to external forces that excite the system. This vector can be decomposed so that $\mathbf{f}(\mathbf{q},t) = \mathbf{F}\mathbf{u}(t)$, where **F** is a $n \times r$ input influence matrix, that describes the input locations, and $\mathbf{u}(t)$ is a $r \times 1$ input vector, where *r* is the number of system inputs.

The system represented by Eq. (1) can be described by a continuous time state space model, as follows:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A_c}\mathbf{x}(t) + \mathbf{B_c}\mathbf{u}(t) \\ \mathbf{y}(t) = \mathbf{C_c}\mathbf{x}(t) + \mathbf{D_c}\mathbf{u}(t) \end{cases} \tag{2}$$

where $\mathbf{x}(t) = \begin{bmatrix}\mathbf{q}(t)^T & \dot{\mathbf{q}}(t)^T\end{bmatrix}^T$ is a $2n \times 1$ state vector and *T* denotes the transposed matrix operation. The $m \times 1$ vector $\mathbf{y}(t)$ is the system output, with $m \le n$. In this paper, the system output is the measured accelerations at instant of time *t*, on different points of the structure.

The $2n \times 2n$ state matrix $\mathbf{A_c}$, the $2n \times r$ input influence matrix $\mathbf{B_c}$, the $m \times 2n$ output influence matrix $\mathbf{C_c}$ and the $m \times r$ direct transmission matrix $\mathbf{D_c}$ are defined as:

$$\mathbf{A_c} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \qquad \mathbf{B_c} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{F} \end{bmatrix} \qquad \mathbf{C_c} = \begin{bmatrix} -\mathbf{G}\mathbf{M}^{-1}\mathbf{K} & -\mathbf{G}\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \qquad \mathbf{D_c} = \begin{bmatrix} \mathbf{G}\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}$$

where **G** is a $m \times n$ matrix that contains information about the location of measurement points relative to the generalized coordinates used.

In practice, measurements are represented by discrete signals in time domain, and this paper deals with an application that simulates an experimental test. Then it was necessary to use the discrete time state space model that is given by:

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A_d}\mathbf{x}(k) + \mathbf{B_d}\mathbf{u}(k) \\ \mathbf{y}(k) = \mathbf{C_d}\mathbf{x}(k) + \mathbf{D_d}\mathbf{u}(k) \end{cases} \tag{3}$$

where all the variables are obtained at equally spaced time intervals Δt for $t = 0, \Delta t, \cdots, k\Delta t, (k+1)\Delta t, \cdots (l-1)\Delta t$ and $\mathbf{x}(k) = \mathbf{x}(k\Delta t)$. The discrete state space matrices are defined by:

$$\mathbf{A_d} = e^{\mathbf{A_c}\Delta t} \qquad \mathbf{B_d} = (\mathbf{A_d} - \mathbf{I})\mathbf{A_c}^{-1}\mathbf{B_c}$$

$$\mathbf{C_d} = \mathbf{C_c} \qquad \mathbf{D_d} = \mathbf{D_c}$$

3. Auto-Regressive Moving Average Vector Model (ARMAV)

The dynamic behavior of a linear, time-invariant and discrete-time vibratory system can also be described by a parametric model called ARMAV(p_1, p_2). This model is defined by the following matrix difference equation, as presented in (Marple, 1987) and (Kay, 1987):

$$\mathbf{y}(k) = -\sum_{i=1}^{p_1} \mathbf{A}_i \mathbf{y}(k-i) + \sum_{i=0}^{p_2} \mathbf{B}_i \mathbf{u}(k-i) \tag{4}$$

where $\mathbf{y}(k)$ is the system output and $\mathbf{u}(k)$ is the system input, considered a stationary Gaussian white noise.

In the Equation (4) the output at any instant k is written as a linear combination of the past and present inputs, and past outputs, weighted by the coefficients of the $m \times m$ matrix $\mathbf{A}_i$ and the coefficients of the $m \times r$ matrix $\mathbf{B}_i$, that represent the auto-regressive (AR) and moving average (MA) parts, respectively. The auto-regressive part describes the global dynamics of the system, while the moving average part is related with the excitations.

In actual measurements, all the obtained data contains noises or disturbances. A model that works well on these situations and makes possible good estimates of system modal parameters, is the ARMAV(p_1, p_2) model, with $p_1 = p_2 = p$, and $p \geq (2n/m)$ (Andersen, 1997).

4. Relationship between the State Space Model and the ARMAV*(p, p)* Model

Andersen (1997) shows that the ARMAV(*p*, *p*) model can be described in state space through a realization, called canonical observability. This realization is built based on the auto-regressive matrices coefficients and the impulse response functions of the ARMAV model.

This way, the relationship between the discrete state space model, given by Eq. (3), and the ARMAV model coefficients is performed through matrices $\mathbf{A_d}$ $mp \times mp$, $\mathbf{B_d}$ $mp \times r$, $\mathbf{C_d}$ $m \times mp$ and $\mathbf{D_d}$ $m \times r$, defined as:

$$\mathbf{A_d} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0} & \mathbf{I} \\ -\mathbf{A}_p & -\mathbf{A}_{p-1} & \cdots & \cdots & -\mathbf{A}_2 & -\mathbf{A}_1 \end{bmatrix} \qquad \mathbf{B_d} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{A}_1 & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{A}_2 & \mathbf{A}_1 & \mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{A}_3 & \mathbf{A}_2 & \mathbf{A}_1 & \mathbf{I} & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{p-1} & \mathbf{A}_{p-2} & \cdots & \cdots & \mathbf{A}_1 & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_1 - \mathbf{A}_1\mathbf{B}_0 \\ \mathbf{B}_2 - \mathbf{A}_2\mathbf{B}_0 \\ \mathbf{B}_3 - \mathbf{A}_3\mathbf{B}_0 \\ \mathbf{B}_4 - \mathbf{A}_4\mathbf{B}_0 \\ \vdots \\ \mathbf{B}_p - \mathbf{A}_p\mathbf{B}_0 \end{bmatrix}$$

$$\mathbf{C_d} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \end{bmatrix} \qquad \mathbf{D_d} = \mathbf{B_0}$$

and the $mp \times 1$ state vector is given by $\mathbf{x}(k) = \begin{bmatrix} \mathbf{y}(k)^T & \mathbf{y}(k+1)^T & \cdots & \mathbf{y}(k+p-1)^T \end{bmatrix}^T$.

It is observed that the matrix $\mathbf{A_d}$ is constituted only by auto-regressive matrices coefficients, while the matrix $\mathbf{B_d}$ is constituted by auto-regressive and moving average matrices coefficients.

The global dynamical characteristics of the system are described by the $\mathbf{A_d}$ matrix. Then, the natural frequencies f_j and the damping ratios ξ_j are estimated from the $\mathbf{A_d}$ eigenvalues α_j, with the relationships:

$$f_j = \frac{|\lambda_j|}{2\pi} \qquad \text{and} \qquad \xi_j = \frac{-real(\lambda_j)}{|\lambda_j|} \tag{5}$$

where $\lambda_j = \dfrac{ln(\alpha_j)}{\Delta t}$ and $j = 1, 2, \cdots, mp$.

The system mode shapes $\mathbf{\Psi}_j$ are obtained through the $\mathbf{A_d}$ eigenvectors $\mathbf{\Phi}_j$. It can be identified as follows:

$$\mathbf{\Psi}_j = \mathbf{C_d}\mathbf{\Phi}_j \tag{6}$$

In general, the number of the ARMAV model eigenvalues is larger than the number of eigenvalues corresponding to the physical system. Therefore, only a subset of the eigenvalues founded represents the structural modes. In this paper, the physical and computational modes are discriminated using some confidence factors. The stability diagrams were used to find the model order and to get the true modes.

5. Identification of the ARMAV Model Parameters

The discrete time state matrix $\mathbf{A_d}$ is composed by the auto-regressive matrices coefficients $\mathbf{A}_i$. Then, the matrices $\mathbf{A}_i$, with $i = 1, 2, \cdots, p$, must be identified so that the modal parameters of the structure in analysis can be estimated.

In order to represent a realist situation, it was considered measured responses $\mathbf{y}(k)$ corrupted by a white noise $\mathbf{w}(k)$ as follows: $\tilde{\mathbf{y}}(k) = \mathbf{y}(k) + \mathbf{w}(k)$. Then, substituting $\tilde{\mathbf{y}}(k)$ in Eq. (4), expanded to order p, we obtain:

$$\tilde{\mathbf{y}}(k) = -\mathbf{A}_1\tilde{\mathbf{y}}(k-1) - \cdots - \mathbf{A}_p\tilde{\mathbf{y}}(k-p) + \mathbf{B}_0\mathbf{u}(k-0) + \mathbf{B}_1\mathbf{u}(k-1) + \cdots + \mathbf{B}_p\mathbf{u}(k-p) + \mathbf{e}(k) \tag{7}$$

where the error is defined by: $\mathbf{e}(k) = \mathbf{w}(k) + \mathbf{A}_1 w(k-1) + A_2\mathbf{w}(k-2) + \cdots + \mathbf{A}_p\mathbf{w}(k-p)$.

Using input and output data measured at several time instants $k = p+1,\ p+2,\ \cdots,\ p+L$, where $L \le L_{max}$ and $L_{max} = l - p$, the Eq. (7) can be written in a matrix form to all instants of time k as:

$$\begin{bmatrix}\tilde{\mathbf{y}}(p+1) & \cdots & \tilde{\mathbf{y}}(p+L)\end{bmatrix} = \begin{bmatrix}\mathbf{A}_1 & \cdots & \mathbf{A}_p & \mathbf{B}_0 & \cdots & \mathbf{B}_p\end{bmatrix}\begin{bmatrix} -\tilde{\mathbf{y}}(p) & -\tilde{\mathbf{y}}(p+1) & \cdots & -\tilde{\mathbf{y}}(p+L-1) \\ \vdots & \vdots & \vdots & \vdots \\ -\tilde{\mathbf{y}}(1) & -\tilde{\mathbf{y}}(2) & \cdots & -\tilde{\mathbf{y}}(L) \\ \mathbf{u}(p+1) & \mathbf{u}(p+2) & \cdots & \mathbf{u}(p+L) \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{u}(1) & \mathbf{u}(2) & \cdots & \mathbf{u}(L) \end{bmatrix} +$$

$$+\begin{bmatrix}\mathbf{e}(p+1) & \cdots & \mathbf{e}(p+L)\end{bmatrix}$$

In a compact form it can be written as:

$$\tilde{\mathbf{Y}} = \boldsymbol{\theta}\,\tilde{\mathbf{S}} + \mathbf{E} \tag{8}$$

where $\tilde{\mathbf{Y}}$, $\boldsymbol{\theta}$, $\tilde{\mathbf{S}}$ and $\mathbf{E}$ are matrices with dimensions $m \times L$, $m \times (2p+1)$, $(mp + rp + r) \times L$ and $m \times L$, respectively.

The parameters of the ARMAV model, that compose the $\boldsymbol{\theta}$ matrix, can be obtained from the solution of the linear system represented by Eq. (8) using the least square method, that minimizes $\mathbf{E}^T\mathbf{E}$. Then, it is obtained an estimate that is given by $\boldsymbol{\theta} = \tilde{\mathbf{Y}}\tilde{\mathbf{S}}^T\left[\tilde{\mathbf{S}}\tilde{\mathbf{S}}^T\right]^{-1}$, (Lawson, 1974).

Therefore, the eigenvalues and eigenvectors of the $\mathbf{A_d}$ matrix can be calculated from $\boldsymbol{\theta}$ matrix. Next, the modal parameters of the structure in study can be evaluated from Eq. (5) and (6).

6. Numerical Simulation

The presented simulation has the main objective of verifying the performance of the ARMAV model in a system identification using least square method. The identification was performed using simulated numerical data, which were generated to characterize a 5 dof mechanical system, represented in Fig. (1), with linear viscous damping.

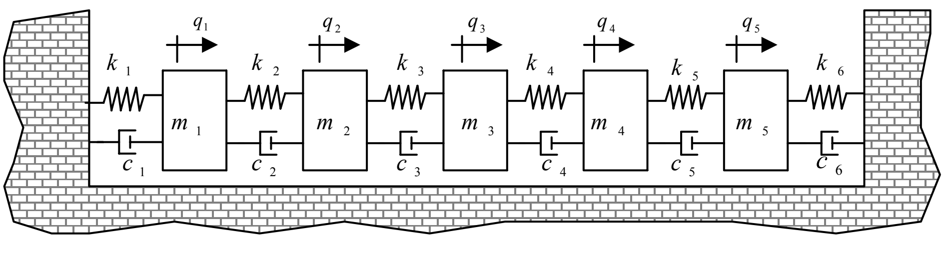

Figure 1 - Mass-spring-damper 5 dof system.

This mechanical system is described by the following mass, damping and stiffness matrices:

$$\mathbf{M}=\begin{bmatrix}0.5 & 0 & 0 & 0 & 0\\ 0 & 2 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0.3 & 0\\ 0 & 0 & 0 & 0 & 0.1\end{bmatrix}\qquad \mathbf{C}=\begin{bmatrix}12 & -2 & 0 & 0 & 0\\ -2 & 7 & -5 & 0 & 0\\ 0 & -5 & 8 & -3 & 0\\ 0 & 0 & -3 & 3.5 & -0.5\\ 0 & 0 & 0 & -0.5 & 0.6\end{bmatrix}\qquad \mathbf{K}=\begin{bmatrix}6330 & -4330 & 0 & 0 & 0\\ -4330 & 6830 & -2500 & 0 & 0\\ 0 & -2500 & 6100 & -3600 & 0\\ 0 & 0 & -3600 & 4600 & -1000\\ 0 & 0 & 0 & -1000 & 3000\end{bmatrix}$$

where the damping matrix is a non-proportional model.

The acceleration responses were generated from a random white noise multiple excitation, actuating as a force on each system mass. The white noise is applied because it stands out the dynamical characteristics of the system when the ARMAV models were used.

To simulate the disturbances in the response data, due to computational inaccuracy and interferences in the sensor measurements, noise levels of 1%, 5% and 10% were added to the measured responses. The noise level was established by a relationship between the noise RMS and the largest RMS among the five system output signals.

The external noises in data constitute the largest source of troubles on the determination of the system model proper order, generating dynamics that does not exist. In this case, the model order over-determination is a necessary trick so that it makes possible the right modal parameters identification. However, this procedure introduces some spurious modes, which do not represent the physical properties of the system, caused by additional degrees of freedom of the mathematical model assumed. In this paper, the proper ARMAV model order was obtained using the frequency stability diagrams, as it can be observed in Fig. (2). In order to plot these stability diagrams, five measurement responses were considered, with and without noises. In all cases, the values of natural frequencies stabilized and so the proper system order was obtained.

The distinguish the physical from the computational modes, in cases where the models are over-determinate, it was used the confidence factor called Weighted Modal Phase Colinearity (MPCW) and some criteria of natural selection such as: elimination of not complex conjugated eigenvalues, negative damping ratios and frequencies out of a specified range frequency, in agreement with the range of actual excitation frequencies.

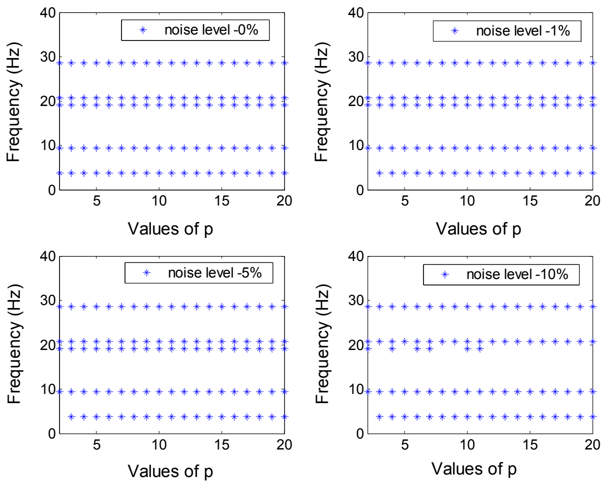

Figure 2 - Frequency stability diagrams.

In all cases analyzed here, we used a vector with 1000 data points for each input and output channel, sampled in a time interval $\Delta t = 0.001$ seconds. The Tables 1 and 2 show the theoretical and identified modal parameters using the ARMAV(2,2) model. In these first cases we considered all the measured responses without noise.

Table 1 - Frequencies and damping ratios of the 5 dof model.

Theoretical Model		**Identified Model - without noise**		**Error(%)**	
f_j **(Hz)**	ξ_j	f_j **(Hz)**	ξ_j	f_j **(Hz)**	ξ_j
3.7489	0.0324	3.7491	0.0325	0.0061	0.1014
9.4084	0.0603	9.4071	0.0641	0.0147	6.3588
19.0538	0.0936	19.0559	0.0509	0.0111	0.2798
20.7020	0.0505	20.7029	0.0938	0.0041	0.6388
28.5345	0.0250	28.5316	0.0256	0.0103	2.5741

Table 2 - Mode shapes of the *5* dof model.

Theoretical Modes				
Mode 1 - (Ψ_1)	**Mode 2 - (Ψ_2)**	**Mode 3 - (Ψ_3)**	**Mode 4 - (Ψ_4)**	**Mode 5 - (Ψ_5)**
0.3784-0.0178i	-0.3161+0.0138i	0.9657-0.00000i	-0.0481-0.0442i	-0.0004+0.0005i
0.5294-0.0075i	-0.3349-0.0127i	-0.1877+0.0317i	0.0321+0.0124i	-0.0012-0.0008i
0.5563-0.0000i	0.5649-0.0177i	-0.0049-0.0242i	-0.2620-0.0162i	0.0286+0.0088i
0.4891+0.0019i	0.6403-0.0000i	0.1387+0.0529i	0.7664-0.0000i	-0.2115-0.0328i
0.1661+0.0020i	0.2411+0.0077i	0.0791+0.0481i	0.5763+0.0787i	0.9764-0.0000i
Identified Modes – without noise				
0.3784-0.0176i	-0.3169+0.0064i	0.9656-0.00000i	-0.0463-0.0472i	-0.0004+0.0005i
0.5293-0.0073i	-0.3349-0.0197i	-0.1881+0.0308i	0.0317+0.0143i	-0.0011-0.0009i
0.5563-0.0000i	0.5647-0.0185i	-0.0048-0.0242i	-0.2618-0.0164i	0.0285+0.0093i
0.4891+0.0017i	0.6400-0.0000i	0.1386+0.0534i	0.7664-0.0000i	-0.2111-0.0368i
0.1661+0.0019i	0.2409+0.0080i	0.0789+0.0485i	0.5762+0.0792i	0.9763-0.0000i

With added noises, the identification of system was performed by ARMAV(*3*,*3*), ARMAV(*4*,*4*) and ARMAV(*4*,*4*) models, for system responses data corrupted by 1%, 5% and 10% noise levels, respectively. As it was expected, the increase of model order was necessary to the system identification.

It was observed that for data with 10% of noise, the third mode, corresponding the natural frequency of 19.05 Hz, was only identified using the following values of $p = 2, 4, 6, 7, 10$ and 11. This happened due mainly to the closeness of the third and fourth modes.

Table 3 - Errors for identified modes with different noise levels.

Error (*MA*) – Identified Modes			
Noise Level - 0%	**Noise Level- 1%**	**Noise Level - 5%**	**Noise Level - 10%**
0.0001	0.6259	2.3417	3.4543
0.0098	0.0254	0.1596	0.4561
0.0165	0.1219	1.2287	5.7731
0.0320	0.1119	0.6426	1.0506
0.0045	0.0192	0.5325	1.0974

To calculate the errors presented in Table (3) and to plot the magnitudes of the mode shapes showed in Fig. (3), it was necessary to transform complex modes in real modes. This task was performed assuming as real mode component the module of the correponding complex mode component, presented in Table (2). The sign of each component of approximate real modes was obtained using the following rule:

$$sign = \begin{cases} + & if \quad 0 \le \beta \le \pi/2 \quad or \quad 3\pi/2 \le \beta \le 2\pi \\ - & if \quad \pi/2 < \beta < 3\pi/2 \end{cases}$$

where β is the corresponding phase angle of each component of the complex mode.

The relative error of each mode was calculated with:

$$MA = \frac{|TMA\text{-}IMA|}{TMA}$$

where *TMA* is the area defined by the curve obtained by the theoretical approximate real mode and *IMA* is the area defined by the curve obtained by the identified approximate real mode.

With these simulated tests, it can be observed that natural frequencies and damping ratios values do not present changes when the output data were corrupted with noises. The relative errors remained the same to those shown in Table (1). However, the mode shapes have significant changes, but it remain inside the limits of acceptable errors, not exceeding 6% in the case with 10% of the noise level, as it can be seen in Table (3).

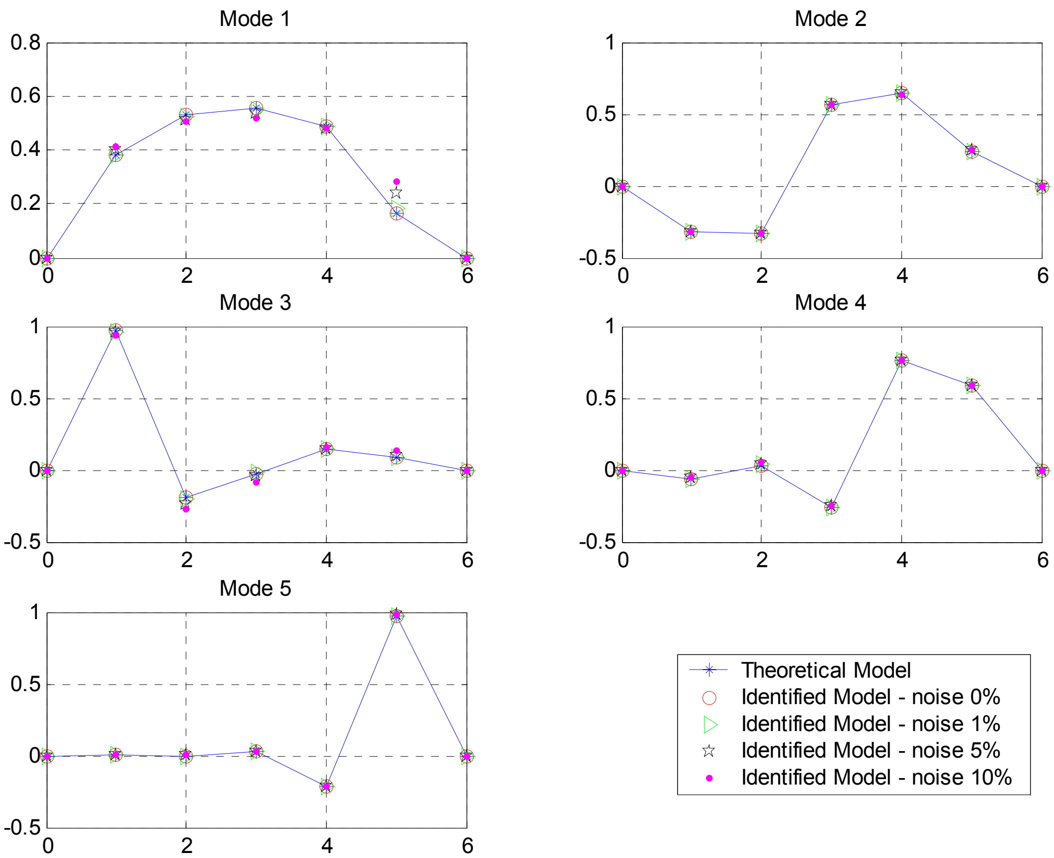

Figure 3 - Comparison between magnitudes of the modes with different noise levels.

In actual applications, only some of mass acceleration points are measured, due to the high cost of the equipments with a high number of channels. Then we also analyzed simulated tests where only some outputs were measured. It could be observed that the quality of the identification depends on the amount of information and the location of the measurements points. The Table (4) shows some of the system identification results obtained considering only 2, 3 and 4 points of measured outputs, with 5% noise level.

Table 4 - Identification errors considering different measured point locations.

Error (%) – Modal Parameters - 4 points of measured response : m_1, m_2, m_3 and m_4			
	f_j **(Hz)**	ξ_j	**Modes - (Ψ_j)**
ARMAV(*4,4*)	0.0126 0.0147 0.0111 0.0041 0.0103	0.0126 0.0147 0.0111 0.0041 0.0103	0.3821 0.1097 0.5034 1.0198 2.6018
Error (%) - Modal Parameters - 3 points of measured response: m_1, m_2 and m_3			
ARMAV(*6,6*)	0.0069 0.0165 0.0110 0.0044 0.0103	0.1354 6.3679 0.2801 0.6391 2.5721	0.4840 0.2573 1.6319 1.1065 18.1958
Error (%) - Modal Parameters - 2 points of measured responses: m_1 and m_2			
ARMAV(*44,44*)	0.3860 0.5818 0.0156 0.0032 0.0101	14.0500 4.0126 0.2530 0.6544 2.5624	0.0525 0.0208 0.4292 1.0579 5.2909

The errors presented in Table (4) show that the identification performed with 4 points of measured responses reached satisfactory results, maintaining the same order $p = 4$ used when all the mass points were measured. However, it was necessary to increase the ARMAV model order from $p = 6$ to $p = 44$, when we considered only 2 points of measured outputs. It could be observed that, in all cases, the errors in frequency identification did not exceed 0.6 %. However, when only 2 points of measurements were used, a damping ratio error of 14 % has been founded.

7. Conclusions

In this paper it was presented an application of the ARMAV model to the identification of the modal parameters of a five degrees of freedom mass-spring-damper system. The numerical simulations showed the efficiency of the least square method applied to modal parameters identification of a 5 dof mechanical system using the ARMAV state space model. With simulated tests, it could be observed that in presence of noises the values of frequencies and damping ratios, identified from an over-determined system, have not significant differences when all measured responses were considered. The vibration modes, however, have some differences but it remains inside an acceptable error range. The simulated model proved the efficiency of the method to identify systems that have close natural frequencies and light damping ratios. Moreover, the presented method showed satisfactory a parameters identification even when only some mass acceleration responses were measured.

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