FREQUENCY DOMAIN AERODYNAMIC RESPONSES FOR AEROELASTIC ANALYSES IN THE UNSTEADY TRANSONIC REGIME

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Abstract. *The paper presents results which constitute part of an ongoing research work aimed at more efficient aeroelastic analyses in the transonic regime based on numerical aerodynamic data. The methodology here proposed is based on the construction of the aerodynamic operators for aeroelastic analyses from reduced-order models obtained from a relatively small number of nonlinear, unsteady and expensive CFD runs. The CFD tool employed in the present work is based on the Euler formulation. The governing equations are integrated by a cell-centered, finite-volume, centered space discretization and a five-stage, hybrid, explicit, Runge-Kutta time-marching scheme. This CFD tool solves flows around two-dimensional lifting surfaces performing harmonic and impulsive movements in both plunging and pitching modes. The computational domain is discretized using unstructured grids and the movement is modeled with a dynamic mesh algorithm. The results presented in the paper concern particularly the NACA 0012 airfoil subjected to impulsive motions in both modes. CFD tools are only capable to offer time domain data. Neverthless, it is well known that impulsive excitations contain information of the complete frequency spectrum. Therefore, with a Fourier analysis of such CFD data, the authors obtained frequency domain results. It means that with this procedure, one can replace many CFD runs, one for each frequency of interest, by a single run of the impulsive motion.*

Keywords: *Frequency domain, aeroelasticity, unsteady aerodynamics, Fourier analysis*

1. Introduction

Aeroelasticity can be defined as the science which studies the mutual interaction between aerodynamic and dynamic forces. However, a broader definition is possible when inertial forces are also focused. The analysis of dynamic characteristics of either complex or simple structures is quite developed nowadays as far as numerical and experimental methods are concerned. Hence, it is correct to state that reliability in aeroelastic calculations is strongly dependent on the correct evaluation of the aerodynamic operator.

Traditionally, the methods developed for determining the aerodynamic operator for subsonic and supersonic regimes are based on linearized formulations which do not present the same satisfactory results in the transonic range. According to Tijdeman (1977), this occurs due to the nonlinearity of transonic flows characterizing a significant change of the flow behavior, even when a profile is subjected to small pertubations. Ashley (1980) reported the use of semi-empirical corrections to the linearized theory results as a means of improving flutter predicitons. Nevertheless, Ashley (1980) himself believed that really satisfactory aeroelastic quantitative predicitons of the transonic regime should be possible only when accurate, three-dimensional, unsteady Computational Fluid Dynamics (CFD) codes were developed. Hence, the methodology here presented, which is based on the ideas of Rausch, Batina and Yang (1990) and Oliveira (1993), intends to obtain the aerodynamic operator for two-dimensional lifting surfaces employing modern CFD techniques.

CFD is a subject that has played an extremely important role in recent studies of aerodynamics. The possibility of numerically treating a broad range of phenomena which occur in flows over bodies of practically any geometry has inumerous advantages over experimental determinations, such as greater flexibility together with time and financial resource savings. However, obtaining more reliable numerical results for a growing number of situations has been one of the major recent challenges in many science fields. Fletcher (1988a) and Hirsch (1994) show that, particulary in aerodynamics, the general phenomena are governed by the Navier-Stokes equations, which constitute a system of coupled nonlinear partial differential equations that has no general analytical solution and that is of difficult algebraic manipulation. Hirsch (1994) coments, among other issues concerning CFD techniques, on how to simplify the mathematical models conveniently in order to ease the numerical treatment of each case. A survey on this subject is also presented by Azevedo (1990). Space and time discretization schemes, as well as convergence acceleration techniques, boundary condition settings and other numerical integration tools are available and largely used in order to solve such models.

After selection of the theoretical model, it is indispensable to define the physical domain where the flows take place, determining the boundary conditions. This flow solution approach demands the discrete representation of the physical domain to make the problems numerically coherent defining a computational mesh of points or regions where the calculations are performed. The mesh generation, as it is vastly documented in the literature, e.g., Fletcher (1988b), and verified by the CTA/IAE work group's own experience, is extremely important and decisive in the accuracy and convergence of the solution. The mesh type is also an essential factor on the CFD tool behavior. Structured meshes have the advantages of being well-behaved, the existence of an intrinsic correspondence between adjacent nodes and a very good control over grid refinement through stretching functions. However, this sort of mesh does not adpat readily to complicated geometries, requiring the adoption of more sophisticated multiblock mesh techniques. On the other hand, unstructures meshes are extremely flexible when it comes to geometric forms and they allow the use of interesting techniques such as solution adaptative refinement.

As stated by Marques (2004), together with the evolution of the work and projects performed by CTA/IAE, the demand for aerodynamic parameters has swelled, mainly those concerning the vehicles developed in this center. Nevertheless, the application of CFD tools in these parameter analyses has always been limited by the need of adequated code development and the lack of computational resources compatible with the work to be performed. Therefore, a progressive approach has been adopted in the development of CFD tools in CTA/IAE and in ITA, as presented by Azevedo (1990), Oliveira (1993), Fico and Azevedo (1994), Azevedo, Fico and Ortega (1995), Azevedo, Strauss and Ferrari (1997), Bigarelli, Mello and Azevedo (1999), Simões and Azevedo (1999) and Bigarelli and Azevedo (2002).

The present work is based on the finite-volume formulation, where a CFD tool is applied with unstructured two-dimensional meshes around lifting surfaces to acquire unsteady responses to harmonic and exponentially-shaped pulse motions. These time domain responses supply the generalized aerodynamic forces necessary as input to the aeroelastic model. The methodology here presented intends to obtain frequency domain responses from the solutions to impulsive motion and, with that information, determine the aeroelastic stability margin with a single expensive CFD run for each structural mode. However, the authors have not reached the final stability margin results yet and only the frequency domain responses are presented. Nevertheless, the authors are confident that the succesfull validation of the presented methodology can be a contribution in the area of reduced-order models for aeroelastic applications.

2. Aerodynamic theoretical formulation

The CFD tool applied in this work is based on the 2-D Euler equations. Due to the use of unstructured meshes and the adoption of the finite volume approach, these equations are written in the Cartesian form. Furthermore, as usual in CFD applications, flux vectors are employed and the equations are nondimensionalized. Hence, they can be written as

$$\frac{\partial}{\partial t}\int\int_V Q dxdy + \int_S (Edy + Fdx) = 0. \quad (1)$$

In Eq. (1), V represents the volume of the control volume or, more precisely, its area in the two-dimensional case. S is its surface, or its side edges in 2-D. Q is the vector of conserved properties, given by

$$Q = \{\rho \;\; \rho u \;\; \rho v \;\; e\}^T . \quad (2)$$

E and F are the inviscid flux vectors in the x and y directions, respectively, defined as

$$E = \left\{ \begin{array}{c} \rho U \\ \rho u U + p \\ \rho v U \\ (e+p)U + x_t p \end{array} \right\}, \quad E = \left\{ \begin{array}{c} \rho V \\ \rho u V \\ \rho v V + p \\ (e+p)V + y_t p \end{array} \right\}. \quad (3)$$

The nomenclature adopted here is the usual in CFD: ρ is the density, u and v are the Cartesian velocity components and e is the total energy per unity of volume. The pressure, p , is given by the perfect gas equation, written as

$$p = (\gamma - 1)\left[e - \frac{1}{2}\rho\left(u^2 + v^2\right)\right]. \quad (4)$$

Once again, as usual, γ represents the ratio of specific heats. The contravariant velocity components, U and V , are determined by

$$U = u - x_t \quad \text{and} \quad V = v - y_t, \quad (5)$$

where x_t and y_t are the Cartesian components of the mesh velocity in the unsteady case. For further details on the theoretical formulation, such as boundary and initial conditions, the interested reader should refer to the work of Marques (2004).

3. Numerical formulation

The algorithm here presented is based on a cell-centered, finite-volume scheme in which the stored information is actually the average value of the conserved properties throughout the entire control volume. Spatial discretization considers a centered scheme, and apropriate artificial dissipation terms (Mavriplis, 1990) are explicitly added in order to control nonlinear instabilities. The numerical solution is advanced in time using a second-order accurate, 5-stage, explicit, hybrid scheme which evolved from the consideration of Runge-Kutta time stepping schemes presented by Jameson, Schmidt and Turkel (1981) and Mavriplis (1990).

Moreover, steady-state solutions for the mean flight condition of interest must be obtained before the unsteady calculation can be started. Therefore, it is also important to guarantee an acceptable efficiency for the code in steady-state mode. In the present work, both local time stepping and implicit residual smoothing are employed to accelerate convergence to steady-state. More details on convergence acceleration techniques are found in the work of Jameson and Baker (1983), Jameson and Mavriplis (1986), Jameson and Baker (1987), Oliveira (1993) and Marques (2004). Further details on the numerical formulation are presented by Oliveira (1993) and Marques (2004).

4. Mesh generation and movement

The meshes used in the present work were generated with the commercial grid generator ICEM CFD©, a very powerfull tool capable of creating sophisticated meshes with very good refinement and grid quality control. Figure 1 shows samples of the meshes around a NACA 0012 profile and a flat plate which were employed to obtain the results here discussed.

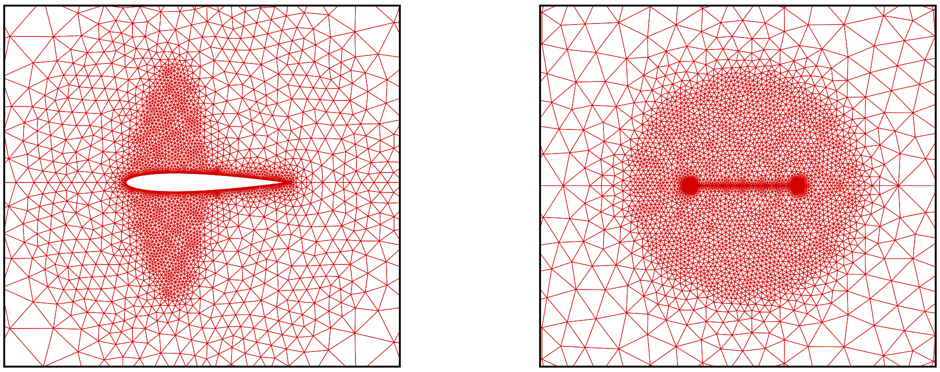

Figure 1. Mesh around (left) NACA0012 profile with 292 wall points and (right) flat plate with 236 wall points.

Unsteady calculations involve body motion and, therefore, the computational mesh should be somehow adjusted to take this motion into account. The approach adopted here is to keep the far field boundary fixed and to move the interior grid points in order to accomodate the prescribed body motion. This is done following the ideas presented by Batina (1989) and Rausch, Batina and Yang (1990), which assume that each side of the triangle is modeled as a spring with stiffness constant inversely proportional to the length of the side. Hence, once points on the body surface have been moved and assuming that the far field boundary is fixed, a set of static equilibrium equations can be solved for the position of the interior nodes (Oliveira, 1993). Control volume areas for the new grid can, then, be computed. The mesh velocity components can also be evaluated considering the new and old point positions and the time step. For further details, the interested reader should refer to the work of Marques (2004).

5. Aeroelastic analysis methodology

As no aeroelastic results have yet been obtained and due to the lack of space, the aeroelastic formulation of the case of interest has been omitted. However the interested reader can find it in the work of Oliveira (1993) and Bisplinghoff, Ashley and Halfman (1955). The unsteady movements related to the aeroelastic phenomena, mainly flutter, can be represented by a series of harmonic motions. Therefore, a large computational cost reduction comes from the use of the indicial method. According to this approach, the aerodynamic response to a harmonic excitation of any frequency can be obtained from Duhamel's integral of the response of the flow to an indicial motion. Following this same idea, and noticing that the transient flow due to an impulsive excitation takes a shorter period of time to die out than those that results from an indicial motion, Oliveira (1993) proposed a similar methodology where the aerodynamic response are evaluated in the frequency domain from the responses to an impulsive excitation in the time domain.

Therefore, the aerodynamic calculations for a determined flight condition are reduced to a single computational run for each structural mode. Moreover, the only hypothesis adopted is that the aerodynamic generalized forces are linear with regard to the displacement modes and amplitudes, which guarantees that this methodology captures the flow nonlinearities and dynamics, except for those related to viscous effects which are not included in an Euler formulation.

However, the theoretical impulsive motion is a singularity and the indicial one leads to the appearance of infinite velocities, which makes both numerically untreatable. Hence, other smoother excitation functions are employed, as shown by Davies and Salmond (1980) and Mohr, Batina and Yang (1989). The motion suggested by Bakhle et al. (1991) is defined as

$$f_p\left(\bar{t}\right) = \begin{cases} 4\left(\frac{\bar{t}}{\bar{t}_{max}}\right) exp\left(2 - \frac{1}{1-\frac{\bar{t}}{\bar{t}_{max}}}\right), & 0 \le \bar{t} \le \bar{t}_{max}, \\ 0, & \bar{t} \ge \bar{t}_{max}, \end{cases} \tag{6}$$

where the overline indicates the dimensionless time and $\bar{t}_{max}$ is the excitation duration. As can be seen in Fig. 2, the function defined in Eq. (6) guarantees a smooth motion.

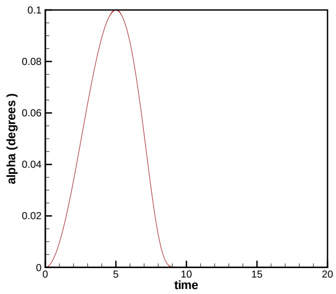

Figure 2. Impulsive excitation in angle of attack.

The methodology consists, then, in obtaining a transfer function in the frequency domain applicable to any desirable input. This transfer function is the frequency domain response to the impulsive excitation. Therefore, this is accomplished using the following steps:

- Obtaining the steady aerodynamic solution for a given Mach number and angle of attack;
- Performing unsteady aerodynamic response evaluations departing from the steady solution given in the previous item. This stage leads to time responses in terms of aerodynamic coefficients as a result to an exponentially-shaped pulse excitation of each of the modes;
- Obtaining the Fourier transform of the time responses applying a Fast Fourier Transform (FFT) algorithm. This is done in the present work employing the FFT capability available in the commercial program Matlab©;
- Aproximating the obtained data with an interpolating polynomial;
- Formulation of the corresponding eigenvalue problem, valid for a determined range of dimensionless velocities, and, finally, flutter speed prediction through a root locus analysis.

As shown by Oliveira (1993), the corresponding frequency domain points resulting from the FFT procedure are given by

$$f = \frac{1}{\Delta t}\frac{i}{N} = \frac{a_\infty}{\Delta \bar{t} c}\frac{i}{N}; \quad i = 0, 1, 2, ..., N_{max}, \tag{7}$$

$$Nmax = \begin{cases} N/2, & \text{if } N \text{ is even} \\ (N-1)/2, & \text{otherwise,} \end{cases} \tag{8}$$

where c is the chord lentgh and a_∞ the freestream speed of sound. Equation (7), rewritten in terms of the reduced frequency based on the half-chord length (b), stands as

$$k = \frac{\omega b}{U_\infty} = \frac{2\pi f b}{U_\infty}, \tag{9}$$

$$k = \frac{\pi}{M_\infty \Delta \bar{t}}\frac{i}{N}; \quad i = 0, 1, 2, ..., N_{max}, \tag{10}$$

where M_∞ is the Mach number referring to the undisturbed conditions.

As the input is not exactly an impulsive excitation, the real impulse response is evaluated using a well-known property of the convolution integral, as given by Brigham (1988),

$$g(t) = f_p(t) * i(t) \rightarrow G(f) = F_p(f)I(f), \quad (11)$$

$$I(f) = \frac{G(f)}{F_p(f)}, \quad (12)$$

where $i(t)$ represents the time response to a impulsive movement and $g(t)$ is the response to the exponentially-shaped excitation given in Eq. (6). The functions in capital letters are Fourier transforms of the corresponding functions in lower case letters. Therefore, after obtaining the FFT of the time response, it has to be divided by the FFT of the input function. Although the input is not the exact impulsive excitation, it is capable of exciting the reduced frequencies of interest in aeroelastic studies.

The frequency domain responses obtained by these steps consist in a set of numerical values, which are not convenient for the solution of the aeroelastic equation given by Oliveira (1993). Therefore, it is convenient to approximate these data using interpolating polynomials, as previously stated. As proposed by Abel (1979) and Oliveira (1993), this polynomial, already in the Laplace domain, is given by

$$[A(s)] = [A_0] + [A_1]\left(\frac{b}{U_\infty}\right)s + [A_2]\left(\frac{b}{U_\infty}\right)^2 s^2 + \sum_{m=3}^{n} \frac{[A_m]}{s + \frac{U_\infty}{b}\beta_{m-2}}, \quad (13)$$

where β_m introduce the aerodynamic lags and are arbitrarily selected from the range of reduced frequencies of interest. Moreover, $[A_m]$ are the approximating coefficient matrices given by a least squares optimization method, where $s = ik$. The method to be applied is to be chosen after criterious evaluation among those available in the literature such as Abel (1979), Eversman and Tewari (1991) and Oliveira (1993). The aerodynamic influence coeficient matrix, $[A]$, is given by

$$[A] = \begin{bmatrix} c_{l_h}/2 & -c_{l_\alpha} \\ c_{m_h} & 2c_{m_\alpha} \end{bmatrix}. \quad (14)$$

6. Results and discussion

Before attempting applications of the proposed methodology, some validation simulations are performed with the CFD tool. This has been done throughout the entire development of this code as can be seen in the work of Azevedo (1992), Oliveira (1993), Simões and Azevedo (1999) and Marques (2004). Once the CFD tool is tested and proves to be a reliable one, the next step is to proceed in obtaining the unsteady responses of interest. The approach selected is to reproduce the results presented by Rausch, Batina and Yang (1990). Therefore, initially, the response of a flat plate to the excitation presented in Fig. 2 at $M_\infty = 0.5$ is determined. This response in terms of the generalized aerodynamic force coefficients is obtained for pitching about the quarter-chord point and for plunging. Due to the lack of space, these time domain responses are omitted and only frequency domain responses are presented here. The corresponding Fourier transforms, together with the frequency domain results given by Rausch, Batina and Yang (1990), are shown in Figs. 3 and 4. These results show a very good agreement between calculations performed by the present authors and the literature data.

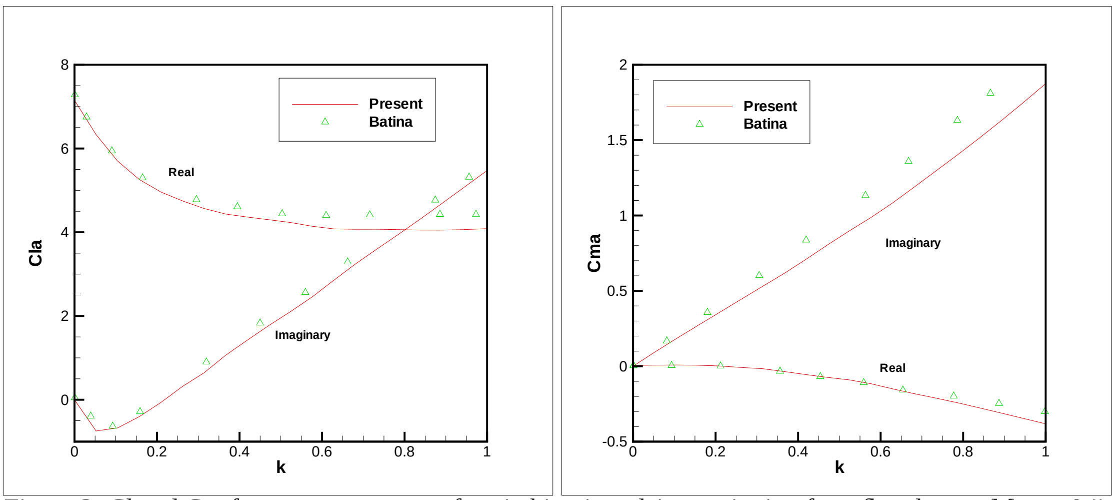

Figure 3. Cl and Cm frequency responses for pitching impulsive excitation for a flat plate at $M_\infty = 0.5$.

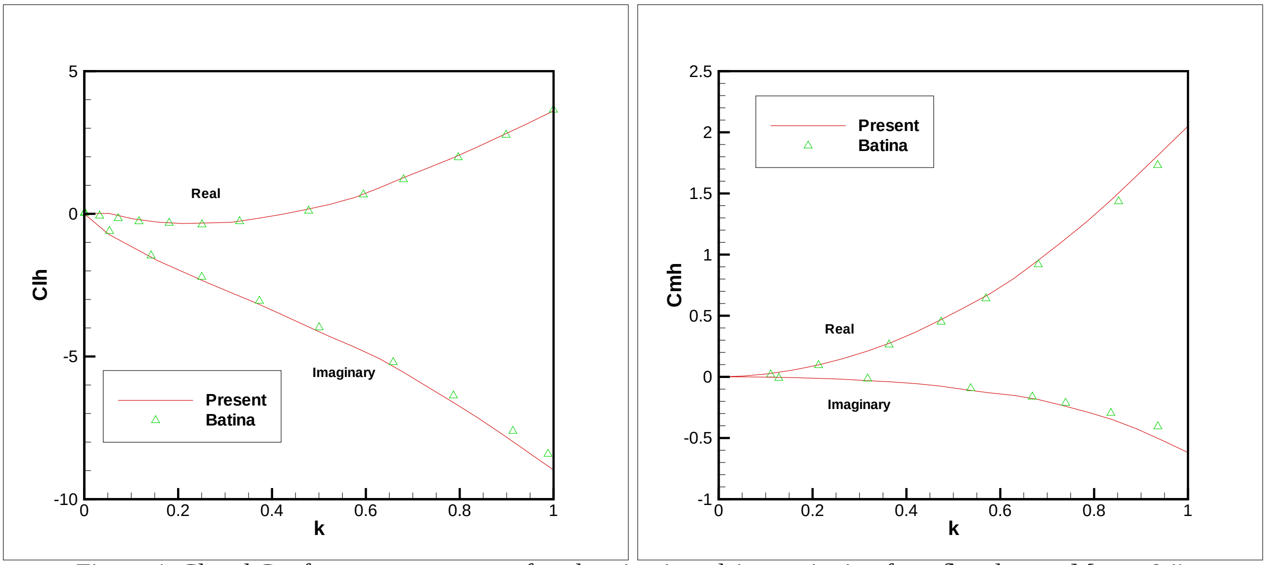

Figure 4. Cl and Cm frequency responses for plunging impulsive excitation for a flat plate at $M_\infty = 0.5$.

However, these results are purely subsonic and this is not the real subject of interest for the present study. Therefore, the authors considered the same sort of analysis for a NACA 0012 airfoil at $M_\infty = 0.8$, for which case results are also found in Rausch, Batina and Yang (1990). The Fourier transforms of the time domain results are shown in Figs. 5 and 6 together with the results presented by Rausch, Batina and Yang (1990). In this case, the results obtained in the present study do not perfectly match the ones found in the literature, but the overall behavior is very close. These differences are currently being investigated. Other results obtained so far indicate that they are not caused by the excitation shape nor other numerical characteristics of the CFD solution, such as the time step. Grid refinement may be one possible explanation, which is currently being numerically evaluated. Despite the differences observed in the transonic results, which the authors believe can be overcome, this work shows that the first three items which compose the methodology for aeroelastic analysis have been successfully implemented.

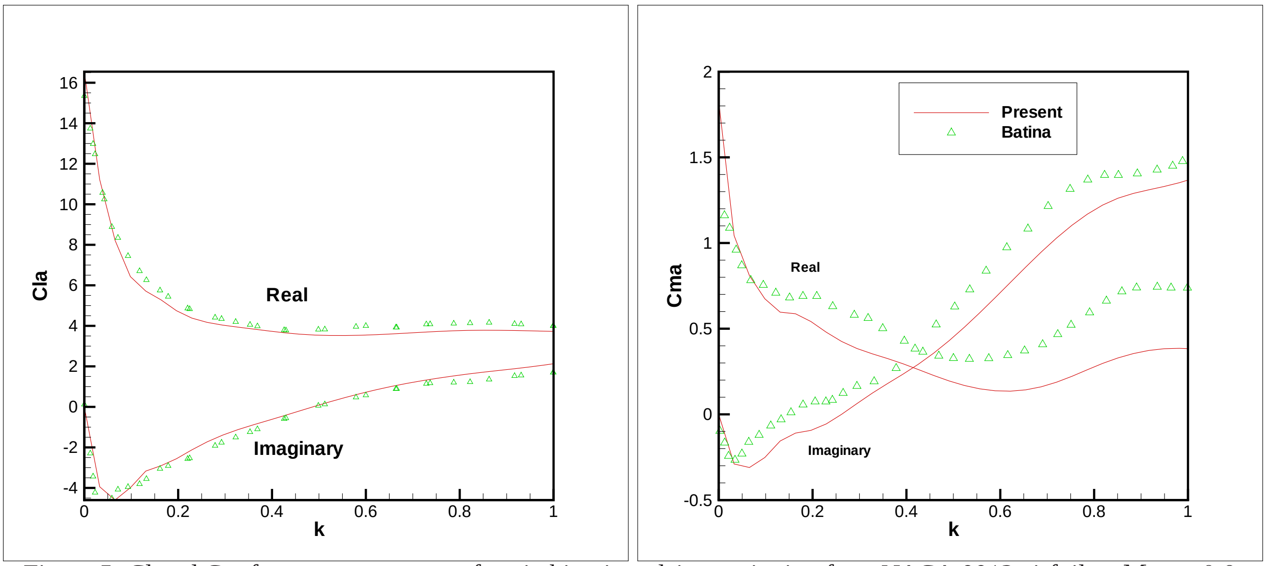

Figure 5. Cl and Cm frequency responses for pitching impulsive excitation for a NACA 0012 airfoil at $M_\infty = 0.8$.

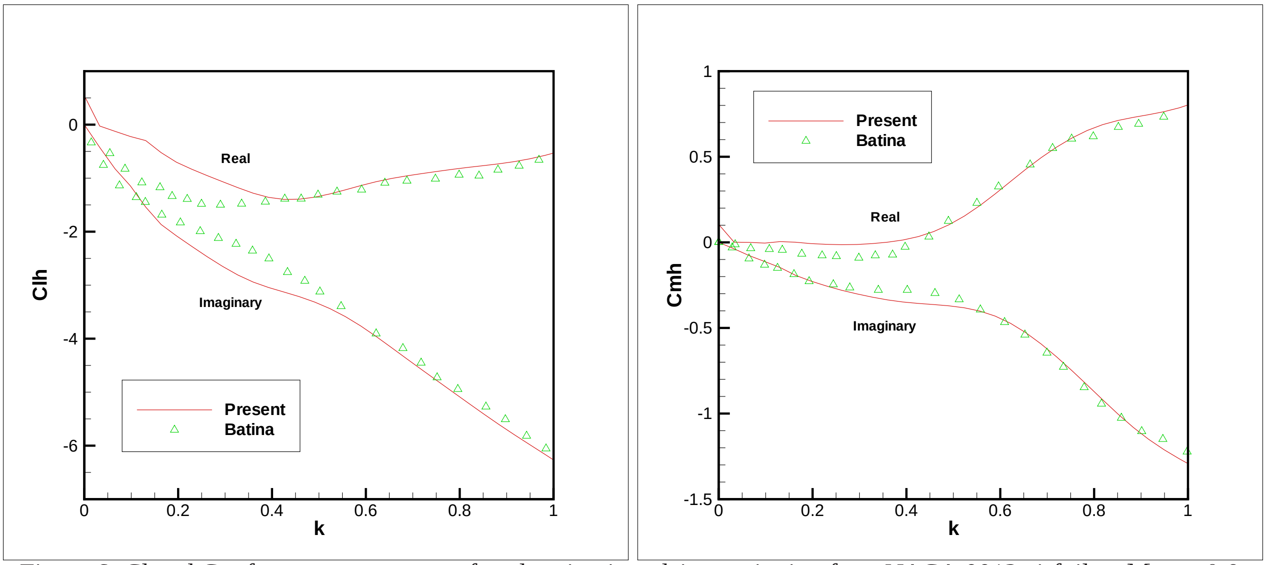

Figure 6. Cl and Cm frequency responses for plunging impulsive excitation for a NACA 0012 airfoil at $M_\infty = 0.8$.

7. Concluding remarks

As one can see in the present results, this is an ongoing work since no aeroelastic problem has been really studied. Nevertheless, the preliminary results obtained so far are excellent and encourage the authors to move forward to successfully achieve the complete verification of the methodology proposed. The small differences found in some frequency domain results are being studied and the authors believe they can soon be overcome. All indications obtained so far tend to point to grid refinement as the source of differences in the transonic results. It is important to notice that the CFD tool developed by the CTA/IAE group has been widely tested and has proved to be a reliable source of the aerodynamic data for the several aerodynamic applications considered in the Institute.

Once validated, this methodology will provide the required capabilities to study aeroelastic stability problems using modern CFD tools. During this development, the authors also desire to address many theoretical questions concerning the use of indicial or impulsive excitation on discrete models which can be very important to many areas, in particular for the development of reduced-order models for aeroservoelastic control laws. Therefore, this work represents a fundamental research for the evolution of numerical aeroelastic studies at CTA/IAE.

8. Ackowledgements

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9. References

Abel, I., 1979, "An Analytical Technique for Predicting the Characteristics of a FlexibleWing Equipped with an Active Flutter-Suppression System and Comparison with Wind-Tunnel Data", NASA Technical Paper 1367, Hampton, VA.

Ashley, H., 1980, "Role of Shocks in the "Sub-Transonic" Flutter Phenomenon", Journal of Aircraft, Vol. 17, No. 3, pp. 187-197.

Azevedo, J.L.F., 1990, "Euler Solutions of Transonic Nozzle Flows", Proceedings of the 3rd National Meeting on Thermal Sciences - III ENCIT, Itapema, SC, Brazil, pp. 243-248.

Azevedo, J.L.F., 1992, "On The Development of Unstructured Grid Finite Volume Solvers for High Speed Flows", Report NT-075-ASE-N/92, Instituto de Aeronáutica e Espaço, São José dos Campos, SP, Brazil.

Azevedo, J.L.F., Fico, N.G.C.R.J. and Ortega, M.A., 1995, "Two-Dimensional and Axisymmetric Nozzle Flow Computation Using the Euler Equations", Journal of the Brazilian Society of Mechanical Sciences and Engineering, Vol. 17, No. 2, pp. 147-170.

Azevedo, J.L.F., Strauss, D. and Ferrari, M.A.S., 1997, "Viscous Multiblock Simulations of Axisymmetric Launch Vehicle Flows", Proceedings of the 15th AIAA Applied Aerodynamics Conference, AIAA Paper No. 97-2300, Atlanta, GA.

Bakhle, M.A., Mahjan, A.J., Keith, T.G. and Stefko, G.L., 1991, "Cascade Flutter Analysis with Transient Response Aerodynamics", Computers and Structures, Vol. 41, No. 5, 1991, pp. 1073-1085.

Batina, J.T., 1989, "Unsteady Euler Airfoil Solutions Using Unstructured Dynamic Meshes", 27th Aerospace Sciences Meeting, AIAA Paper No. 89-0115, Reno, NV.

Bigarelli, E.D.V. and Azevedo, J.L.F., 2002, "On Turbulence Models for 3-D Aerospace Applications", Proceedings of the 9th Brazilian Congress of Thermal Engineering and Sciences, Paper CIT02-0341, Caxambu, MG, Brazil.

Bigarelli, E.D.V., Mello, O.A.F. and Azevedo, J.L.F., 1999, "Three Dimensional Flow Simulations for Typical Launch Vehicles at Angle of Attack", Proceedings of the 15th Brasilian Congress on Mechanical Engineering - COBEM 1999, Águas de Lindóia, SP, Brazil.

Bisplinghoff, H.L., Ashley, H.L. and Halfman, R.L., 1955, *Aeroelasticity*, Adison-Wesley, Cambridge, MA.

Brigham, E.O., 1988, *The Fast Fourier Transform and its Applications*, Prentice Hall, Englewood Cliffs, NJ.

Davies, D. and Salmond, D.J., 1980, "Indicial Approach to Harmonic Perturbations in Transonic Flow", AIAA Journal, Vol. 18, No. 8, pp. 1012-1014.

Eversman, W. and Tewari, A., 1991, "Consistent Rational-Function Approximation for Unsteady Aerodynamics", Journal of Aircraft, Vol. 28, No. 9, pp. 545-552.

Fico, N.G.C.R. and Azevedo, J.L.F., 1994, "Numerical Investigation of Supersonic Flow in a Slotted Wind Tunnel", Proceedings of the 15th Iberian Latin-American Congress on Computational Methods for Engineering - XV CILAMCE, Belo Horizonte, MG, Brazil.

Fletcher, C.A.J., 1988a, *Computational Techniques for Fluid Dinamics - Fundamental and General Techinques*, Vol. I, Springer Verlag, Berlin.

Fletcher, C.A.J., 1988b, *Computational Techniques for Fluid Dinamics - Specific Techinques for Different Flow Categories*, Vol. II, Springer Verlag, Berlin.

Hirsch, C., 1994, *Numerical Computation of Internal and External Flows - Fundamentals of Numerical Discretization*, Vol. I, Wiley, Chichester, NY.

Jameson, A. and Baker, T.J., 1983, "Solution of the Euler Equation for Complex Configurations", AIAA Paper 83-1929.

Jameson, A. and Baker, T.J., 1987, "Improvements to the Aircraft Euler Method", AIAA 25th Aerospace Sciences Meeting, AIAA Paper 87-0452, Reno, NV.

Jameson, A. and Mavriplis, D., 1986, "Finite Volume Solution of the Two-Dimensional Euler Equations on a Regular Triangular Mesh,. AIAA Journal, Vol. 24, No. 4, pp. 611-618.

Jameson, A., Schmidt, W. and Turkel, E., 1981, "Numerical Solution of the Euler Equations by Finite Volume Methods Using Runge-Kutta Time-Stepping Schemes", AIAA 14th Fluid and Plasma Dynamics Conference, AIAA Paper 87-0452, Palo Alto, CA.

Marques, A.N., 2004, "Simulation of Unsteady Aerospace Flows Using Unstructured Meshes", Graduation Project, Instituto Tecnológico de Aeronáutica, São José dos Campos, SP, Brazil (in Portuguese).

Mavriplis, D.J., 1990, "Accurate Multigrid Solution of the Euler Equations on Unstructured and Adaptative Meshes", AIAA Journal, Vol. 28, No. 2, pp. 213-221.

Mohr, R.W., Batina, J.T. and Yang, H.T.Y., 1989, "Mach Number Effects on Transonic Aeroelastic Forces and Flutter Characteristics", Journal of Aircraft, Vol. 26, No. 11, pp. 1038-1046.

Oliveira, L.C., 1993, "A State-Space Aeroelastic Analysis Methodology Using Computational Aerodynamics Techniques", Master Thesis, Instituto Tecnológico de Aeronáutica, São José dos Campos, SP, Brazil (in Portuguese).

Rausch, R.D., Batina, J.T. and Yang, H.T., 1990, "Euler Flutter Analysis of Airfoil Using Unstructured Dynamic Meshes", Journal of Aircraft, Vol. 27, No. 5, pp. 436-443.

Simões, C.F.C. and Azevedo, J.L.F., 1999, "The Influence of Numerical Parameters on Unsteady Airfoil Inviscid Flow Simulations Using Unstructured Dynamic Meshes", Proceedings of the 15th Brazilian Congress of Mechanical Engineering - COBEM 1999, Águas de Lindóia, SP, Brazil.

Tijdeman, H., 1977, "Investigation of tbe Transonic Flow Around Oscillating Airfoils", NLR-TR-77090U, National Aerospace Laboratory, The Netherlands.

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