

COMPARISON OF MODELS FOR THE STEADY-STATE ANALYSIS OF TILTING-PAD THRUST BEARINGS

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Abstract. *Prediction of the minimum oil film thickness and the maximum temperature on the surface of the bearing pad is crucial in the design and dimensioning of bearings. Friction loss, oil bath temperature and pad deflection are other parameters of interest. Depending on the desired information a numerical model requires different levels of detail. The two dimensional Reynolds equation for pressure in the oil film can be solved isothermally or considering viscosity variations in two or three dimensions, requiring solution of the equations for thermal equilibrium in oil and pad. Knowing the temperature distribution the deflection of the pad due to pressure and thermal bending can be calculated using a flat plate approximation. At the five free sides of the pad heat transfer can be modelled. The temperature distribution at the inlet to the pad can be calculated through equilibrium of thermal energy for the groove between pads and the oil bath temperature from energy equilibrium for the entire bearing. The main theoretical contribution of this paper is the elaboration and comparison of 7 different mathematical models of increasing complexity. The results are compared to experimental data for steady-state operation of a 228 mm outer diameter bearing. It is found that for the given bearing a two dimensional model is sufficient to estimate the minimum oil film thickness and the maximum temperature on the pad surface. Three dimensional modelling does not improve the quality of the results.*

Keywords: *lubrication, tilting-pad bearing, thermo-elastohydrodynamic, mathematical model*

1. Introduction

The maximum oil temperature is an important parameter for reliable operation of slider bearings since the babbitt coating on a bearing pad loses strength and will experience creep at high temperatures. If the thickness of the oil film gets too close to the surface roughness the pad and collar surfaces may touch leading to mixed lubrication and possible bearing damage. The minimum oil film thickness is therefore also of critical importance. Being able to accurately predict these parameters based on operating conditions is therefore a crucial task in the design and dimensioning of bearings.

The heat generated through viscous dissipation in the lubricating oil is removed partly by the flow of the lubricant and partly by conduction to the surrounding solids. The oil velocity tends to zero at the bearing pad leading to much higher temperatures than close to the collar. The bearing pad will deflect due to elasticity and thermal bending caused by temperature gradients in the pad. The influence of thermal bending is often negligible in small bearings while in large bearings the deflection can be of the same order of magnitude as the minimum oil film thickness.

Many authors have dealt with modelling of hydrodynamic thrust bearings. Thermal deflections of the pads were first considered by Sternlicht, Carter & Arwas (1961) in a 2 dimensional model assuming adiabatic flow. Huebner (1974) presented a 3 dimensional analysis performed on a sector shaped thrust bearing. He followed the general approach suggested by Dowson (1962). A 2 dimensional analysis by Ettles (1976) allowed for pad deflection, heat conduction to pad and runner and used a hot oil carry over factor to determine the change in temperature from the trailing edge of one pad to the leading edge of the next. Vohr (1981) suggested using a heat balance formulation to determine the temperature at the inlet to the pads. Kim, Tanaka & Hori (1983) presented the first 3 dimensional model performed on a tilting-pad thrust bearing. They however neglected pad deflections. Ettles & Anderson (1991) extended the work presented in 1976 to 3 dimensions and gave a detailed discussion of heat transfer correlations for the back surfaces of pads.

This paper presents and compares models of increasing complexity considering the steady-state behavior of tilting-pad thrust bearings under fully flooded operating conditions. The momentum equations are reduced and integrated across the oil film to give equations for the velocity components. These are introduced in the continuity equation and integrated across the oil film to give the Reynolds equation for 2 dimensional pressure in the oil film. The Reynolds equation is solved subject to a viscosity distribution calculated through the energy equation. The energy equation can be integrated across the oil film to give a 2 dimensional viscosity distribution or solved in all 3 dimensions to give a 3 dimensional viscosity distribution. The bearing pad deflects due to thermal gradients. This effect can be modelled using a 2 dimensional flat plate approximation for deflection and 1 or 3 dimensional heat transfer through the pad. The leading edge oil temperature is a boundary condition to the energy equation. It can be calculated through equilibrium of thermal energy for the groove.

The models presented are:

1. An isothermal hydrodynamic(HD) model only solving the Reynolds equation.
2. A 2D adiabatic thermo-hydrodynamic(THD) model solving the 2D energy equation.
3. A 2D THD model in which heat transfer to the collar and 1D heat transfer through the pad is considered.
4. A 2D thermo-elasto-hydrodynamic(TEHD) model in which deflection of the pad is introduced.
5. A 3D adiabatic THD model.
6. A 3D THD model solving the 3D energy equation for the oil and 3D heat transfer through the pad.
7. A 3D TEHD model.

All models require specification of the collar temperature.

Simulated values of film thickness, oil temperature and friction loss are compared to experimental results presented by Glavatskikh (2000, 2001). A sketch of a bearing pad and the coordinate system used are presented in Fig. 1.

2. Governing Equations

The bearing is assumed to work under conditions where the following assumptions can be adopted.

- | | |
|--|---|
| (i) The fluid is Newtonian and incompressible. | (iv) Viscosity varies with temperature only. All other fluid properties are constant. |
| (ii) The flow is laminar. | (v) Deflections of the collar surface are negligible. |
| (iii) Body forces and effects of inertia are negligible. | |

2.1. Fluid film

Subject to the assumptions (i) to (v) and through an order of dimension analysis the governing equations for the fluid film are reduced to the standard form employed in lubrication theory:

The continuity equation and the momentum equations:

$$0 = \frac{1}{r} \frac{\partial}{\partial r} [rv_r] + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}, \quad \frac{\partial p}{\partial r} = \frac{\partial}{\partial z} \left[\mu \frac{\partial v_r}{\partial z} \right], \quad \frac{1}{r} \frac{\partial p}{\partial \theta} = \frac{\partial}{\partial z} \left[\mu \frac{\partial v_\theta}{\partial z} \right], \quad 0 = \frac{\partial p}{\partial z} \quad (1)$$

The momentum equations can be integrated twice across the oil film to give the velocity components. With the boundary conditions that v_r , v_θ and v_z are zero at $z = h$, that v_r and v_z are zero and that v_θ is equal to ωr at $z = 0$ they become:

$$v_r = \frac{\partial p}{\partial r} \int_0^z \frac{z dz}{\mu} - \frac{\partial p}{\partial r} \frac{F_1}{F_0} \int_0^z \frac{dz}{\mu}, \quad F_0 = \int_0^h \frac{dz}{\mu}, \quad F_1 = \int_0^h \frac{z dz}{\mu} \quad (2)$$

Nomenclature			T	=	temperature	[K]	
B	=	arc length of pad at mean radius	[m]	T_a	=	oil bath temperature	[K]
c	=	oil thermal capacity	[J/kg/K]	T_c	=	collar temperature	[K]
D	=	flexural rigidity of pad	[Nm]	T_{in}	=	oil inlet temperature	[K]
E	=	Young's modulus for bearing pad	[N/m ²]	T_{le}	=	leading edge temperature	[K]
F_z	=	load per bearing pad	[N]	v_r, v_θ, v_z	=	velocity components in r, θ and z directions	[m/s]
h	=	oil film thickness	[m]	V_r, V_θ	=	Kirchhoff stresses	[N]
h_0	=	film thickness at pivot point	[m]	\dot{V}_{oil}	=	oil flow through bearing	[m ³ /s]
H	=	convection heat transfer coefficient	[W/m ² /K]	w	=	deflection of pad in direction of z_p	[m]
k	=	oil thermal conductivity	[W/m/K]	z	=	coordinate in the direction of film thickness	[m]
k_p	=	pad thermal conductivity	[W/m/K]	z_p	=	coordinate in the direction of pad thickness	[m]
k_1, k_2	=	coefficients for viscosity relation	[-]	α	=	coeff. of linear thermal expansion	[1/K]
M_x, M_y	=	moments of pressure around pivot point	[Nm]	α_r	=	roll angle of pad	[rad]
M_T	=	thermal bending moment on pad	[Nm]	α_p	=	pitch angle of pad	[rad]
M_r, M_θ	=	bending moments on pad	[Nm]	θ	=	angular coordinate	[rad]
$M_{r\theta}$	=	twisting moment on pad	[Nm]	θ_0	=	pad angle	[rad]
p	=	pressure	[N/m ²]	θ_p	=	angular coordinate at pivot point	[rad]
\dot{Q}	=	heat flow	[W]	θ_{gr}	=	angular extent of groove	[rad]
\dot{Q}_{gen}	=	generated heat in bearing	[W]	μ	=	oil viscosity	[Ns/m ²]
r	=	radial coordinate	[m]	ν	=	Poisson's ratio for pad material	[-]
r_1	=	pad inner radius	[m]	ρ	=	oil density	[kg/m ³]
r_2	=	pad outer radius	[m]	ω	=	angular velocity of collar	[rad/s]
r_p	=	radial coordinate at pivot point	[m]				
t_p	=	bearing pad thickness	[m]				

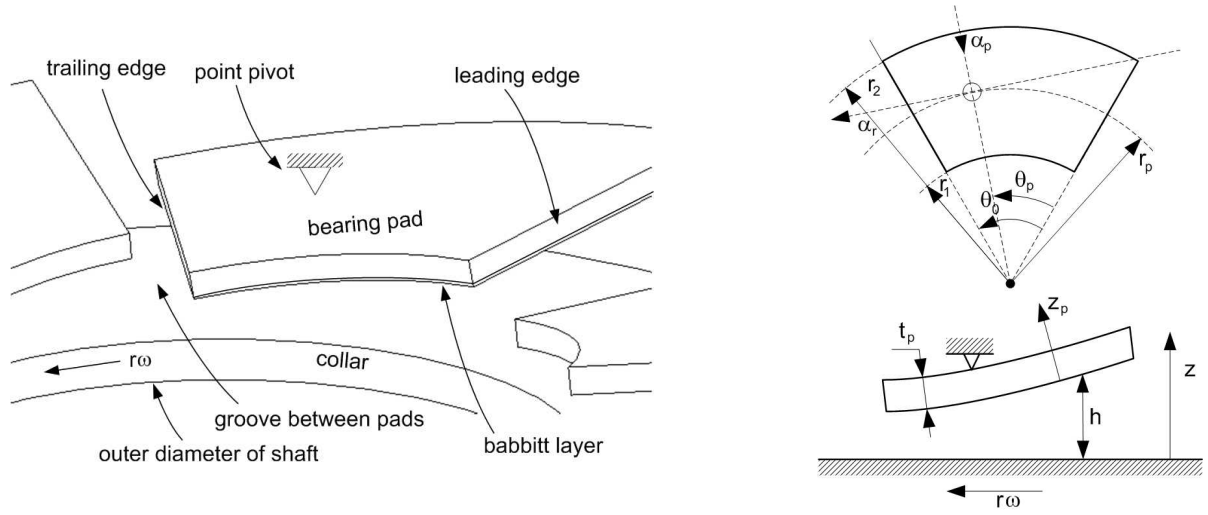


Figure 1: Left: An illustration of a bearing pad and the terminology used in the paper. Right: The coordinate system used.

$$v_\theta = \frac{1}{r} \frac{\partial p}{\partial \theta} \int_0^z \frac{z dz}{\mu} - \left(\frac{r\omega}{F_0} + \frac{1}{r} \frac{\partial p}{\partial \theta} \frac{F_1}{F_0} \right) \int_0^z \frac{dz}{\mu} + r\omega \quad (3)$$

$$v_z = -\frac{1}{r} \frac{\partial}{\partial r} \int_0^z r v_r dz - \frac{1}{r} \frac{\partial}{\partial \theta} \int_0^z v_\theta dz \quad (4)$$

The velocity components can be introduced in the continuity equation and integrated across the oil film thickness to yield a generalized Reynolds equation as suggested by Dowson (1962):

$$\frac{1}{r} \frac{\partial}{\partial r} \left[G_1 r \frac{\partial p}{\partial r} \right] + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left[G_1 \frac{\partial p}{\partial \theta} \right] = \omega \frac{\partial}{\partial \theta} \left[\frac{G_0}{F_0} - h \right], \quad G_0 = \int_0^h \int_0^z \frac{dz}{\mu} dz, \quad G_1 = \int_0^h \int_0^z \frac{z dz}{\mu} dz - \frac{F_1}{F_0} G_0 \quad (5)$$

Viscosity can be considered constant or to vary in two or three dimensions. Considering the temperature to be constant in the z -direction the integrals F_0 , F_1 , G_0 and G_1 can be calculated analytically simplifying the Reynolds equation considerably. Subject to a known viscosity distribution and the boundary condition that pressure should be zero at the edges of the bearing pad Eq. 5 can be solved to give the pressure distribution in the oil film.

The equation for conservation of energy reduces to:

$$\rho c \left[v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right] = k \frac{\partial^2 T}{\partial z^2} + \mu \left[\left(\frac{\partial v_r}{\partial z} \right)^2 + \left(\frac{\partial v_\theta}{\partial z} \right)^2 \right] \quad (6)$$

Considering viscosity to be constant in the z -direction and integrating across the oil film gives a 2D formulation:

$$\rho c \left[-\frac{h^3}{12\mu} \frac{\partial p}{\partial r} \frac{\partial T}{\partial r} \right] + \left[-\frac{h^3}{12\mu r^2} \frac{\partial p}{\partial \theta} + \frac{h\omega}{2} \right] \frac{\partial T}{\partial \theta} = \frac{h^3}{12\mu} \left(\frac{\partial p}{\partial r} \right)^2 + \frac{h^3}{12\mu r^2} \left(\frac{\partial p}{\partial \theta} \right)^2 + \frac{\omega^2 r^2 \mu}{h} + S_T \quad (7)$$

where S_T contains the energy added through heat transfer at $z = 0$ and at $z = h$. S_T is determined in section 2.2. The boundary conditions on temperature are:

$$T(r, 0, z) = T_{le}(r), \quad T(r, \theta, 0) = T_c, \quad T_{oil}(r, \theta, z=h) = T_{pad}(r, \theta, z_p=0) \quad (8)$$

The leading edge temperature, T_{le} is assumed to be uniform in z . Determination of T_{le} is discussed later. No boundary condition is needed in the r -direction as the oil flows out of the domain at both radial boundaries.

2.2. Heat Transfer

Heat conduction in pad:

$$0 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z_p^2} \quad (9)$$

The boundary conditions at the back of the pad can be formulated:

$$H(T - T_a) = -k_p \frac{\partial T}{\partial z_p} \Big|_{z_p=t_p}, \quad H(r, \theta, t_p) = 25.5(r\omega)^{0.7} \mu^{-0.2} B^{-0.4} \quad (10)$$

where T_a is the oil bath temperature, and $H(r, \theta, t_p)$ is the convection coefficient at the back of the pad as stated by Ettles & Anderson (1991). They show $H(r, \theta, t_p)$ to be accurate within $H_{-50\%}^{+100\%}$ for pads of greatly varying size having either

line or point pivots. They state that experiments indicate the values of H at the leading, trailing and inner radial surfaces to be twice as high and the value at the outer radial surface to be four times as high. These values are used.

On the oil/pad-surface the boundary conditions on temperature are

$$T_{pad}(r, \theta, 0) = T_{oil}(r, \theta, h), \quad k \frac{\partial T_{oil}}{\partial z} \Big|_{z=h} = k_p \frac{\partial T_{pad}}{\partial z_p} \Big|_{z_p=0} \quad (11)$$

Heat transfer through the pad can be treated in a simplified way by only modelling heat transfer at the back of the pad and treating conduction in the pad as a 1D problem. This formulation is used in combination with the 2D integrated energy equation for the oil film. The oil temperature in this formulation is considered uniform in the z -direction and the Nusselt number, $Nu = 7.55$ (Kays & Crawford, 1993) for a fully developed velocity profile between parallel plates is used to calculate the convection heat transfer coefficients at pad and collar. The S_T -term in Eq. 7 thereby becomes:

$$S_T = \left[\frac{4h}{k_{oil}Nu} + \frac{t_p}{k_p} + \frac{1}{H_{back}} \right]^{-1} (T - T_a) - \frac{k_{oil}Nu}{4h} (T - T_c) \quad (12)$$

Following the general procedure of Vohr (1981) T_a and T_{le} can be determined so as to fulfill equilibrium of thermal energy for the entire bearing and for the groove between pads. Assuming that all the generated heat flows into the oil bath and is mixed with the cold oil, the oil bath temperature T_a is determined from the heat generated in the oil due to viscous shear \dot{Q}_{gen} :

$$T_a = T_{in} + \frac{\dot{Q}_{gen}}{c\rho\dot{V}_{oil}}, \quad \dot{Q}_{gen} = \int_A r\omega\tau_w dA = \int_A r\omega\mu \frac{\partial v_\theta}{\partial z} \Big|_{z=0} dA \quad (13)$$

The heat balance in the groove is not readily determined. Heat is convected to the collar at the trailing edge of the pad where the exit oil is hot. At the leading edge of the pad however the collar may conduct heat to the oil. Heat is convected from the hot exit oil to the colder oil bath. All these phenomena are difficult to quantify and few experimental results are available in the literature. Vohr (1981) states an average experimental value of the groove convection coefficient $H_{gr} = 2960 \text{ W/m}^2\text{K}$ for a bearing of $r_1 = 0.43 \text{ m}$, $r_2 = 0.585 \text{ m}$ and $\theta_{gr} = 6^\circ = 0.105 \text{ rad}$ operating at 150 rpm. Vohr's result is extrapolated using laminar flow theory stating the convection coefficient to be proportional to the square root of velocity divided by the distance between pads. The groove heat transfer \dot{Q}_{gr} can then be determined.

$$\dot{Q}_{gr} = \int_{A_{gr}} H_{gr}(T_c - T_a) dA, \quad H_{gr} = 2960 \sqrt{\frac{0.105 \text{ rad}}{\theta_{gr}} \frac{\omega}{15.71 \text{ rad/s}}} \quad (14)$$

The thermal energy of the oil entering the bearing at the leading edge can be determined:

$$\dot{Q}_{oil,le} = \dot{Q}_{oil,te} - \dot{Q}_{gr}, \quad \dot{Q}_{oil,le} = \int_{A_{le}} (T - T_a) c\rho v_\theta dA, \quad \dot{Q}_{oil,te} = \int_{A_{te}} (T - T_a) c\rho v_\theta dA \quad (15)$$

The expression states energy conservation on a control volume. With the additional assumption that no thermal energy is transported in the radial direction in the groove T_{le} can be found.

2.3. Deflection of bearing pad

The deflection of the pad is modelled using a flat plate formulation following Boresi, Schmidt & Sidebottom (1993):

$$\nabla^2(\nabla^2 w) = \frac{p}{D} - \frac{1}{D} \nabla^2 M_T, \quad \text{where } D = \frac{Et_p^3}{12(1-\nu^2)} \text{ and } M_T = \frac{E}{1-\nu} \int_0^{t_p} (z_p - \frac{t_p}{2}) \alpha T dz_p \quad (16)$$

In the case of 1D heat transfer the temperature varies linearly through the pad and the integral determining M_T can be solved analytically. A pad with a point pivot is restricted by $w = \partial w / \partial r = \partial w / \partial \theta = 0$ at the pivot point. At the free edges of the plate the boundary conditions are that the Kirchhoff shear stresses and bending moments are zero, and at the four corners the twisting moments are zero:

$$\begin{aligned} 0 &= V_\theta(r, 0), & 0 &= M_\theta(r, 0), & 0 &= V_\theta(r, \theta_0), & 0 &= M_\theta(r, \theta_0), & 0 &= V_r(r_1, \theta), & 0 &= M_r(r_1, \theta), \\ 0 &= V_r(r_2, \theta), & 0 &= M_r(r_2, \theta), & 0 &= M_{r\theta}(r_1, 0), & 0 &= M_{r\theta}(r_1, \theta_0), & 0 &= M_{r\theta}(r_2, 0), & 0 &= M_{r\theta}(r_2, \theta_0) \end{aligned} \quad (17)$$

where

$$\begin{aligned} M_r &= -D \left[\frac{\partial^2 w}{\partial r^2} + \nu \left(\frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right) \right] - M_T, & V_r &= -D \left[\frac{\partial}{\partial r} (\nabla^2 w) + (1-\nu) \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial^2 w}{\partial \theta^2} \right) \right] - \frac{\partial M_T}{\partial r}, \\ M_\theta &= -D \left[\frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} + \nu \frac{\partial^2 w}{\partial r^2} \right] - M_T, & V_\theta &= -D \left[\frac{1}{r} \frac{\partial}{\partial \theta} (\nabla^2 w) + (1-\nu) \frac{\partial^2}{\partial r^2} \left(\frac{1}{r} \frac{\partial w}{\partial \theta} \right) \right] - \frac{1}{r} \frac{\partial M_T}{\partial \theta}, \\ M_{r\theta} &= -D(1-\nu) \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial w}{\partial \theta} \right) \end{aligned} \quad (18)$$

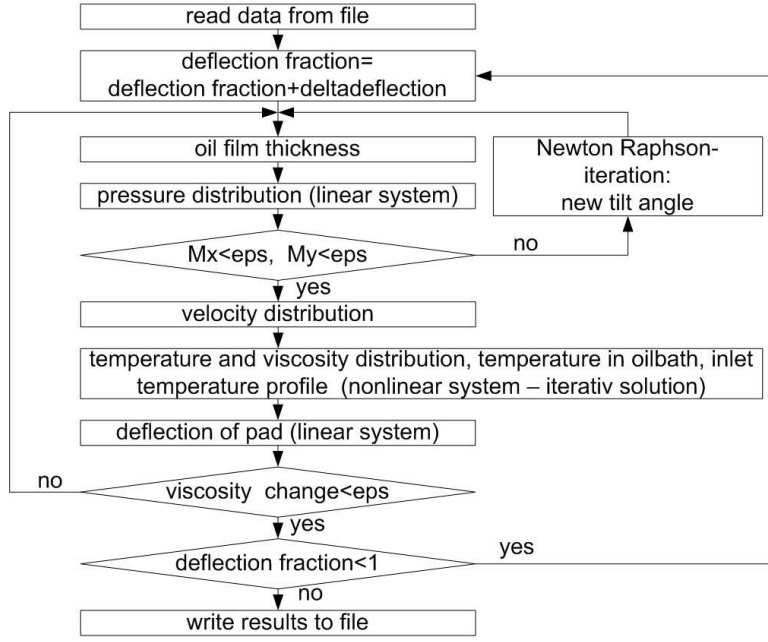


Figure 2: Flow chart for program

2.4. Bearing equilibrium

The film thickness can be calculated as a geometrical relation between the roll and pitch angles and the film thickness at the pivot point. Linearizing to only contain first order terms yields:

$$h = h_0 - (r_p - r \cos(\theta - \theta_p))\alpha_r - r \sin(\theta - \theta_p)\alpha_p + w \quad (19)$$

The pressure distribution can be integrated over the pad area to give the resulting force and the moments around the pivot point. The moments must be zero at the equilibrium position of the pad.

$$F_z = \int_A p dA, \quad 0 = M_x = \int_A p r \sin(\theta - \theta_p) dA, \quad 0 = M_y = \int_A p (r \cos(\theta - \theta_p) - r_p) dA \quad (20)$$

Solving these equations for a given pressure distribution, pad deflection and load F_z yields the distribution of the film thickness over the pad.

2.5. Viscosity relation

Viscosity is calculated as a function of temperature using an expression by Roelands(Hamrock et al. (2004)).

$$\mu = 10^{[k_1 \cdot (1 + \frac{T - 273.15}{135})^{k_2} - 4.200]} \quad (21)$$

3. Solution Method

Coordinates are transformed using the following transformations: $\bar{\theta} = \theta/\theta_0$, $\bar{r} = r/r_1$, $\bar{z} = z/h$, $\bar{z}_p = z_p/t_p$, which for the fluid form a nonorthogonal coordinate system. Second order finite differences are used for solving Eq. (5) and (16). Equations (6) and (7) are solved using a second order finite volume formulation for gradients in the \bar{z} -direction while upwinding is used for the convective terms in the $\bar{\theta}$ and \bar{r} -directions. Integrals are evaluated using the composite trapezoidal rule. The problem is solved using an iterative procedure as depicted in the flowchart in Fig. 2. The Newton-Raphson method is used for adjusting the pitch and roll angles, α_p and α_r until the moments in Eq. (20) around the pivot point are below some threshold. Proper variable transformations(Huebner, 1974) set the Reynolds equation and the equations for moment equilibrium on a form in which they are independent of oil film thickness, h . It is therefore only necessary to iteratively adjust α_p and α_r .

The discretized Reynolds equation and plate equation constitute linear systems of equations which can be solved using direct means. A sparse equation solver from a reputable linear algebra library is chosen. The discretized energy equations for oil and pad and their boundary conditions constitute a nonlinear system of equations. It is chosen to solve for temperature, viscosity and oil bath temperature at the same time using a point iterative Gauss-Seidel SOR solver.

The first sweep of the equations causes the temperature distribution to be overestimated. This causes the pad deflection to be overestimated. As a result the oil film thickness distribution and viscosity distribution in the next sweep may cause the energy equation to diverge. To overcome this problem only a fraction of the calculated deflection is applied to the oil film distribution. An outer loop increases the fraction to one.

4. Results and discussion

Simulation results are compared to experimental data presented by Glavatskikh (2000, 2001) considering a 228 mm outer diameter six pad bearing. The pads are supported by spherical pivots and coated with a babbitt layer less than 1 mm thick. Data for the considered bearing, operating conditions and measurements are given in Tab. 1 and Fig. 3.

Simulations of the bearing are done using a grid of 30×30 control volumes in the radial and circumferential directions - and in the case of 3D models 20 control volumes in the z -direction of the oil film and 20 in the z_p -direction of the bearing pad. A uniform collar temperature of 61°C is used. Results are stated in Tab. 2. A grid convergency study showing the convergence of maximum temperature, minimum oil film thickness and power loss is shown in Tab. 3. Illustrations of the numerical results are given in Fig. 4. In Tab. 2 a measured value of power loss is stated. It represents the total power loss in the bearing and contains losses from the friction between the pads and the collar and losses stemming from the churning of the oil in the oil bath. The calculated power loss does not include churning losses. It can therefore not be directly compared to the measured value.

The 3D-TEHD model gives a power loss 18 % lower than the measured value. The oil film thickness at the trailing edge is 13 % too high. At the leading edge it is 15 % too small. Calculated temperatures at the leading edge are higher than the measured ones. At the trailing edge they are lower than the measured ones. The temperature rise from the leading to the trailing edge is 23 % smaller than the one measured. The temperature rise from bearing inlet to the trailing edge of the pad is 23.8 K which is 7 % less than the measured value.

Possible reasons for the discrepancies are discussed in the following. The treatment of the heat transfer in the groove is crude and gives rise to errors in the leading edge oil temperature. The oil temperature at the leading edge is considered uniform in z . Most certainly the oil will be colder at the pad surface than at the collar surface, so the description used overestimates the temperature at the pad surface. This may explain the errors in the pad temperatures at the leading edge. Convection coefficients at the free sides of the pad may be too high. The convection coefficients used are mean values which are constant in the circumferential direction. The flow at the back and at the inner and outer radii of the pad is driven by the motion of the collar and shaft and is a boundary layer flow which develops from the leading edge of the pad. The convection coefficients are therefore higher at the leading than at the trailing edge. These two points may explain the too small temperature rise from the leading to the trailing edge. The discrepancies in the oil film thickness show that the calculated pitch angle, α_p is too small. This may partly be caused by a calculated pad deflection which is too small. The pad is considered to be a flat plate of uniform thickness while in reality it has cutouts (Glavatskikh, 2000) at the leading and trailing edges giving larger deflections. Also the pad is considered to be initially flat while it may be slightly crowned due to machining inaccuracies. The pressure at the leading edge is set to zero. The velocity boundary layer at the collar surface is however much larger than the oil film thickness at the leading edge. Therefore the oil is stagnated as it approaches the pad leading to an inlet pressure build up before the leading edge. Assuming an inlet pressure larger than zero results in a larger pitch angle reducing the error at the leading edge. Simulations have shown that the combined effects of inlet pressure build up and increased deflection due to the effects mentioned above cannot however fully explain the errors in oil film thickness. The errors in temperature which are discussed above may also influence the pitch angle.

In comparison to the results of the 3D-TEHD model the isothermal model overestimates the minimum oil film thick-

number of pads	6
inner radius	57.15 mm
outer radius	114.3 mm
pivot radius	85.725 mm
pad angle	50.0°
pivot angle	30.0°
pad thickness	28.58 mm
oil type	VG46
viscosity at 40°C	39.0 mPas
viscosity at 100°C	5.4 mPas
density	855.0 kg/m^3
thermal capacity	2090 J/kg/K
thermal conduc.	0.13 W/m/K
axial load	52265 N
shaft speed	1500 rpm
Inlet temperature	40°C
Oil flow	15 L/min

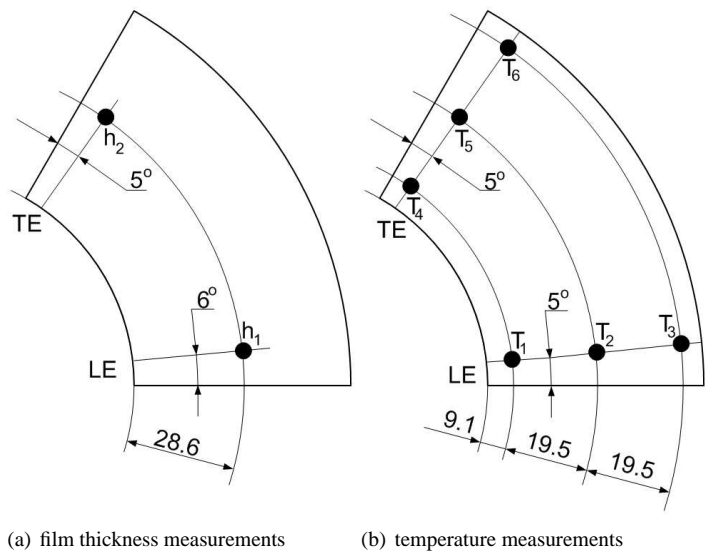


Figure 3: Locations of measurements are shown. Temperatures are measured in the pad 3 mm below the babbitt layer.

Table 1: Data for the bearing considered.

	experimental value	HD-ISO ¹	2D-THD ¹ adiab.	2D-THD	2D-TEHD	3D-THD ¹ adiab.	3D-THD	3D-TEHD
T_c [°C]	61			61.0	61.0		61.0	61.0
T_{max} [°C]			77.2	71.1	69.9	78.6	68.4	68.9
$T_{max,babbitt}$ [°C]				69.6	68.3		67.2	66.5
T_1 [°C]	52			50.6	49.1		56.1	54.0
T_2 [°C]	53			49.6	48.3		56.6	53.9
T_3 [°C]	51			49.7	48.3		54.9	52.1
T_4 [°C]	62			61.8	60.7		62.4	61.7
T_5 [°C]	68			63.3	62.3		65.6	64.9
T_6 [°C]	67			65.4	64.2		65.8	64.8
h_1 [μm]	58	43.9	49.1	47.4	56.0	47.5	35.7	49.2
h_2 [μm]	20	26.9	24.4	24.0	24.8	24.0	21.2	22.5
h_{max} [μm]		53.0	60.2	58.2	71.5	58.3	43.1	63.6
h_{min} [μm]		23.8	19.4	19.8	21.3	19.5	19.1	20.1
Power loss [kW]	3.2			2.61	2.43		2.93	2.63

Grid convergency study	
15 × 15 × (10 + 10)	
h_{min} [μm]	20.38
T_{max} [°C]	70.27
Power loss [kW]	2.617
30 × 30 × (20 + 20)	
h_{min} [μm]	20.12
T_{max} [°C]	68.94
Power loss [kW]	2.629
60 × 60 × (40 + 40)	
h_{min} [μm]	20.04
T_{max} [°C]	67.87
Power loss [kW]	2.635

Table 2: Comparison of experimental and numerical results. $T_{le} = \frac{1}{2}(T_c + T_{in})$ in the models marked with¹. In the other models T_{le} is determined by energy conservation as described in the text. $T_c = 61^\circ\text{C}$ is the condition necessary to close the numerical problem.

Table 3: Grid convergency study conducted on the 3D-TEHD model.

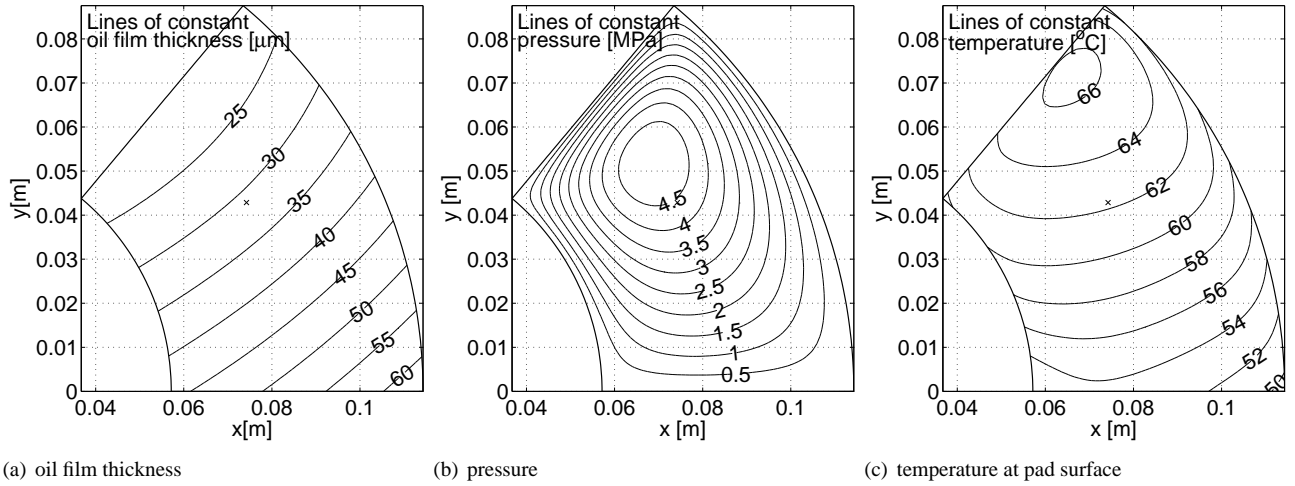


Figure 4: Results from the 3D-TEHD simulation of the bearing considered by Glavatskikh (2000, 2001).

ness by 19 %. Considering the simplicity of the model this result is quite good. The 2D models only slightly overpredict the maximum babbitt temperature and minimum oil film thickness compared to the the 3D models. They seem to overestimate the temperature rise along the pad and estimate a pitch angle which is much higher than the one calculated in the 3D-models. A pitch angle very close to the one which is experimentally determined. The 2D and 3D adiabatic models predict oil film and temperature distributions which only differ slightly. The difference between the models as heat transfer is included is therefore a result of the different accuracy in modelling heat transfer. And the seemingly more accurate oil film thickness calculations in the 2D case must therefore be considered to be an accidental effect of less accurate boundary conditions on the energy equation in the z-direction.

Both 2 and 3 dimensional modelling provide good results for h_{min} although the predicted pitch angles are very different. The reason is that h_{min} is insensitive to changes in boundary conditions which strongly influence the pitch angle. A higher pitch angle demands a higher mean oil film thickness because of equilibrium with the applied load. Raising the pitch angle while keeping the minimum oil film thickness constant provides a higher mean film thickness. This effect in combination with a higher side leakage causes h_{min} to be insensitive to raising α_p for the bearing in question.

In order to determine minimum oil film thickness and maximum babbitt temperature a 2D model seems to be sufficient for the considered bearing.

The bearing is small compared to bearings which are used in for instance hydro power plants. In larger bearings the

effects of thermal deflections of the pads are much larger. In the considered bearing deflection of the pad causes the oil film at the inner and outer corners at the trailing edge to be 2 μm or 10 % larger than the minimum located approximately at the mean radius. In larger bearings the deflection may cause the oil film to be 100 % larger at the two corners than at the mean radius. 3D calculations of temperature may then be necessary for an accurate prediction of pad deflection and oil film thickness.

A 2D integrated description is meaningful only if streamlines are nearly parallel. If the pitch angle is large reverse flow may occur at the leading edge. In cases where reverse flow is significant a 2D model may not accurately determine the temperature distribution.

For the bearing in question the 3D model does not improve the results obtained by a 2D model. Considerable work must be done in improving the boundary conditions in order to profit from the increased complexity of a 3D description.

5. Conclusion

Numerical models have been developed in order to determine temperature distribution and oil film thickness in a tilting-pad thrust bearing. The Reynolds equation has been solved isothermally and subject to 2D or 3D viscosity distributions. The deflection of the bearing pad due to pressure and thermally induced moments has been modelled using a flat plate approximation. The thermally induced moments are calculated from the temperature distribution in the pad which has been solved using 1D and 3D models. The inlet temperature distribution has been determined through an energy balance for the groove between pads. The collar temperature must be set as a boundary condition.

Simulations have been carried out to compare with experimental data available in the literature. These are data for steady-state operation of a bearing of 228 mm outer diameter.

Simulations show that an isothermal model predicts the minimum oil film thickness within 30 % of the actual value. 2D and 3D models both determine the minimum oil film thickness to be within 15 % and the temperature rise from inlet to trailing edge to be within 7 % of the measured values.

The 3D models do not improve the accuracy. More detailed treatment of the boundary conditions on pressure and temperature is necessary in order to profit from the increased complexity of a 3D model.

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