UNSTEADY COMPRESSIBLE AERODYNAMICS CAUSED BY VERTICAL GUSTS AND VORTICES

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Abstract. *Aerodynamic effects are studied (lift and pressure coefficients), on a thin profile penetrating into a sharp edge gust (for several gust velocity ratios), and also the interaction of the profile with a convected (from convection phenomenon) vortex, passing over the profile – a phenomenon known in literature as AVI (Airfoil Vortex Interaction). Such predictions are mainly relevant in helicopters project, where the calculation of these loads is the key to the calculation of aeroelastic response and rotor aeroacoustics. The present work uses a numerical approach based on vortex singularity. The results now avaiable in literature are based on approximated exponential equations, or computed via Computational Fluid Dynamics (CFD). Thus, the method intends a more accurate computation compared to those of approximated equations, and quite faster than those done via CFD. Results are obtained for subsonic and supersonic flow in compressible environment.*

Keywords: *Compressible, Gust, BVI, AVI, Vortex.*

1. Introduction

Along the past decade, the Generalized Vortex Lattice Method was developed for the unsteady motion, initially for subsonic (Soviero, 1993) and later for Supersonic (Soviero and Ribeiro, 1995) and transonic flows (Soviero and Pinto, 2001). In all previous works, the profile motion (heaving or pitching) was restricted to the harmonic motion and, in this way, the calculation was done on the frequency domain.

If the objetive is to obtain the aerodynamic loads from arbitrary motions, the only practical way to get them is, according to Bisplinghoff *et al*. (1955), using a superposition method (Fourier`s integral) of the results obtained for harmonic motions. However, such a methodology is not adequate for sudden movements, which can happen during maneuvers of airplanes of high performance, sharp edge gusts or fast deflections of command surfaces, such as the ailerons. In these cases the number of terms in the series needed to describe forces and moment coefficients, can become prohibitively large due to slow convergence behavior of the solution describing the studied motion.

Therefore, the study and development of a numerical method that allows the obtention of aerodynamic forces and moments for a profile in arbitrary motion, consists on attractive tasks. A methodology based on vortex singularity (Hernandes and Soviero, 2004), was developed to obtain the aerodynamic forces for any motion. In this work, Hernandes and Soviero have calculated the so called indicial response of the profile, those being the unit step function and the sharp edge gust. The present work uses the same methodology, adapted to the employed boundary conditions.

In this work, the motions of the sharp edge gust and the interaction of the profile with a vortex convecting over it, are studied. In the sharp edge gust the profile is submitted to a uniform vertical gust, that has its gust propagation velocity varied. The interaction study of the profile with a vortex convecting (from convection phenomenon) over it, also with variations in vortex propagation velocity, is known in literature as AVI (airfoil-vortex interaction) and is a particular case (two-dimensional) of the already known BVI (blade-vortex interaction).

For incompressible flow, the indicial lift of a profile submitted to a sudden change in the angle of attack (step function) was obtained by Wagner (1925). The corresponding solution for the sharp edge gust in incompressible flow, was obtained by Küssner (1936). Bisplinghoff *et al.* (1955), presents a synthesis of these results. In the study of aerodynamic loads in helicopters rotors, in situations where the hypothesis of incompressibility is employable, the Wagner and Küssner functions are used, together with the Duhammel Integral (Bisplinghoff *et al*., 1955), where it is possible through indicial functions (Wagner or Küssner), to obtain solutions for arbitrary motions in incompressible flow. The study of gusts with variable propagation speed, where intermediate solutions are obtained among the results of Wagner and Küssner, was developed by Miles (1956) and Drischler-Diederich (1957), both studies make use of the Wagner function approaches to obtain results. A numerical method to solve any motion incompressible flow, was developed by Soviero and Lavagna (1997).

The compressible flow may be divided in subsonic and supersonic. For supersonic flow, there are avaiable in literature analytical solutions for the problem of sudden change in the angle of atack (Lomax *et al*., 1952) and for the sharp edge gust problem (Heaslet and Lomax, 1947). Bisplinghoff *et al*. (1955), present a summary of these results.

Numerical results for both motions were obtained by Hernandes and Soviero (2004). In supersonic flow, studies of gusts with variable propagation velocity were not observed, nor the study of AVI (and BVI). Despite most of the application of gusts study and AVI (and BVI) are in the development of helicopters rotors, where supersonic flow is not employed in usual way, there exists in the development of supersonic aircrafts an application for these studies, being therefore the supersonic flow considered in present work, and filling a blank space existing in literature.

In subsonic flow, the first studies were developed by Lomax *et al.* (1952), who obtained analytical solutions for pressure distribution, lift and pitch-up moment for the step function and, in analogous way, for the sharp edge gust by Heaslet and Lomax (1947). However, the analytical results were only found for brief instants of time after the beginning of the motion, being the rest of this motion completed by approximated functions, normally exponential functions. A numerical model for arbitrary compressible motions, based on the linearized acceleration potential, was developed by Long and Watts (1987). Numerical solutions of the Navier-Stokes equations were proposed by Shaw and Qin (2000). There are many studies that uses indicial functions approach to obtain other motions, normally making use of superposition (Beddoes, 1984; Leishman,1996). Leishman (1997), through the use of reverse flow theorems, obtained results for gusts with variable propagation speed. Sitaraman and Baeder (2004), obtained results for the AVI problem through computational fluid dynamics (CFD) and superposition of indicial functions. Both studies are used here comparatively to the results obtained for validation of these. The gusts and AVI studies for profiles in subsonic flow, has its straight application in the development of helicopters rotors.

2. Approach

2.1. Sharp Edge Gust

Let us consider a thin profile, with zero angle of attack, immersed in a uniform flow with velocity U . The profile, is then submitted to a sharp edge gust of uniform intensity w_g (in the present work, it was considered the simplifying hyphotesis of $w_g = U$, without loss of generality), that moves relatively to the uniform flow with velocity U_g . It is set a gust velocity ratio, such that:

$$\lambda = \frac{U}{U + U_g} \tag{1}$$

Since the velocity under wich the gust comes into the profile, is the sum of the velocities, U , and of the relative gust velocity, U_g , we have that the gust moves over the profile with velocity $U + U_g = \lambda^{-1} U$. The boundary condition over the profile, is then given by the normal velocity over it, being in the parts hit by the gust, $U_n = w_g = U$, and in parts still not hit by the gust $U_n = 0$. Fig. 1, illustrates the gust problem.

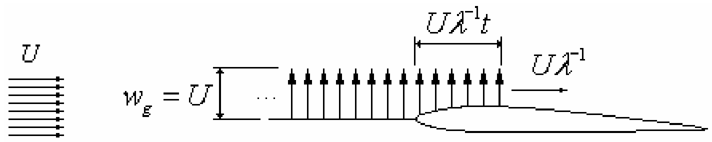

Fig 1. Sharp edge gust.

Two typical cases of gust velocity ratio, λ , shall be commented. When we have $\lambda = 0$ ($U_g \to \infty$), the solution of the problem results from the resolution of the step function problem (where, at an instant $t = 0^+$, the whole profile is submitted to the gust, similarly to a sudden change in the angle of attack). And when $\lambda = 1$, that is the classic sharp edge gust, where the gust moves together with the flow over the profile. Both solutions are known in literature as indicial responses, and were already object of previous study by the authors (Hernandes and Soviero, 2004).

2.2. Aifoil Vortex Interaction (AVI)

The AVI problem, consists on studing the interaction of a vortex convecting in an uniform compressible flow with velocity U under a profile. Are objects of study, the aerodynamic loads generated by this vortex under the profile. In the model studied, the vortex is considered to be thrown thirty chords of distance ahead of the profile, with vortex intensity gone through a dimensionless process by uniform flow and chord of $\Gamma/Uc = 0.2$, and still, the vortex located down the $1/4c$ profile. These values were chosen, in order to allow comparison to another reference (Sitaraman and Baeder, 2004). Fig. 2, shows the AVI problem schematic.

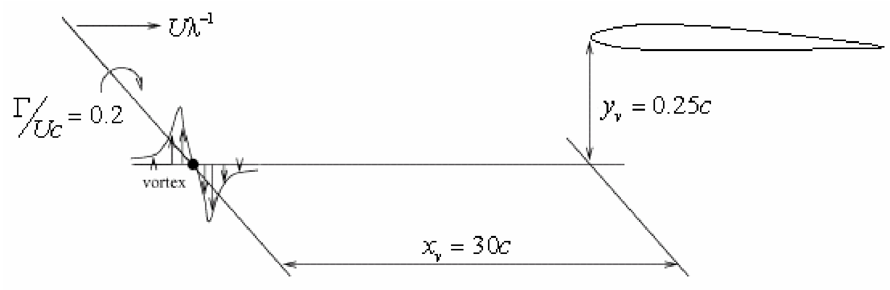

Figura 2. AVI schematics.

Since the vortex is thrown to a relatively large distance (in terms of non-stationary effects) of the profile, $30c$, it is considered the simplifying hypothesis of incompressible vortex, since we may consider its effects in small initial instants, and when next to the profile it would be reaching the steady flow. Fig. 3, shows the vortex-induced velocity that moves in flow velocity, in a position situated at thirty chords from its origin, that is, the leading edge of the considered AVI schematic. The figure shows that at this distance, the compressible and incompressible vortex-induced velocity are the same, allowing us to adopt the hypothesis of incompressible vortex without mistakes. However, this fact constitutes a limitation of the model related to the lauching position of the vortex, being these effects the aim of future studies. It is still considered, the point vortex and without suffering deformation when next to the profile (there are studies on this deformation – Lee and Smith, 1991).

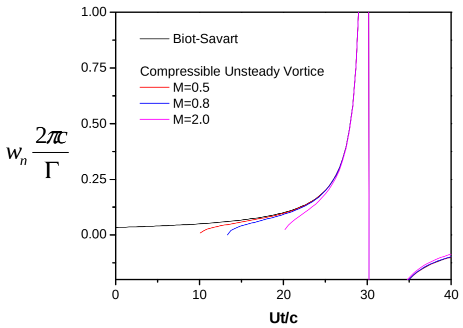

Fig 3. Velocity induced by a vortex convecting with uniform free stream flow velocity at a point situated at thirty chords ahead of its origin.

In analogous way, for the sharp edge gust is defined a velocity ratio, λ, of same formulation and physical sense. The boundary condition is given by the incompressible point vortex equation, in such a way that normal velocity induced over the profile is given by:

$$w_v = -\frac{\Gamma}{2\pi}\frac{x_v - x}{(x_v - x)^2 + y_v{}^2} \quad , \quad \begin{array}{l}\Gamma = 0.2Uc \\ x_v = 30c \quad y_v = 0.25c\end{array} \tag{2}$$

3. Methodology

It is used the model developed by Hernandes and Soviero (2004), subjected to boundary conditions described for sharp edge gust problems with variable velocity and AVI.

Are hyphoteses of the model, the non-viscous fluid, therefore, the forces act normal to the surface (with no tangential forces); irrotational flow ($\nabla X \vec{V} = 0$); and it is assumed the concept of small perturbations ($u', v', w' << U$). The study is then restrict to the velocities potential equation for non-steady flow:

$$\phi_{tt} + 2U\phi_{xt} + U^2\phi_{xx} = a^2\nabla^2\phi \tag{3}$$

The profile is divided in a convenient number of n panels. As the profile is subject to the boundary condition ($U_n \neq 0$), a potential jump of perturbation, $\delta\phi$, associated to each panel surges over the profile (in panels where the condition $U_n \neq 0$ is filled), and that can be determined through the Piston Theory (Bisplinghoff *et al.*, 1955). At a following immediate instant, these jumps are replaced by pairs of counter-rotating vortices of intensity Γ and $-\Gamma$ (where Γ is numerically equal to $\delta\phi$).

The velocity U_n, after initial condition, is constituted of initial boundary condition summed to the normal velocities induced by vortices emited in previous instants. The vortices introduced in substitution to the potential jumps, are defined in two kinds: the bound vortices, and the free vortices (that moves with the freestream flow velocity).

The sequence of events can be understood with the help of Fig. 4. Over the panels, at any instant of time, there are velocity potential jumps generated by the boundary condition at that instant summed to the induced velocities from the vortices emitted at all previous time steps. In panels junction, a balance among the emited vortices is done, resulting in a summation at the left ends of each panel. The potential jumps, are intimately related to the impulsive share of the motion, and the generated vorticity to the circulating share.

The sequence of events for supersonic flow, is analogous to the subsonic flow, differing only in the aspect that there is no emition of vortices impulsively created by the trailing edge, being all vortices bounded to the profile. The counter-rotating vortices originated from velocity perturbation potential jumps for supersonic flows, are all bounded to the profile, for Kutta´s condition need not to be obeyed in this flow. For subsonic flow, the trailing edge vortex of the profile (of intensity $-\Gamma_1^k$, placed at the right end of the panel $j = 1$) is, by imposition of the model, free to automatically create a shedding and satisfy Kelvin´s theorem.

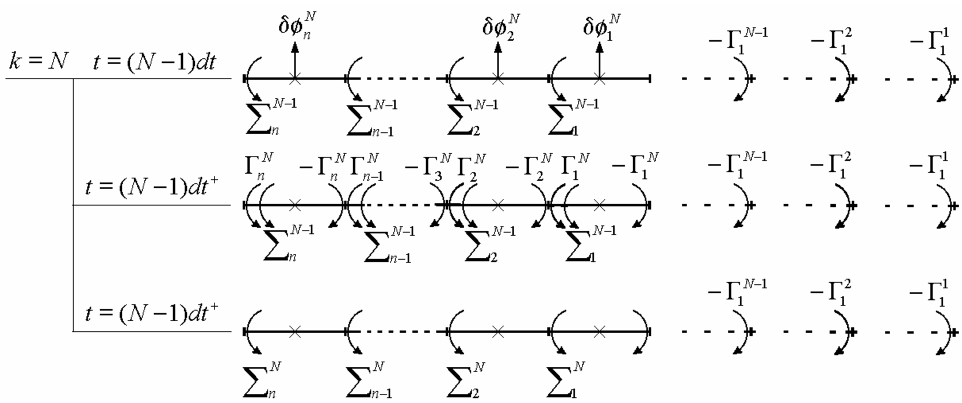

Fig 4. Numerical model

The solution of the problem is obtained from the solution of a linear system, where we have a matrix of coefficients $[A]$, such that multiplied by the matrix of potential jumps $[\delta\phi]^k$ for a considered instant of time k, results in the boundary condition matrix $[W]^k$ (normal velocity over the panels).

$$[A][\delta\phi]^k = [W]^k \tag{4}$$

Matrix $[A]$, is associated to the influence of the generated vortices at a given instant k and its influence at the same instant. We still must sum up to the elements of the main diagonal, the share regarding the impulsion of the own panel given by $1/_{2adt}$.

For subsonic flow, the matrix of velocities $[W]^k$ is a function of the velocity normal to the profile due to the motion ($U\alpha$), adding the velocities induced by the vortices emited at instants previous to the one considered. For the right work of the method in supersonic flow, it is essential to consider the fundamental element of the method – the vortex singularity. In the subsonic flow, it’s possible to calculate the vortex-induced velocity at any point of the affected area by it. In the supersonic flow, the origin of the vortex is singular, not being possible to calculate the induced velocity at this point. Therefore, it is necessary to define the contribution of this singularity for the velocity field. This concept, is explored by Miranda *et al.* (1977). The share related to the induced velocity due to this singularity, is defined from the induced velocity in steady flow.

From the solution of the system (matrix $[\delta\phi]^k$), we can calculate the aerodynamic coefficients.

4. Results and Discussion

The aerodynamic coefficients are presented, numerically calculated for both flows, subsonic and supersonic. Numerical results are compared to solutions available in literature. For gusts study, the results are compared to Leishman (1997) and Lomax (1954) in subsonic flow and to Bisplinghoff *et al.* (1955) in supersonic flow, this last limited to $\lambda = 0$ and $\lambda = 1$, due to the lack of results obtained in literature. For the AVI study, the results are compared in subsonic flow to Sitaraman and Baeder (2004), not being found any reference of the AVI studies (or BVI) in supersonic flow.

Fig. 5, shows the results obtained for sharp edge gust with variable gust propagation velocity in subsonic and supersonic flows, compared to other references. It is observed a strict correspondence in values obtained with the references, as for subsonic as for supersonic flow.

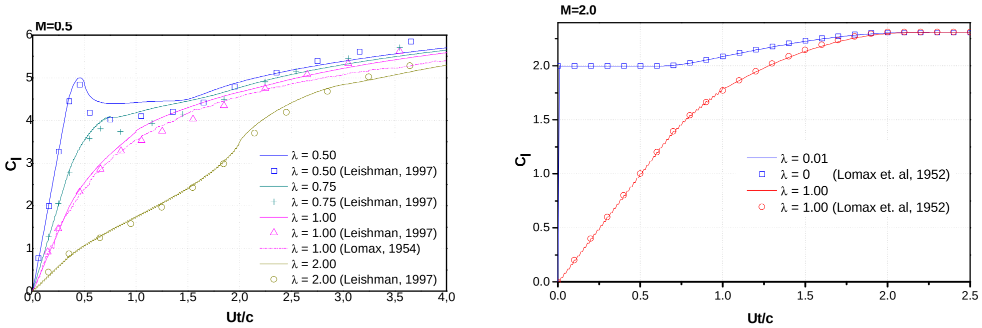

Fig 5. Sharp edge gust compared to references. Left, subsonic.flow. Right, supersonic flow.

Fig. 6 shows, for initial instants of the motion, the results obtained in subsonic and supersonic flows of the profile submitted to a sharp edge gust with variable gust propagation velocity. Curves of the lift coefficient and the variation of the aerodynamic center of the profile, are presented. Two curves from the graphics of lift coefficient are remarkable: $\lambda = 0.01$ (using 0.01 for numerical limitation, having the intention to show the behavior for $\lambda = 0$) and $\lambda = 1$, that represents the motions of sudden change in the angle of attack and sharp edge gust, indicial solutions found in literature. It is verified that, the coefficient λ varies form 0 to 2, a series of intermediate curves between the curves for these coefficients are obtained. In subsonic flow, the peak value of the lift coefficient is associated to the non-circulating portion (or impulsive) that tends to extinguish, predominating the circulating portion in the following instant – where it is verified the continuous growth of lift. From the lift curves, it is observed that as the gust propagation velocity raises (increase of λ), the peak from non-circulating portion is reduced, at the same time that it lasts longer, and this behaviour is clear for $\lambda = 2$. The curves for supersonic flow show similar behaviour, where an increase of λ is translated in a slower growth of circulation. The last graphics, show the variation in time of the profile pressure center along the chord. In subsonic case, the final position of the pressure center is in the quarter half of the chord, from the leading edge. In supersonic flow, the final position is in the half way of the profile chord. It is noticed that a raise in λ, implies on a higher time for the steady flow to be reached. Again, this behaviour is explained by the fact that a raise in λ, makes the circulating portion to grow in a slower way, taking therefore to a higher time until steady state is reached. It is also verified, in higher magnitude in higher λs, a small oscilation in the curves. This oscilation, is numerically originated and occurs due to a oscilation in the pressure jump coefficient distribution curve over the profile (Hernandes and Soviero, 2004), but it is not meaningfull to invalidate results. The variation of the pressure center, must be taken into account in simplified versions of the Vortex Lattice Method, where a unit discretization is used along the chord, as in Weissinger method.

Fig. 7, shows an evolution of the pressure jump coefficient in time over the profile. It is clearly seen, a line that begins in the origin and ends in position $Ut/c = \lambda$, which is the point where the gust has run the profile as a whole. Following this condition, in subsonic flow, it is noticed a predominant non-steady region followed by a well defined standard region, where there exists practically only the growing circulating portion. In supersonic case, after the gust hit the whole profile, there exists a region of high gradients, achieving then the steady flow well visible in the figures.

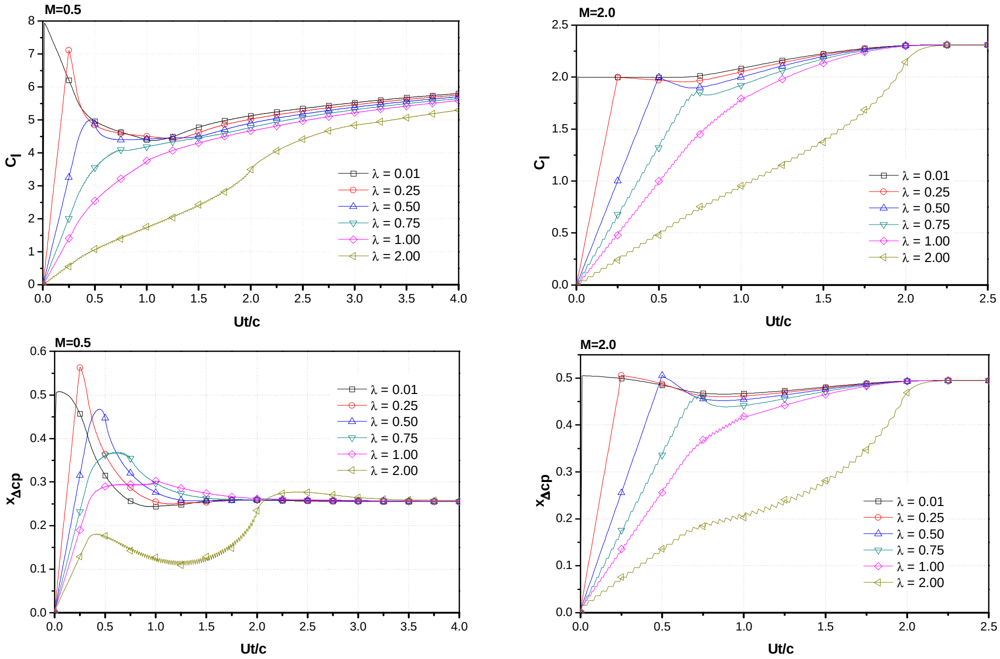

Fig 6. Sharp edge gust for several λ. Left, subsonic flow. Right, supersonic flow.

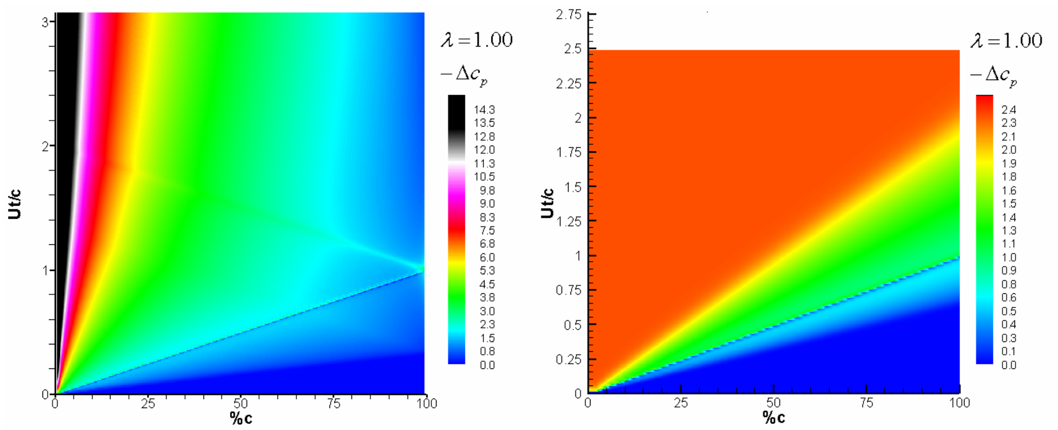

Fig 7. Distribution of pressure jump coefficient over the profile for sharp edge gust. Left, subsonic flow (M=0.5). Right, supersonic flow (M=2.0).

The results obtained for the AVI problem are showed in Figs. 8 to 10. Fig. 8, shows the lift coefficients obtained comparing to the values obtained by Sitaraman and Baeder (2004). A good agreement in results was verified. A small divergence in the positive peak value is observed , being that smaller in the values obtained via CFD and higher in the values obtained indicially (approximated functions), these last use exponential functions that normally cannot correctly translate the non-steady peaks – present in regions dominated by impulsive portion, these situated in regions next to the vortex position. It is possible to obtain a small improvement in the curve, approximating of the CFD curve in peak region, by raising the number of panels. However, this small difference is only noticed in the peak region and also has a high computational cost.

Fig. 9, shows the aerodynamic lift coefficients and the evolution of the pressure center for subsonic and supersonic flows, for several values of vortex propagation velocity. It is observed that, the higher the values of λ, in subsonic flow, lower will be the peaks and it takes longer for the lift to return to its initial value, fact explained by the behaviour that a raise in λ implyes on a higher smoothing of impulsive portion and slower growth of circulating portion. In supersonic flow, this behavior is not so evident. Still, it is shown in Fig. 9 the evolution in time of the pressure center of the profile, that obviously has its behaviour directly affected by the vortex position over the profile.

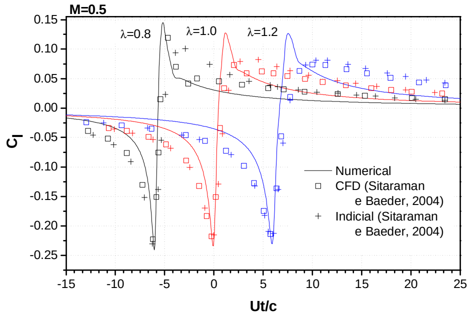

Fig 8. AVI compared to the reference.

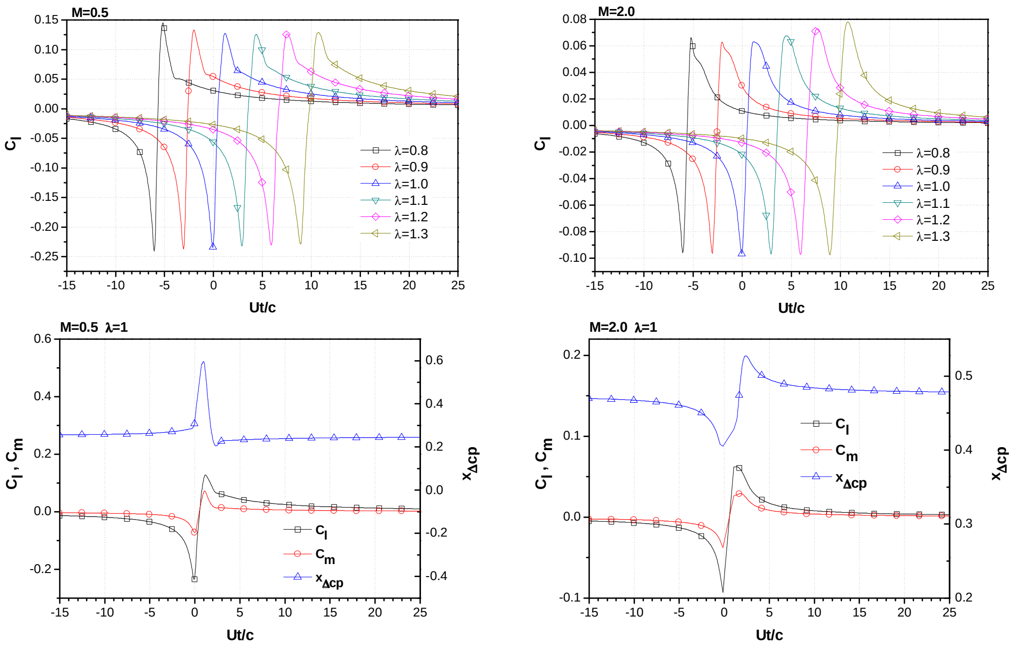

Fig 9. AVI for several λ. Left, subsonic flow. Right, supersonic flow.

Variations in time of pressure jump over the profile, are shown in Fig. 10, with emphasis to the period that the vortex passes over the profile, where the change on the lift signal occurs, noticed in pressure distribution by inversion of the curves. Two moments are pointed out in order to evidence this fact: $Ut/c = -1$, where the vortex is placed at one chord from leading edge and $Ut/c = 2$, where the vortex is at one chord downstream the trailing edge.

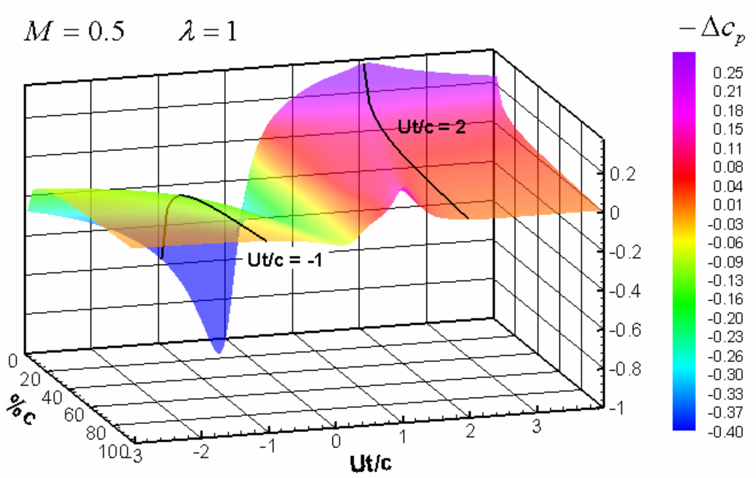

Fig 10. Pressure jump coefficient distribution over the profile, for AVI problem.

5. Conclusions

Aerodynamic forces were numerically obtained for a profile subject to sharp edge gust and the solution of the AVI problem, both considering gust propagation velocities and variable vortices. Comparisons with the literature were done, in order to validate the results and show the accuracy of the method. The results show the compressibility effects in these problems, as well as how these motions are affected by the propagation velocity of the gust or vortex. The study of the interaction airfoil-vortex, consists on fundamental basis of studies in noise reduction of helicopters, being this study necessary to the development of more silent rotors. Still, the results obtained by the present method are far faster (about 10000 times faster) than if we obtained these same results via (CFD), and more precise than the methods that use an approach from approximated equations, therefore, being an excellent tool in preliminary project of aircrafts and helicopters.

6. Acknowledgments

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