FLIGHT TEST MANEUVERS OPTIMIZATION FOR PARAMETER IDENTIFICATION

Nei Salis Brasil Neto
Instituto Tecnológico de Aeronáutica
Praça Eduardo Gomes, 50, Vila das Acácias – 12228-900
São José dos Campos – SP - Brasil
sbrasil@ita.br

Luis Carlos Sandoval Góes
Instituto Tecnológico de Aeronáutica
Praça Eduardo Gomes, 50, Vila das Acácias – 12228-900
São José dos Campos – SP - Brasil
goes@ita.br

Elder Moreira Hemerly
Instituto Tecnológico de Aeronáutica
Praça Eduardo Gomes, 50, Vila das Acácias – 12228-900
São José dos Campos – SP - Brasil
hemerly@ita.br

Abstract. *This paper concerns parameter identification flight test maneuvers development and proposes an optimization technique that demonstrates the input signal importance for the quality and reliability of the entire system identification process applied to flight dynamics. The technique's goal is to reach better levels of identificability through optimized excitation signals specification. Regarding the efficiency of asymptotically unbiased estimators, such as maximum likelihood, the sensitivities of the output equations to the system parameters are maximized. The sensitivity matrix and Cramer-Rao lower bounds concepts are used as optimization criteria to generate excitation signals which should minimize the parameter estimation uncertainties. This approach depends only on the a priori knowledge about the dynamic model and has as starting point square waves bang-bang signals. Concerning the optimization procedure, a genetic algorithm is used, aiming at providing global solutions. The efficiency of this technique is shown through a Monte Carlo analysis of the parameter estimation process, using a system identification tool developed by ITA and EMBRAER. The linear longitudinal aircraft dynamics was used to generate simulated data, enabling comparison of the sub-optimal and optimal inputs. The parameter estimation reliability improvement is shown and the need for specification of optimized flight test maneuvers for parameter identification becomes clear.*

Keywords*: Flight Test Maneuver Optimization, Stability and Control Derivatives, System Identification*

1. Introduction

By following the $\mathbf{M^4V}$ methodology for the system identification procedure some excitation techniques and flight test maneuvers are studied. The analyses are not restricted on conventional maneuvers specification, and extrapolate for flight test maneuver development and optimization techniques.

During the analyses, it is verified that the flight test maneuvers for system identification and parameter estimation are players as important as the data acquisition system, concerning the entire system identification process reliability. These two components must be jointly treated, especially because they present high impact on the information content of the flight test data for parameter estimation.

The conventional excitation techniques analysis and the development and optimization of excitation signals are based on the information content of the flight test data. Basically, the maneuver specification must improve the system identification algorithm efficiency and facilitate the increase on the identification levels, regarding practical and operational constraints. Concerning the operational constraints, the flight test maneuver for parameter estimation must be specified in order to maintain flight conditions inside the flight safety envelope. This includes, for example, control surfaces deflection, load factor, aircraft attitude, flight velocity and angle of attack and sideslip angle limitations.

Furthermore, the mathematical model constraints must be taken into consideration. Therefore, the small perturbation assumptions play the most important role. The flight test maneuvers for parameter estimation must be specified such that discontinuities are not reached, mainly concerning high excursions on angle of attack and sideslip angle.

To summarize, the main goal of the maneuver specification is to excite the dynamic system so that the output equations sensitivities for the system parameters are maximized. The approach presented on this study is based on the *a priori* knowledge of the dynamic and measurement system to develop an experiment that appropriately excites the system natural modes (Morelli, 1997) (Gupta and Hall, 1975) (Stepner and Mehra, 1973). In this way, it is necessary to concentrate the excitation signal energy near the system modes. At this point, it becomes clear that very good excitation signal will be specified if the a priori model is very good. The maneuver specification and development techniques must, therefore, be robust for erroneous a priori modeling.

The approach for maneuver development and optimization technique is proposed to facilitate the excitation signal specification, through an optimization algorithm, that maximizes the entire system identification process reliability. The system identification process reliability is directly related to the flight test data information content. The information content is computed on the Fisher's Information Matrix, which is a function of the measurement model and of the flight test maneuvers, through the output sensitivities to the system parameters. Analysis of the Cramer-Rao inequality, one of the most important result of the theory of accuracy estimation (Maine and Illiff, 1985) (Maine and Iliff ,1981) (Goodwin and Paine, 1977), can prove that the lower bound for parameter estimation covariance is the inverse of the information matrix. The equality on the Cramer-Rao inequality holds when the estimator is asymptotically unbiased and efficient, just as the maximum likelihood estimator. In this way, the proposed optimization criterion for system identification flight test maneuver development is based on the Fisher's Information Matrix.

In section 2, the theoretical background and the problem formulation for system identification maneuver optimization will be presented. The results for the development process of excitation signal for aircraft longitudinal dynamics are shown in section 3. In addition, the results of a Monte Carlo simulation of the estimation process are presented, aiming comparison between conventional and optimal inputs. The parameter estimation reliability improvement is shown and the need for specification of optimized flight test maneuvers for parameter identification becomes clear.

2. Excitation Signal Development and Optimization

The system identification flight test maneuver development and optimization technique is based on the information content of the aircraft flight test response data. The information content can be computed by using the Fisher Information Matrix, which is a function of the dynamic system output sensitivities to the system parameters and of the measurement noise.

The parameter estimation accuracy assessment can be done through the information matrix. With reasonable assumptions, the Cramer-Rao inequality theorem proves that, for asymptotically unbiased and efficient estimators, the inverted information matrix contain the lower bound for the parameter estimation covariance (Goodwin and Paine, 1977) (Maine and Iliff 1981) (Ljung, 1987).

In this way, the system identification flight test maneuver development and optimization depends just on the information matrix. The information matrix depends on the output sensitivities to the system parameters and on the measurement noise. Therefore, the proposed development and optimization process does not depend on the estimation algorithm. It just depends on a priori knowledge of the system. The feedback of the estimation results to the a priori dynamic model can occur and the excitation signal optimization can be more and more efficient.

2.1. Problem Formulation

The aircraft flight dynamics is supposed to be represented by the small perturbations dynamic model:

$$\begin{aligned} \dot{x}(t) &= A(\theta)x(t) + B(\theta)u(t) \\ y(t) &= C(\theta)x(t) + D(\theta)u(t) \\ y_m &= y_i + \upsilon_i \ ,i = 1, 2, 3, ..., N \end{aligned} \tag{1}$$

Where the measurement variables are contaminated with white Gaussian measurement noise $\upsilon(i)$, described by:

$$E\{v_i\} = 0 \ \text{ e } \ E\{v_i v_j^T\} = R\delta_{ij} \tag{2}$$

and *N* denotes de length of the measurement vector. The output sensitivities to the system parameters are defined by:

$$S_i = \frac{\partial y_i}{\partial \theta} \tag{3}$$

The sensitivites of the ith output appear at the ith sensitivity matrix line. The output sensitivities to the jth parameter appear at the jth column of S.

The sensitivity matrix can be computed by:

$$\frac{d}{dt}\left[\frac{\partial x}{\partial \theta_k}\right] = A\frac{\partial x}{\partial \theta_k} + \frac{\partial A}{\partial \theta_k}x + \frac{\partial B}{\partial \theta_k}u$$

$$\frac{\partial x}{\partial \theta_k}(0) = 0 \tag{4}$$

$$\left[\frac{\partial y}{\partial \theta_k}\right] = C\frac{\partial x}{\partial \theta_k} + \frac{\partial C}{\partial \theta_k}x + \frac{\partial D}{\partial \theta_k}u \text{ , k=1,2,...,p}$$

where p is the length of the system parameter vector.

Concerning system identification through an asymptotically unbiased and efficient estimator, the minimum parametric estimation standard deviation are computed from the diagonal elements of the dispersion matrix (Maine and Iliff, 1981) (Morelli, 1993). The dispersion matrix is defined as the inverse of the Fisher Information Matrix:

$$D = M^{-1} \tag{5}$$

where:

$$M = \sum_{i=1}^{N} S_i^T R^{-1} S_i \tag{6}$$

The Equation (4) are derived from the differentiation of Eq. (1) with respect to parameter vector θ. The information matrix depends on the input vector through the sensitivities. The input vector influences the sensitivities directly, as a forcing function, and indirectly, as an influence on the states.

The dispersion matrix can also be represented as:

$$D = \left[\sum_{i=1}^{N}\left(\frac{\partial y_i}{\partial \theta}\right)^T R^{-1}\left(\frac{\partial y_i}{\partial \theta}\right)\right]^{-1} \tag{7}$$

From the principal diagonal of D, the uncertainty levels for the parameter estimation (standard deviation) are computed as follows:

$$\sigma_k = \sqrt{d_{kk}} \text{ , k=1,2,...,p} \tag{8}$$

In addition, the correlation coefficients between the estimated parameters are:

$$\rho_{\theta_k \theta_l} = \frac{d_{kl}}{\sqrt{d_k d_l}} \text{ ,l=1,2,3,...,p} \tag{9}$$

where d denotes the elements of D.

At this point, the link between the parameter estimation reliability and the excitation signal is established. The dispersion matrix is a function of the input vector through the output sensibilities to the system parameters. Besides, the parameter estimates standard deviation and correlation coefficients are computed from Eq. (8) and Eq. (9).

The present work establishes the dispersion matrix as the main element for the system identification flight test maneuver development and optimization criterion. Both the principal diagonal elements and the off principal diagonal elements of D can be considered in the process, depending on the required objectives.

2.2. Aspects concerning the optimization process

The development of an input that excites as much as possible the dynamic response of the aircraft without the perfect knowledge about the system modes, while practical constraints have to be respected, is a difficult problem. The main obstacle for researches is the practical implementation of the resultant signals (Stpner and Mehra, 1973) (Gupta and Hall, 1975). In the other hand, the computational problems involve the optimization criterion selection, the optimization algorithm application for global solution and multiple input problems resolution.

Recently, an optimize flight maneuver development technique that accesses the aforementioned problems was developed (Morelli, 1993). The inputs are square waves globally optimized which respect to the imposed practical constraints. The flight maneuvers are bang-bang signals, or full positive, or full negative, or zero. Concerning the optimization, dynamic programming techniques are applied, aiming for globally optimized solutions. The approach considers two problems: the smaller time maneuver, which minimizes the maneuver time and the uncertainty levels to reach a priori defined goals; and the smaller Cramer-Rao maneuver, that minimizes the uncertainty levels with a fixed maneuver time.

The present work approach is similar to the second option above. The square waves bang-bang excitation signals are generated to minimize the uncertainty levels at a given maneuver time. Regarding the optimization, an genetic algorithm is used, aiming for globally solutions.

The system identification flight test maneuver development must account for flight safety and operational and mathematical constraints. The process must guarantee that the flight envelope will be respected and that the mathematical assumptions will be maintained. In this way, it is important that input and output variables - as surface deflection, linear acceleration, angle of attack and sideslip angle - are constrained. In previous works (Gupta and Hall, 1975) (Mehra, 1974) (Stepner e Mehra, 1973), the constraints were indirectly imposed through the input signal energy, computed as follow:

$$\int_0^T u(t)^T u(t) dt = E \tag{10}$$

This approach is an indirect manner to limit the outputs. It is necessary, however, to verify later on whether the output response is kept under the limits or not.

In addition, another constrained imposition technique is applied in this work: the direct limitation to the output response. Therefore, the cost function will be minimized respecting the imposed output variation constraints.

Regarding the optimization criterion, the cost function composition is done considering the principal diagonal elements of the dispersion matrix, as follow:

$$J = Tr[D] \tag{11}$$

Another dispersion matrix norm can be used. In addition, the introduction of a weighting matrix that facilitates the access of most interesting parameters can be considered. From Equation (11), the weighted cost function can be written as:

$$J = Tr[WD] \tag{12}$$

where W is the weighting matrix.

It is interesting to point out that even though the system has multiple parameters, the excitation signal development can be performed concerning just one or a small group of them, which is of interest. The importance of this approach becomes clear when there are identificability problems involving parameters with low influence on output response.

Regarding the excitation signal configuration for the optimization, the variables involved in the process are the switching times and the amplitude. The switching time's number can be selected by the user. The amplitude is bounded between a minimum and maximum value also selected by the user. These characteristics determine the input complexity and whether it can be practically implemented or not.

3. Results

The results shown in this study were all generate through the SYSID. This system identification tool was developed by ITA and EMBRAER as a part of a research and development project funded by FAPESP.

One hundred runs Monte Carlo simulation of the estimation process was performed, aiming at statistically comparing the different applied flight test maneuvers. The Monte Carlo estimation data were analysed statistically by using the following procedure: Gaussian probability density functions were fitted to the data to provide a systematic approach and a mathematical basis for extrapolation. In the same case study, conventional and optimized signal excitations were applied. The resultant estimation distributions were compared, providing a measure for the different estimation scatters. Then, the cumulative density function was used to determine the probability of estimates in a given interval. This approach has tremendous advantages over theoretical uncertainty levels assessment, because it measures the actual performance of the estimation algorithm.

The simulated deterministic data were contaminated with zero mean Gaussian white noise with the covariance determined by R . The noise introduction is necessary to provide stochastic analysis through Monte Carlo simulation.

Previously, conventional inputs were analysed. The conventional inputs analysed were doublet, 2-1-1, 3-2-1-1 and 2-3-1-1 and they were specified to provide its maximum power on the system modes. Then the optimized input was compared to the conventional input that provided the best stability and control derivatives extraction.

3.1 Case study

This case study regards an aircraft longitudinal short period model that was previously analysed by (Mehra, 1974), by (Chen, 1975) and by (Morelli, 1993). The main objective now is to present the basic aspects of the system identification flight test maneuver development and optimization technique. The focus is on the resultant signals implementation capability and the advantages of the optimized signal in comparison to the conventional inputs.

$$\begin{bmatrix} \dot{\alpha} \\ q \end{bmatrix} = \begin{bmatrix} Z_\alpha & 1 \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta e} \\ M_{\delta e} \end{bmatrix} \delta e \tag{13}$$

where α, q e δe are, the angle of attack, the pitch velocity and the elevator deflection. The output model is:

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix} \tag{14}$$

The measurement equations are:

$$\begin{bmatrix} y_{m_1}(i) \\ y_{m_2}(i) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1(i) \\ y_2(i) \end{bmatrix} + \begin{bmatrix} \upsilon_1(i) \\ \upsilon_1(i) \end{bmatrix} \quad ,i=1,2,3,...,N \tag{15}$$

where υ is the zero mean Gaussian measurement noise with covariance described by:

$$R = \begin{bmatrix} 2.0 & 0 \\ 0 & 1.0 \end{bmatrix} \tag{16}$$

3.1.1 Conventional 3-2-1-1 input

Through the conventional inputs comparison, was observed that the conventional input that provided the best stability and control derivatives extraction was the 3-2-1-1. Therefore, the results obtained from this signal excitation will be compared to the results obtained from optimized input. The 3-2-1-1 time history and system response are shown in fig. 1.

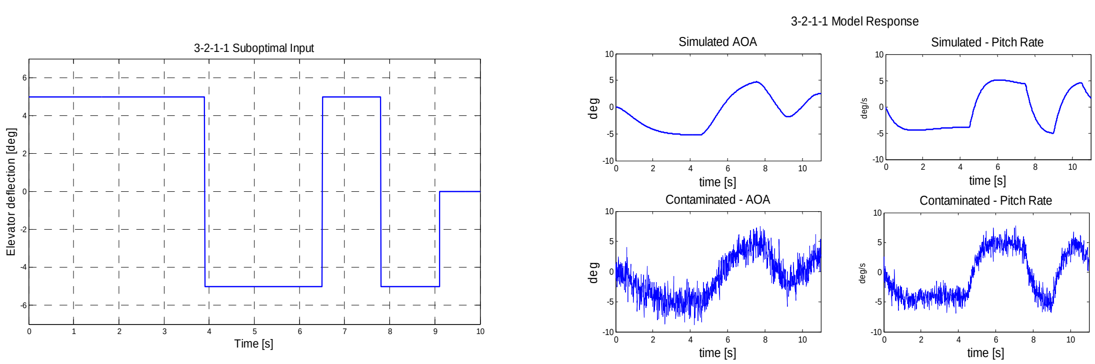

Figure 1. Conventional 3-2-1-1 input time history and system response

3.1.2 Input optimization

The 3-2-1-1 system response presented an angle of attack variation between $\pm 5^o$. To provide comparison, the optimized input was constrained in order to maintain the angle of attack response between $\pm 5^o$:

$$-5 \le y_1(t) \le 5^o \tag{17}$$

Other two constraints were imposed, concerning the input amplitude and the total time available for the maneuver.:

$$\begin{aligned} &3 \le \delta_e \le 15^o \\ &T \le 30s \end{aligned} \tag{18}$$

The number of switching times selected was 10. The initial population for the genetic algorithm was composed by forty individuals, each one containing eleven variables: ten switch times and the amplitude. The initial population respect the following:

$$\begin{aligned} &a = 9 + randn[-6:6] \\ &st_1 = 1 + randn[-1:4] \\ &st_n = st_{n-1} + (4 + randn[-4:4]), \text{ n=2,3,...,10} \end{aligned} \tag{19}$$

Where $\text{randn}[-x:x]$ denotes a zero mean Gaussian distribuition between $\pm x$, a denotes the input amplitude and st_n denotes the switching times. It is important to point out that the initial population function respect all the imposed constraints.

The resultant optimized input was such that the trace of the dispersion matrix was minimized. The table 1 shows the Cramer-Rao lower bounds for the parameter covariance, regarding the optimized input and the 3-2-1-1 input.

Table 1 . Cramer-Rao lower bounds comparison

Input	Z_α	$Z_{\delta e}$	M_α	M_q	$M_{\delta e}$
Optimal Input 1	0.0245735	0.0173289	0.0275996	0.0417865	0.0319524
3-2-1-1	0.0390869	0.0268282	0.0443046	0.0652885	0.0489506

The applied optimal input was performed through an external pilot interface which was range limited between the optimal input amplitude. The time histories of the resultant optimal input, of the applied optimal input and of the system response are shown in the figure 2.

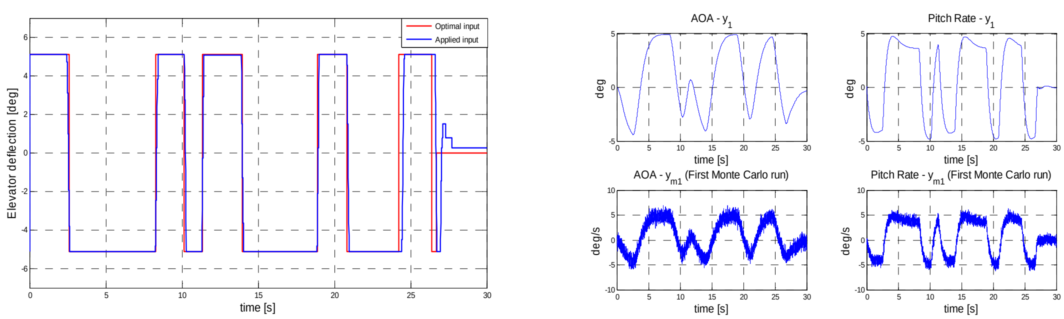

Figure 2. Optimized input time history and system response

3.1.2 Monte Carlo simulation results

The Monte Carlo simulation results are shown in figure 3. The histograms and the fitted probability density function of the one hundred runs for the estimation procedure of the case study parameters are shown, aiming at results comparison between the conventional 3-2-1-1 input and the optimized input.

The bias and scatter of the parameters estimation obtained through the optimized input data were small than the parameters estimation obtained through the conventional 3-2-1-1 input, except for $Z_{\delta e}$. The results are summarized in table 2.

Table 2 . Monte Carlo simulation results

True Parameters	Input	Mean/ Relative Std Dev. (1 sigma)	5% Maximum Estimative Error Probability	Input	Mean/ Relative Std Dev. (1 sigma)	5% Maximum Estimative Error Probability
$Z_{\alpha} = -0.737$	Optimal Input 1	-7.420e-001/ 1.3093e+000	**9.99e-001**	3-2-1-1	-7.468e-001/ 3.041e+000	**8.6734699e-001**
$Z_{\delta e} = 0.005$		4.292e-003/ 1.965e+001	**1.55e-001**		4.797e-003/ 1.199e+001	**3.0637165e-001**
$M_{\alpha} = -0.562$		-5.580e-001/ 2.429e+000	**9.51e-001**		-5.482e-001/ 4.693e+000	**6.5066688e-001**
$M_{q} = -1.588$		-1.603e+000/ 2.423e+000	**9.44e-001**		-1.616e+000/ 5.071e+000	**6.4601702e-001**
$M_{\delta e} = -1.660$		-1.670e+000/ 1.658e+000	**9.95e-001**		-1.673e+000/ 3.621e+000	**8.2269613e-001**

3. Concluding remarks

Comparisons between conventional inputs and optimized inputs for aircraft aerodynamic parameter estimation were performed in this work. The conventional inputs were shaped in order to increase the signal power spectral density in the range of the natural frequencies of the system of interest. In contrast, the optimized input was obtained by directly minimizing the Cramer-Rao lower bounds for the parameter estimation covariance. In general, regarding the bias and scatter of the process distribution, it becomes clear that the optimized input provided better estimation results.

The necessity for the optimized flight test maneuver development approach during the parameter estimation flight test campaign planning is evident. The observed increase in the parameter estimation process efficiency by itself can improve the entire flight test campaign viability and provide better results for aircraft envelope expansion, aircraft certification, control system design, simulation, and other applications. Therefore, it is highly recommended that aerodynamic parameter estimation flight test activities be performed as interactively as possible, with feedbacks to the a priori models and input design techniques, in order to maximize the process efficiency and reliability.

3. Acknowledgements

This work was sponsored by FAPESP, through the partnership project EMBRAER/ITA, as a part of the PICTA Program.

4. References

Chen, R.T.N. 1975, "Input Design for Arcraft Parameter Identification: Using Time-Optimal Control Formulation", AGARD CP-172, paper 13.

Goodwin, G.C. and Payne, R.L., "Dynamic System Identification: Experiment Design and Data Analysis", Ed. Richard Bellman, New York, 1997.

Gupta, N.K. and Hall, E.H.Jr. 1975, "Input Design for Identification of Aircraft Stability and Control Derivatives", NASA CR-2493.

Maine, R.E. and Ilifff, K.W. 1981, "The Theory and Practice of Estimating Accuracy of Dynamic Flight-Determined Coefficients", NASA RP-1077.

Maine, R.E. and Ilifff, K.W. 1985, "Identification of Dynamic Systems: Theory and Formulation", NASA RP-1138.

Mehra, R.K., 1974, "Optimal Inputs for Linear System Identification", IEEE Transactions on Automatic Control, vol. AC-19, no. 3.

Morelli, E.A., 1997, "Flight Test Validation of Optimal Input Design and Comparison to Conventional Inputs", AIAA paper 97-3711.

Stepner, D.E. and Mehra, R.K. 1973, "Maximum Likelihood Identification and Optimal Input Design for Ientifying Aircraft Stability and Control Derivatives", NASA CR-2200.

5. Responsibility notice

The authors are the only responsible for the printed material included in this paper.

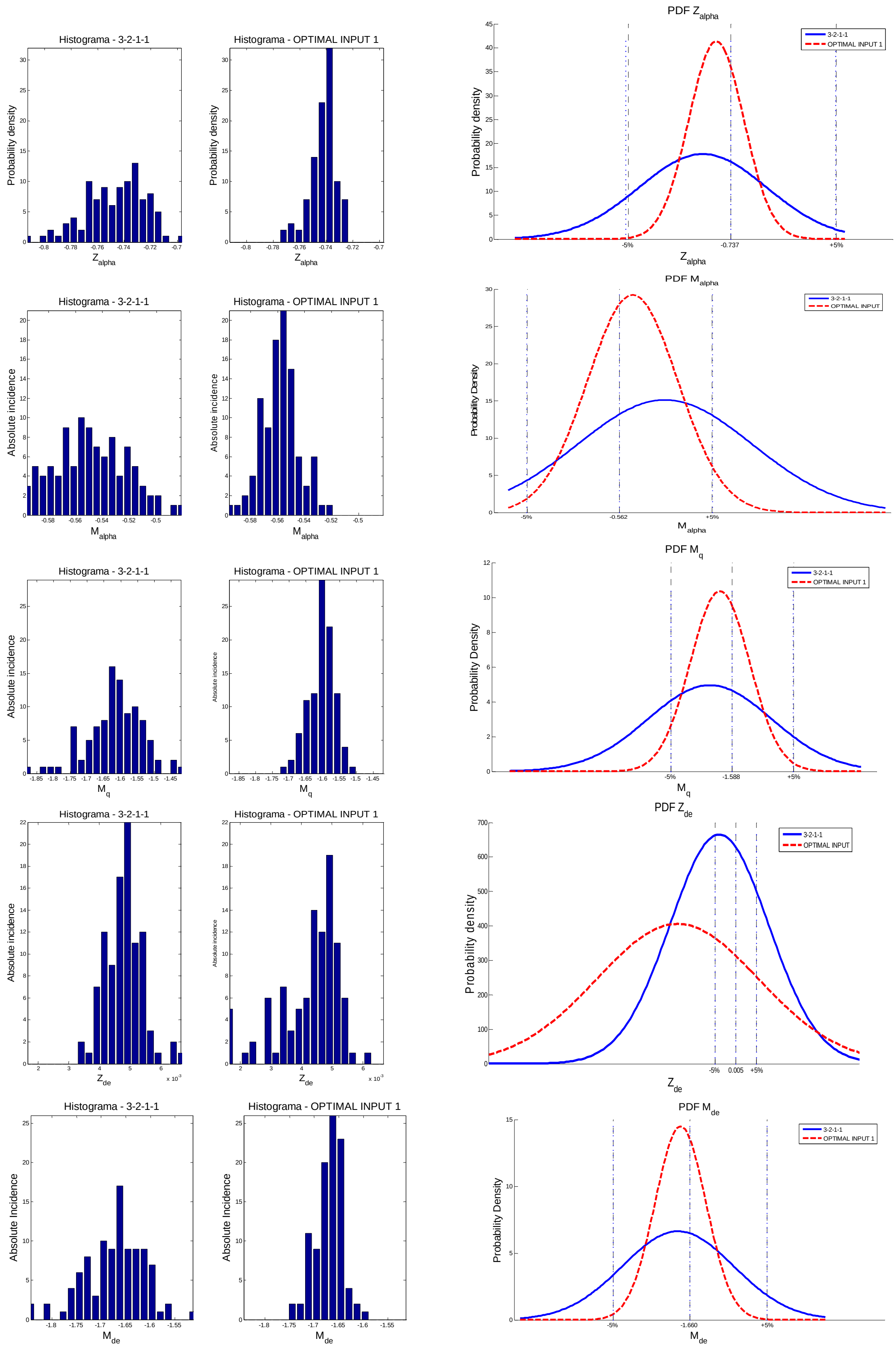

Figure 3. Monte Carlo Simulation Results: histograms and probability density functions