A COMPARISON BETWEEN NUMERICAL AND LABORATORY DATA FOR TWO-DIMENSIONAL STEEP TOPOGRAPHIC ELEVATION

Juliana B. R. Loureiro
Programa de Engenharia Mecânica (PEM/COPPE/UFRJ)
C.P. 68503, 21945-970, Rio de Janeiro, Brasil.
jbrloureiro@mecanica.coppe.ufrj.br

Marcel de Barros Parreira
Programa de Engenharia Mecânica (PEM/COPPE/UFRJ)
C.P. 68503, 21945-970, Rio de Janeiro, Brasil.
marcelparreira@gmail.com

Fernando T. Pinho
Centro de Estudos de Fenômenos de Transporte (Universidade do Minho).
4800-058, Guimarães, Portugal.
fpinho@dem.uminho.pt

Alexandre T. Alho
Departamento de Engenharia Naval e Oceanica (PENO/UFRJ)
Universidade Federal do Rio de Janeiro, Brasil.

Atila P. Silva Freire
Programa de Engenharia Mecânica (PEM/COPPE/UFRJ)
C.P. 68503, 21945-970, Rio de Janeiro, Brasil.
atila@mecanica.coppe.ufrj.br

Abstract. *Numerical simulation results for a neutrally stratified boundary layer over a steep topographic elevation are compared with original laboratory data. The simulations were carried out with a commercially available computational fluid dynamics software using both eddy viscosity and shear stress models. The focus of the present work is on evaluating the ability of the various turbulence schemes in predicting the fluctuating fields over a large hill, where separation is expected to take place. The experiments were conduted in a water channel using laser Doppler anemometry. The dataset provides detailed measurements of longitudinal and vertical mean velocities, along with its fluctuating components for the whole flow field over the hill, including the recirculation region. These features make this dataset as an excellent test of the models' performance. The main objective of the present work is to evaluate the capacity of widely used CFD softwares in simulating complex atmospheric flows.*

Keywords*: Numerical simulation, flow over hills, turbulence, laser-Doppler anemometry, experimental measurements.*

1. Introduction

The study of flows over an arbitrary terrain is a crucial step to understand how the boundary layer behaves in general. Indeed, what can be seen as a purely scientific desire, i.e. to fully comprehend the boundary layer behaviour, is ultimately the solution for all the practical problems concerning meteorological and aeronautical applications.

In micrometeorology, there is a constant need to understand and forecast the widest events of atmospheric phenomena, since a combination of them can pose significant hazards to navigation or aviation, for example. As far as the wind engineering is concerned, it is of prime importance to calculate the speed up factor, i.e. the magnitude and location of the maximum increase in velocity over the hill, to correctly locate wind turbines or other structures to be positioned.

Early research into modelling turbulent flow over smooth topographic elevation, in particular the studies introduced by Hunt and his co-workers, were decisive for the implementation of the former numerical codes capable of predicting flow over changing terrain with reasonable accuracy. However, severe restrictions posed on these calculations, since the validity of the linearised theory proposed by Jackson and Hunt (1975) was limited to low hills of moderate slope.

Considering that many sources of air pollution are located in complex terrain, detailed and accurate information about the flow field is needed so that reasonable predictions of gas concentration can be made. Despite the huge demand for a better understanding of the separation process downstream of steep slopes, only a small number of works have focused on this problem. Actually, the recent developments in technology, which allows increasing computational power, are believed to change this situation. Simplified linear models were turned into more refined nonlinear codes which could then provide reasonable predictions for flows over steeper slopes.

From this standpoint, the objective of the present work is to evaluate the capacity of widely used computational fluid dynamics software, using different turbulence models, in simulating complex atmospheric flows. Naturally, the conclusions of this assessment may be useful not only to practical atmospheric applications, but also to planning an experimental campaign. This work presents numerical simulations for a neutrally stratified boundary layer over a steep hill, and the results are compared with original laboratory data. The computations were carried out using two different eddy viscosity and three shear stress models. The main focus of the present work is on evaluating the ability of the various turbulence schemes in predicting the fluctuating fields over a large hill, where separation is expected to take place. The laboratory data used for numerical validation were conducted in a water channel environment using laser Doppler anemometry. This dataset provides detailed measurements of longitudinal and vertical mean velocities, along with its fluctuating components for the whole flow field over the hill, including the recirculation region. These features make this dataset an excellent test for the models' performance.

As explained by Belcher and Hunt (1998), the boundary layer over a flat, open land is a very slowly varying turbulent flow, so that it can be modeled throughout its depth using turbulence models based on eddy –viscosity concepts. However, the changes induced by the presence of the hill imply that the flow field is distorted over a short longitudinal distance, during a small time interval. Consequently, the turbulence field can no longer reach an equilibrium condition instantaneously, and is therefore said to be governed by the equations of the rapid distortion theory, Batchelor and Proudman (1954). This poses severe difficulties for a numerical simulation of the flow field, for the working turbulence model must be capable of incorporating all turbulence features down to the wall.

2. A brief literature review

A handful of numerical and modelling studies have been carried out to develop and analyse numerical models capable of simulating flow over steep slopes. In the following, we present some representative work of previous contributions.

Concerned with the effects that hills provoke on turbulence, Zeman and Jensen (1987) developed a new model where the von Mises transformation was applied to the mean momentum equations and the second-order closure type turbulence equations were solved. All predictions were compared with data from the Askervien Hill project.

A numerical study of the mean wind speed and turbulence features in and above a forest canopy covering a two-dimensional hill was performed by Kobayashi et al (1994). Using a finite volume numerical algorithm with a highly accurate numerical scheme constructed from a class of total variation diminishing schemes (TVD), the authors managed to eliminate false diffusive errors. To close the averaged Navier-Stokes, the extended k-ε model of Svensson and Häggkvist (1990) that considers two extra terms to account for the drag caused by the canopy was used. The predictions were compared with the reported LDV wind tunnel measurements of Ruck and Adams (1991). The calculations showed good agreement with the data, being capable of predicting the presence or not of a separated flow region. Typical grid size was 100x60. The law of the wall was applied to specify the wall boundary condition.

An evaluation of ten different local turbulence closure procedures was carried out by Hurley (1997) for flow over a hill. His analysis included one- and two-equation models. The results showed that small differences were found for the mean flow properties whereas large differences were noted for the velocity variances and the dissipation rate.

The flow over a two-dimensional hill was simulated by Ying and Canuto (1997) through a second-order closure model. The simulations are compared with data obtained from a wind-tunnel experiment. The results were also compared with lower level turbulence models, including an eddy-viscosity model and an algebraic Reynolds stress model. For boundary condition specification, the standard logarithmic law of the wall profile was adopted. The authors, in addition, specify conditions for k-ε and all the terms in the Reynolds stress tensor. The overall conclusion was that second-order closure models seem to be capable of capturing the fundamental physical mechanisms in the flow over hills.

Castro and Apsley (1997) performed computations for the flow and dispersion over two-dimensional hills of various slopes in a neutrally stable boundary layer. The results were compared with laboratory data. The authors showed that a suitably modified k-ε model generally produced good agreement for the mean behaviour, but lower values for the turbulent kinetic energy and the lateral plume spread. Corrections in the standard k-ε model allowed the authors to account for streamline curvature effects. For a hill with a sufficiently large slope, enough to give rise to a large separated region, the levels of concentration were also found to be well predicted. For hills with lower slopes that provoked intermittent separation, less satisfactory results were observed. To specify the wall boundary conditions, wall functions were invoked. Typical grid size was 160x200.

A non-linear numerical model simulation of the flow over Blashaval hill was performed by Hewer (1998). For most of the simulations, the Reynolds stresses were modeled through a first-order mixing-length closure model. An alternative closure expressed the turbulent viscosity in terms of the turbulent kinetic energy. Overall, the author concluded that the non-linear model simulations of neutral boundary layer flow over a hill of moderate slope provided predictions as good as those provided by the linear models. On the lee slope, wind speeds were over predicted by the non-linear models, which, however, were more accurate than the linear models. The difficulties in dealing with the lee side of the hill arose from inaccuracies in the simulation of flow separation.

Kim and Patel (2000) tested five two-equation turbulence models against a host of situations that ranged from a neutral boundary layer on a flat surface to flow over obstacles and in valleys. The models use the concepts of isotropic eddy viscosity and wall functions and were assessed by comparison with wind-tunnel and field data. The wall function that was used was the classical logarithmic profile. The best overall performance based on such diverse criteria as the prediction of mean velocity profiles, of turbulent kinetic energy profiles, of Reynolds stresses, of speed-up ratio, or of length of separated flow region, was achieved by the RNG-based k-ε model.

Castro et al. (2002) simulated the flow of a neutrally stratified flow over the Askervein Hill using a k-ε model. Boundary conditions were specified through the classical law of the wall. The computational grid with the highest resolution had 155x155x31 node points. Results showed that at the hill top, the speed-up was 10 % less than the experimental value. The recirculation region in the lee of the hill was captured through a time-dependent formulation and a third order discretization of the advection terms. The characteristic roughness near the hill top was reduced to improve the agreement between the numerical and the experimental data.

Large-eddy simulations were also used by Iizuka and Kondo (2004) to describe turbulent flow over a two-dimensional steep hill. Four sub-grid scale (SGS) models were tested using two different ground surface conditions. The two different ground conditions relied on the classical logarithmic law for specification. The computed data were compared with the experimental data of Ishihara et al. (2001). The results provided by the dynamic SGS models were noted to be in very poor agreement with the experimental data and this fact was associated mainly to the inaccurate estimation of the near ground surface model coefficient. To improve accuracy, a hybrid SGS model was introduced by the authors. This new model was proved to give the best results.

Two dimensional steep hills in both neutral and stably stratified flow conditions were also studied by Ross et al.(2004). Turbulence models that used one-and-a-half and second-order closure schemes were used to predict the mean and turbulent quantities of the flow. The numerical predictions were compared to new wind tunnel experiments carried out for two hills with different slopes, one of which was steep enough to cause flow separation. The data, obtained through laser Doppler anemometry included mean and turbulent properties of the flow. The numerical simulations were conducted in a 2-D domain with 128x80 grid points. The wall flow region was treated accordingly to the procedure of Ying and Canuto (1997). The authors report a reasonable prediction for mean flow characteristics for all flow conditions. However, large differences are observed in the separated flow region in the lee side of the hill.

The ability of non-linear eddy-viscosity and second-moment models to describe the flow over two- and three-dimensional hills was investigated by Wang et al. (2004). Five turbulence models were analyzed: two cubic eddy-viscosity models, an explicit algebraic Reynolds-stress model, a quadratic eddy-viscosity model and a Reynolds-stress-transport model. The one major objective of the paper was to examine the flow separation patterns that occur on the lee side of 2D- and 3D-hills. The authors report that in 2D-flow the predicted separation differs greatly from one model to the other, with just one non-linear model performing well. In 3D-flow, none of the models were found to give a good representation of the complex multi-vortical separation pattern. For the 2D- and 3D-flows typical grid sizes were 700x90 and 110x105x80 nodes respectively.

3. Numerical simulations

All simulations were carried out using the commercially available software CFX. The model solves the steady state Reynolds averaged Navier-Stokes equations (RANS) in a Cartesian coordinate system, with a finite-element method. Following previous grid-dependence testes, a non-uniform body-fitted grid comprising 720700 nodes and 350000 elements has been used.

Inlet conditions consisted in a power law mean velocity profile, using an exponent 1/7 with the free stream velocity (U_∞ = 0.0482 m/s) and the boundary layer thickness (δ = 0.10 m) prescribed from the experimental data. Turbulent intensity for the incoming flow was defined as 2%, based once again in the experiments. At the outlet, the flow was allowed to recirculate, and the turbulent intensity was automatically calculated by the model. At the side walls a symmetry condition was imposed. At the ground level along with the hill surface, a smooth wall and the no-slip boundary condition was used. The main computational parameters are outlined in Table (1).

3.1 Turbulence models

Five turbulence models are investigated herein, namely, two eddy-viscosity models: (i) a standard k-ε model, (ii) a k-ω model accounting for shear stress transport; and three Reynolds stress models: (iii) a SSG model, (iv) a LRR model and finally (v) a baseline Reynolds stress model (BSL). Only a brief description of each method will be offered on the following. For further details the reader is referred to the original sources.

In analogy to the laminar regime, the eddy viscosity models assume the hypothesis that the turbulent stresses are related to the mean velocity gradients by an enhanced constant of proportionality, named the eddy or turbulent viscosity. This new constant can be computed in different ways. Among the options, two-equation turbulence models are widely used since they strike a good balance between numerical effort and computational accuracy. In addition to the Reynolds averaged equations, transport equations are separately solved to give a velocity and a length scale. A standard k-ε model (Launder and Spalding, 1974) assumes that the turbulent viscosity is linked to the turbulent kinetic energy and to the turbulence dissipation via Eq. (1):

Table 1. Computed cases and boundary conditions.

	Boundary Conditions
Inlet (Mean velocity)	$U/U_\infty = (z/\delta)^{1/7}$
Inlet (Turbulent intensity)	0.02
Side walls	Symmetry condition
Ground level and hill	No-slip and smooth wall
Turbulence models	k-ε, k-ω, SSG, LLR, SST

$$\mu_t = C_\mu \rho \frac{\kappa^2}{\varepsilon}, \tag{1}$$

where C_μ is a constant. The values of k and ε comes directly from the transport equations Eq. (2) and (3):

$$\frac{\partial(U_i\kappa)}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_k}\frac{\partial\kappa}{\partial x_i}\right) - \overline{u_i u_j} S_{ij} - \varepsilon, \tag{2}$$

$$\frac{\partial(U_i\varepsilon)}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial\varepsilon}{\partial x_i}\right) - \frac{\varepsilon}{\kappa}\left(C_1 \overline{u_i u_j} S_{ij} + C_2\varepsilon\right), \tag{3}$$

where C_1, C_2, σ_ε and σ_k are constants and S_{ij} is the mean strain-rate tensor.

However, one of the limitations of this model is the requirement for high resolution near the wall. For this reason, it is a common practice to use k-ε with boundary conditions specified by the use of wall functions.

An advantage of the k-ω formulation is exactly the near-wall treatment, which can accept higher values of z^+, the non-dimensional distance from the wall. In a similar way, the k-ω method assumes that the turbulent viscosity is a function of the turbulent kinetic energy and the turbulent frequency, as given by Eq. (4):

$$\mu_t = \rho \frac{\kappa}{\omega}. \tag{4}$$

The transport equation for the turbulent kinetic energy and the turbulent frequency is given by Eq. (5) and Eq. (6), respectively:

$$\frac{\partial(U_i\kappa)}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_k}\frac{\partial\kappa}{\partial x_i}\right) - \overline{u_i u_j} S_{ij} - \beta'\kappa\omega, \tag{5}$$

$$\frac{\partial(U_i\omega)}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_\omega}\frac{\partial\omega}{\partial x_i}\right) - \frac{\omega}{\kappa}\left(C_3 \overline{u_i u_j} S_{ij} + C_4\beta\omega^2\right), \tag{6}$$

where C_3, C_4, σ_ω are constants and S_{ij} is the mean strain-rate tensor. Despite the good wall treatment, the main weakness of the k-ω formulation proposed by Wilcox (1986) is the strong sensitivity to free stream conditions. To solve this problem, Menter (1994) introduced a blending function between the k-ω formulation near the surface and the k-ε model in the outer region, which is usually called baseline k-ω model. But, indeed, this method failed to correctly predict the onset and amount of flow separation from smooth surfaces. The main reason for the model failure was that it did not account for the transport of turbulent shear stresses, and consequently it over predicted the eddy-viscosity. Considering these limitations, the present work decided to test the k-ω based shear stress transport model.

The second approach to model the turbulent terms that appear in the Reynolds averaged equations is based on the solution of transport equations for all the components of the Reynolds stress tensor and the dissipations rate. The intrinsic modelling of the stress anisotropies and the exact production term make the Reynolds stress models more appropriate for complex flows.

The standard Reynolds stress model is based on the ε equation. The equation for the transport of Reynolds stresses is given in Eq. (7):

$$\frac{\partial}{\partial x_k} U_k \rho \overline{u_i u_j} = P_{ij} + \phi_{ij} + \frac{\partial}{\partial x_k}\left[\left(\frac{2}{3} c_s \rho \frac{\kappa^2}{\varepsilon}\right)\frac{\partial \rho u_i u_j}{\partial x_k}\right] - \frac{2}{3}\rho\varepsilon\delta_{ij}, \quad (7)$$

where ϕ is the pressure-strain correlation, c_s is a constant and P_{ij} is the production term.

In addition to the standard Reynolds stress transport schee, modifications in the model's constants have been proposed, giving birth to a variety of SST models. The present work have used, besides the baseline Reynolds stress model, the LLR model of Launder, Reece and Rodi (1975) and he SSG of Speziale, Sakar and Gatski (1991).

4. Experimental Validation

The experimental data used for validation of the numerical computations presented in this work were obtained from LDA measurements of a neutrally stratified boundary layer simulated in a water channel environment. The model hill followed a "Witch of Agnesi" profile, an extensively used curve in literature, e. g. Britter et al. (1981) and Arya et al. (1987). The defining parameters of the hill is its height, H = 60 mm, and its characteristic length, L_H = 150 mm, which is defined by the longitudinal distance from the top corresponding to half hill height. These values yield an aspect ratio of 5, a base length of 600 mm and a maximum slope of 18,6°; which characterizes the model as a steep elevation where separation is expected to take place. Profiles were measured for 13 different positions distributed along the topography, as depicted in Figure (1). Measurements were refined downstream of the hill top in order to accurately discriminate the recirculation region.

With respect to the experimental conditions, the model was placed at 8 m downstream of the water channel inlet, assuring an oncoming fully developed flow. The free stream velocity of the incident flow was 0.0482 m/s, with a water level 236 mm.

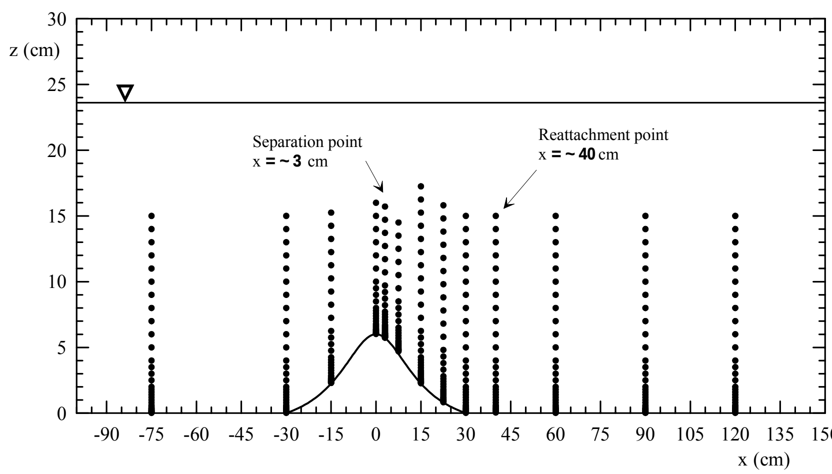

Figure 1: Spatial distribution of the experimental data used for validation of numerical computations.

5. Results

As above explained, five different turbulence models were tested for the case of neutrally stratified turbulent boundary layer over steep surface elevation. Results will be shown first for the two eddy-viscosity models: k-ε and k-ω based SST model, and then for the three shear stress transport models: SSG model, LLR model, and BSL Reynolds stress model. Hereafter, all the data are shown in reference to a Cartesian coordinate system located at the symmetrical axis of the hill, as shown in Figure (1). Presentation of data will be particularly split into two blocks: data for the flow field upstream of the separation point (first three stations) and data for the recirculation region straight downstream of the hill top (next 6 stations).

5.1 Eddy viscosity models

It was found that both the mean velocity and turbulent fields predicted with the k-ε model are in poor agreement with the experimental results, for the whole simulated domain. The model is clearly unable to resolve the sharp velocity gradients of the incident flow field in the near-wall region, what is perfectly noticeable on the top of the hill. Consequently, the k-ε model fails completely to predict the recirculation region. Since all the simulated profiles presented a large discrepancy to the experimental data, no graphical comparison will be shown in the present work. On the other hand, the results provided by k-ω based SST model are in surprisingly good accordance with experiments. Figure (1) shows results of mean longitudinal velocity calculations with the k-ω based SST model using standard constants, for the region upstream and at the top of the hill. Open symbols stands for the experiments while closed symbols denotes the numerical simulations.

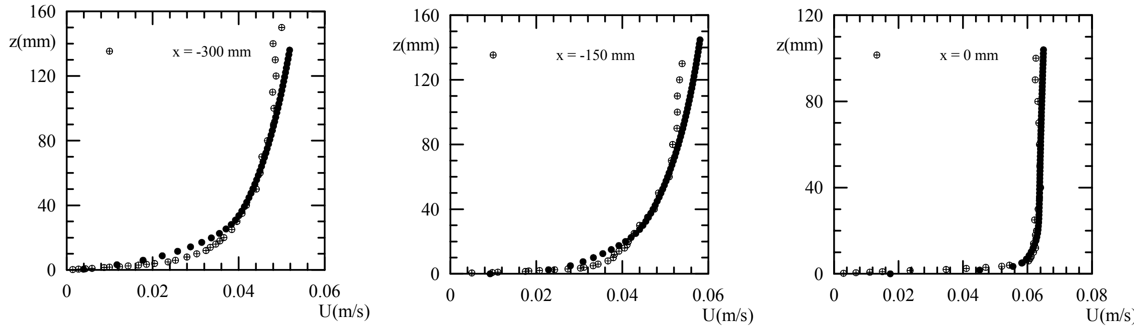

Figure 1: Longitudinal velocity profiles upstream and at the hill top, k-ω based SST model.

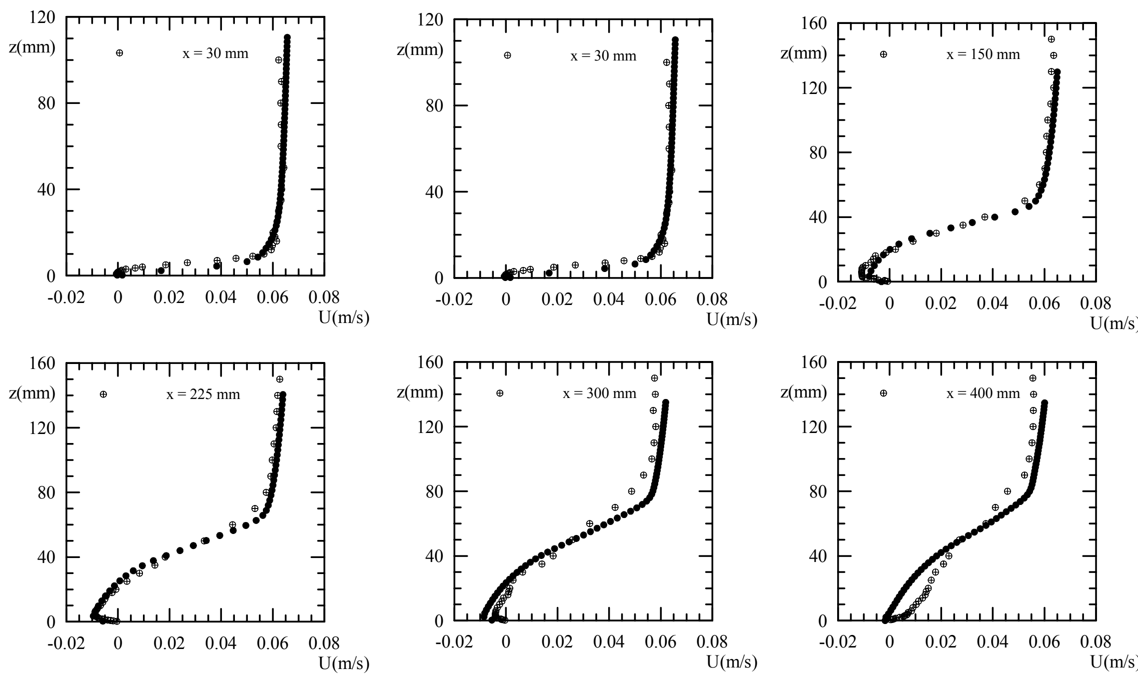

Figure 2: Longitudinal velocity profiles in the recirculation region, k-ω based SST model.

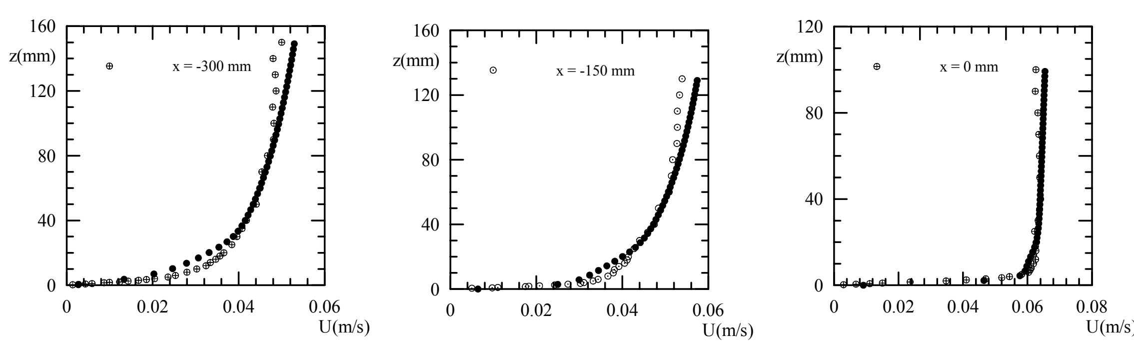

Figure 3: Longitudinal velocity profiles upstream and at the hill top, BSL Reynolds stress model.

5.2 Shear stress models

Results obtained from the simulations with baseline Reynolds stress model (BSL) are shown in Figures (3) to (7). Figure (3) presents the profiles measured upstream of the hill crest, where a progressive acceleration of the flow can be noticed, reaching a maximum on the top. The computed velocity profiles inside the recirculation region are shown in Figure (4). It can be noticed that the BSL model managed to reflect the large near-wall velocity gradients, showing good agreement with the experiments even at the hill crest. This closure model was also successful in predicting the separation point and the flow inside the recirculation region, but failed to correctly predict the reattachment point. Figure (5) shows the turbulent longitudinal velocity profiles located upstream of the hill crest. Hereafter, $\sigma_u^{1/2}$ denotes the longitudinal turbulent velocity component (root-mean-squared value) and $\sigma_w^{1/2}$ the vertical component. Despite the good ability in predicting the mean velocity flow field, the BSL Reynolds stress model completely failed in predicting the turbulent flow field.

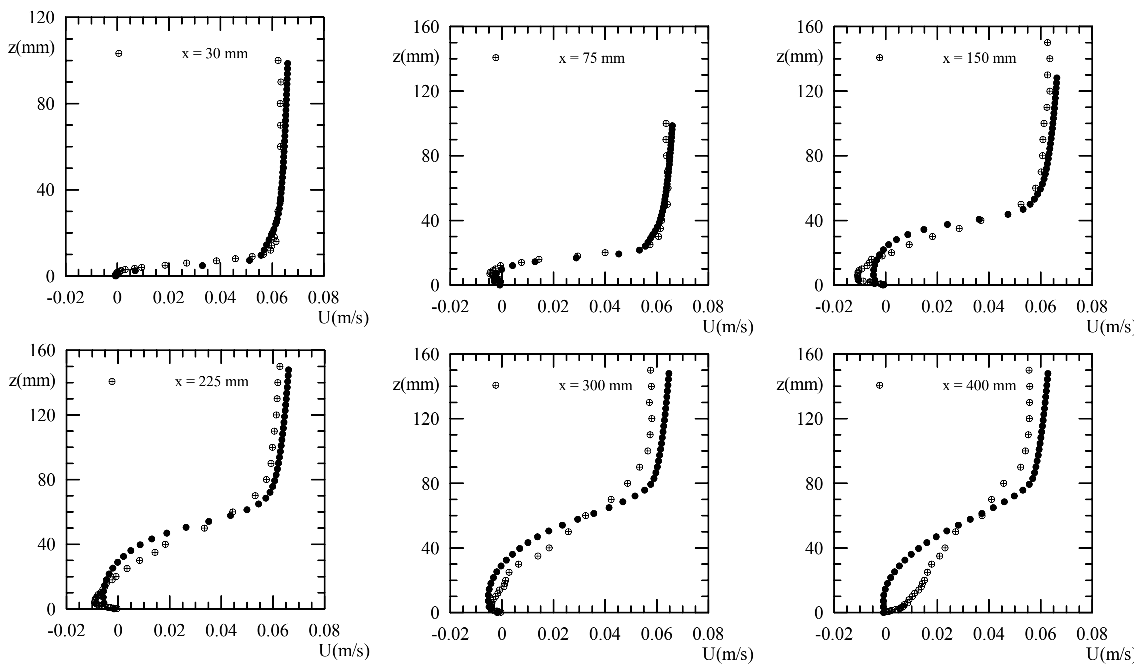

Figure 4: Longitudinal velocity profiles in the recirculation region, BSL Reynolds stress model.

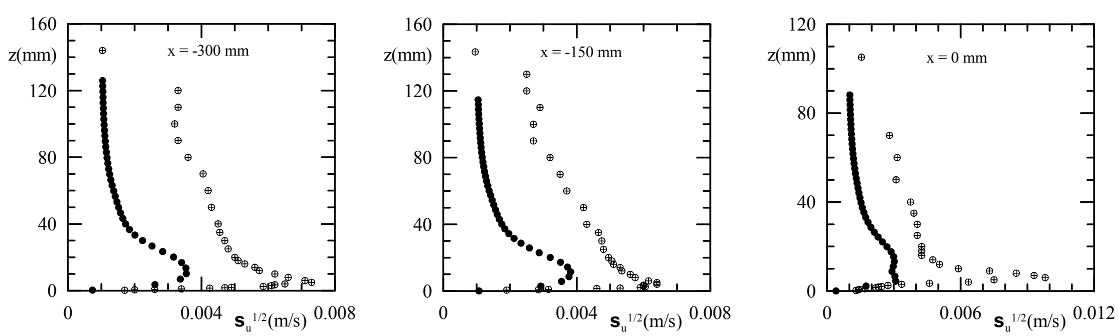

Figure 5: Upstream longitudinal turbulent velocity profiles, BSL Reynolds stress model.

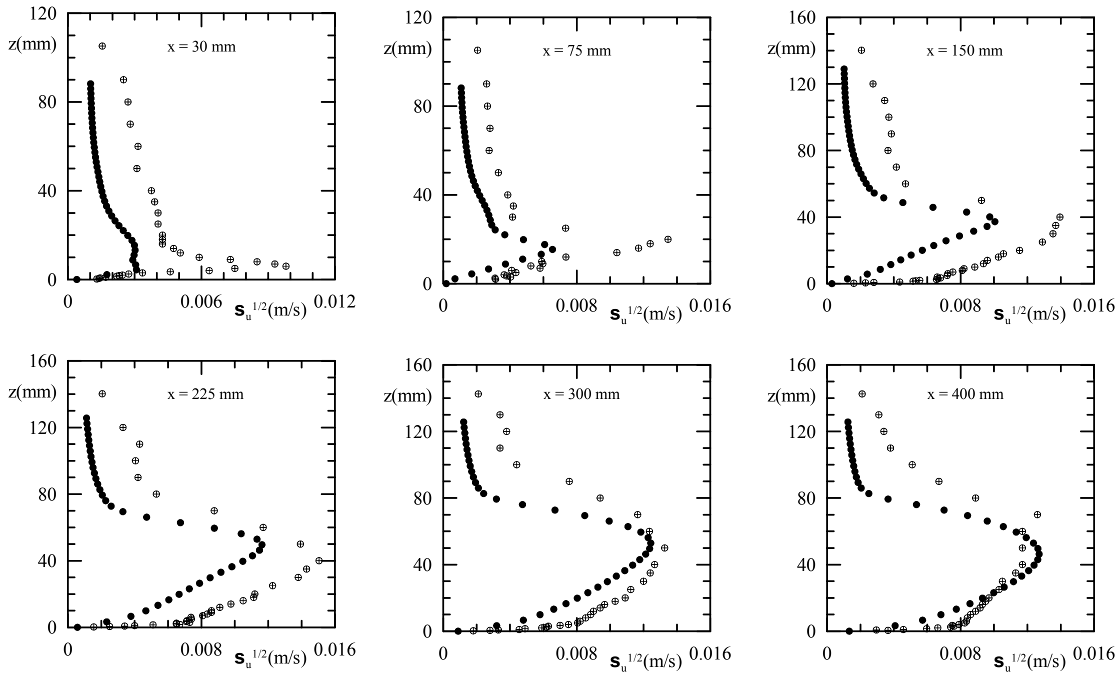

Figure 6: Longitudinal turbulent velocity profiles in the recirculation region, BSL Reynolds stress model.

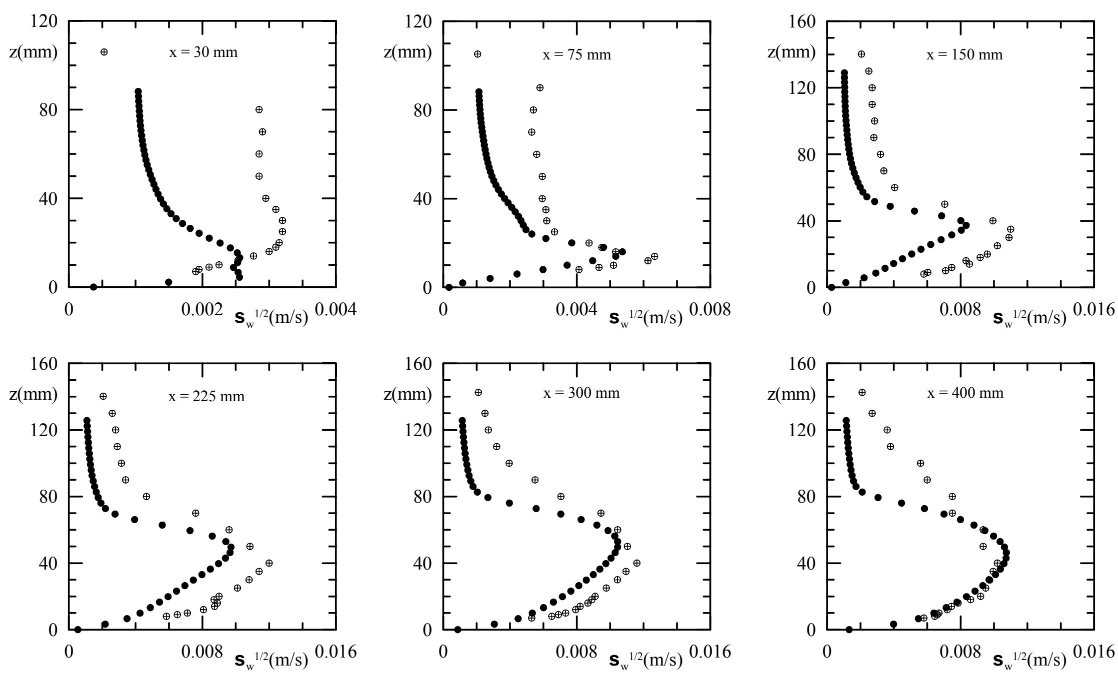

Figure 7: Vertical turbulent velocity profiles in the recirculation region, BSL Reynolds stress model.

It must be remembered that the initial turbulent intensity prescribed in the computations were representative of the experimental results. In fact, further attempts of increasing this inlet condition to correct this underestimation of the turbulent fluctuations were entirely unsuccessful. High inflow turbulence levels distorted even the mean flow prediction, and modified the pattern of the fluctuation profiles.

Results obtained from the SSG turbulence model are presented in Figure (8) to (10). The computations for the LLR model were found to be in close agreement with the numerical data provided by the SSG scheme. Actually, the difference between these two models was less than 2% for all the simulated profiles. Due to this outcome, only the results for the SSG model will be presented here. From figures (8) to (10) it can be inferred that both methods failed to predict the mean and turbulent velocity filed, showing some resemblance with the results of the k-ε model.

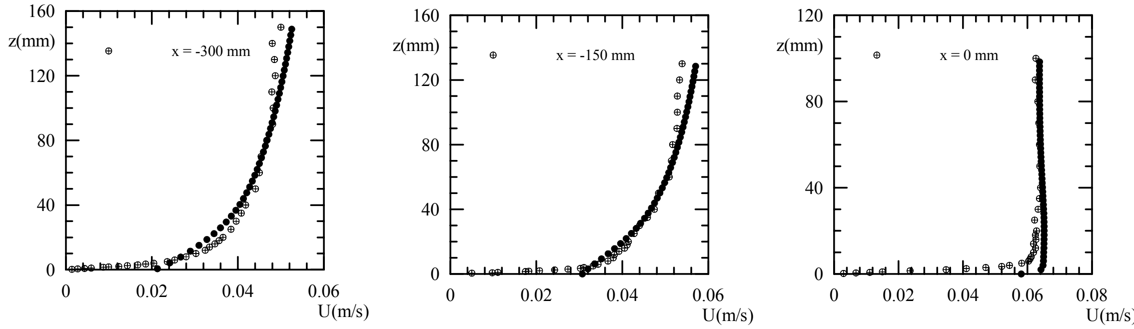

Figure 8: Longitudinal velocity profiles upstream and at the hill top, SSG model.

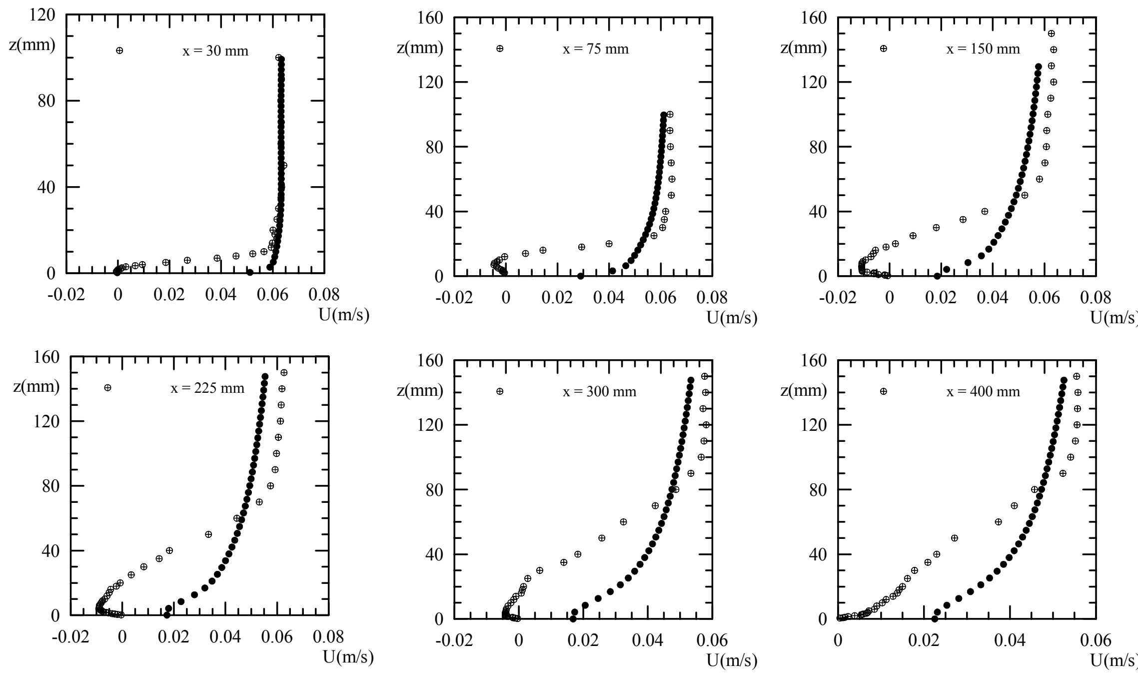

Figure 9: Longitudinal velocity profiles in the recirculation region, SSG model.

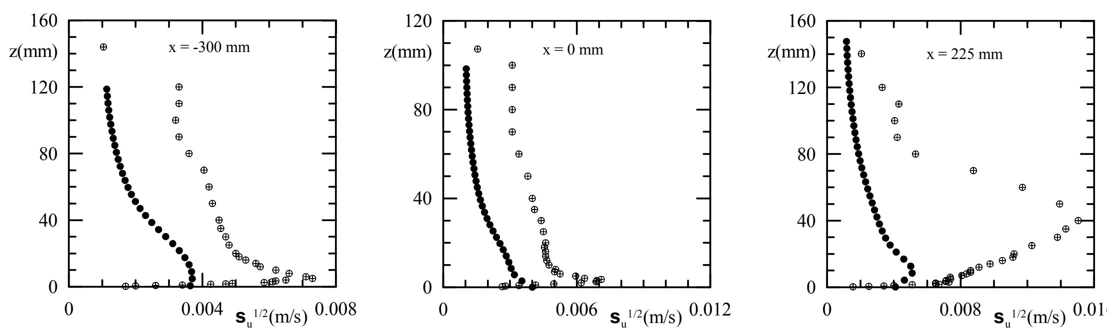

Figure 10: Representative turbulence velocity profiles, SSG model.

6. Final remarks

The present work has striven to evaluate the capacity of widely used computational fluid dynamics software, using different turbulence models, in simulating complex atmospheric flows. This work has presented numerical simulations for a neutrally stratified boundary layer over a steep hill, and the results were compared with original laboratory data. The computations were carried out using two different eddy viscosity and three shear stress models.

The computations allow for the conclusion that the best overall performance, among the five models tested, can be achieved only with the k-ω based SST model and the BSL Reynolds stress model. Both of these turbulence models were capable of predicting the large velocity gradients which occur in the upstream region, as well as the separation region downstream of the top. However, some inaccuracies were found when the location of the reattachment point is concerned.

The k-ε, the SSG and the LLR models, were found to be unable of resolving the sharp mean velocity gradients, and completely failed in predicting the occurrence of a recirculation region.

With regard to the turbulent field, all the evaluated models failed to some extent. None of closure schemes were capable of reflecting the real velocity fluctuating behaviour. Indeed, BSL Reynolds stress model showed agreement with the experimental data, especially for some profiles located inside the recirculation region. Apart from these locations, the fluctuations were highly underestimated for the upstream region and at the hill top.

The laboratory data used for numerical validation were conducted in a water channel environment using laser Doppler anemometry. This dataset provided detailed measurements of longitudinal and vertical mean velocities, along with its fluctuating components for the whole flow field over the hill, including the recirculation region. These features made this dataset an excellent test for the models' performance.

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8. References

Arya, S. P. S., Capuano, M. E. e Fagen, L. C.; "Some Fluid Modelling Studies of Flow and Dispersion Over Two-Dimensional Low Hills", Atmospheric Environment, vol. 21, 753-764, 1987.

Batchelor, G. K. and Proudman, I.: "The effect of rapid distortion of a fluid in turbulent motion", Q. J. Mech. Appplied Math., vol. VII, 83—103, 1954.

Belcher, S. E. e Hunt, J. C. R.; "Turbulent Flow over Hills and Waves", Annu. Rev. Fluid Mech., vol. 30, 507-538, 1998.

Britter, R. E., Hunt, J. C. R. e Richards, K. J.; "Airflow Over a Two-dimensional Hill: Studies of Velocity Speedup, Roughness Effects and Turbulence", Q. J. R. Meteorological Society, vol. 107, 91-10, 1981.

Castro, I. P. and Apley, D. D.: "Flow and dispersion over topography: a comparison between numerical and laboratory data for two-dimensional flows", Atmospheric Environment, vol. 31, 839-850, 1997.

Castro, F. A, Palma, J. M. L. M. and Silva Lopes, A.: "Simulation of the Askervein flow. Part 1: Reynolds averaged Navier-Stokes equations k-ε turbulence model", Boundary Layer Meteorology, vol. 107, 501-530, 2002.

Hewer, F. E.: "Non-linear numerical model predictions of flow over an isolated hill of moderate slope", Boundary Layer Meteorology, vol. 87, 381-408, 1998.

Hughes, T. J. R. and Brooks, A.: "A multi-dimensional upwind scheme with no crosswind diffusion", Element Methods for Convection Dominated Flows, ASME-AMD, vol. 34, 1979.

Hurley, P. J.: "An evaluation os several turbulence schemes for the prediction of mean and turbulent fields in complex terrain"', Boundary Layer Meteorology, vol. 83, 43—7, 1997.

Iizuka, S. and Kondo, H.: "Performance of various sub-grid scale models in large-eddy simulations of turbulent flow over complex terrain", Atmospheric Environment, vol. 38, 7083-7091, 2004.

Ishihara, T, Fujino, Y, and Hibi, K. "Wind tunnel study of separated flow over a two-dimensional ridge and a circular hill", J. Wind Eng, vol. 89, 573-576, 2001.

Jackson, P. S. and Hunt, J. C. R.; "Turbulent Wind Flow Over a Low Hill", Q. J. R. Meteorological Society, vol. 101, 929-955, 1975.

Kim, H. G. and Patel, V. C.: "Test of turbulence models for wind flow over terrain with separation and recirculation", Boundary Layer Meteorology, vol. 94, 5-21, 2000.

Kobayashi, M. H., Pereira, J. C. F. and Siqueira, M. B. B.: "Numerical study of the turbulent flow over and in a model forest on a 2D hill", J. Wind Engng. Ind. Aero., vol. 53, 357-374, 1994.

Launder B. E. and Spalding, D. B.: "The Numerical Computation of Turbulent Flows", Computer Methods in Applied Mechanics, vol. 3, 269-289, 1974.

Launder B. E., Reece, G. J. and Rodi, W.: "Progress in the developments of a Reynolds stress turbulence closure", J. Fluid Mechanics, vol. 68, 537-566, 1975.

Loureiro, J. B. R., Pinho, F. T. And Silva Freire, A. P.: "Experimental investigation of turbulent flow over a steep hill using laser-Doppler anemometry", 18th International Congress of Mechanical Engineering, November, Ouro Preto, MG, 2005.

Menter, F. R.: "Two-equation eddy-viscosity turbulence models for engineering applications", AIAA Journal, vol. 32, 8, 1994.

Ross, A. N., Arnold, S., Vosper, S. B., Mobbs, S. D., Nixon, N. and Robins, A. G.: "A comparison of wind-tunnel experiments and numerical simulations of neutral and stratified flow over a hill", Boundary Layer Meteorology, vol. 113, 427-459, 2004.

Speziale, C. G., Sakar, S. and Gatski, T. B.: "Modelling the pressure-strain correlation of turbulence: an invariant dynamical systems approach", J. Fluid Mechanics, vol. 227, 245-272, 1991.

Ying, R. and Canuto, V. M.: "Numerical simulation of the flow over two-dimensional hills using a second-order turbulence closure model", Boundary Layer Meteorology, vol. 85, 447-474, 1997.

Wang, C., Jang, Y. J. and Leschziner, M. A.: "Modelling two- and three-dimensional separation from curved surfaces with anisotropy-resolving turbulence closures", Int. J. Heat and Fluid Flows, vol. 25, 499-512, 2004.

Wilcox, D. C.: "Multiscale model for turbulent flows", AIAA 24th Aerospcae Sciences Meeting, 1986.

Zeman, O. and Jensen, N. O.: "Modification of turbulence characteristics in flow over hills", Q. J. R. Meteorologycal Society, vol. 113, 55-80, 1987.