A NOTICEABLE QUESTION OF THE WATER ENTRY PROBLEM: THE SPLIT OF KINETIC ENERGY DURING THE INITIAL STAGE

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Abstract. *This paper deals with a challenging question in the classical water entry problem. The kinetic energy distribution during the initial stage of the vertical impact of a rigid body onto the flat free surface of an ideal liquid, at constant velocity, is addressed. When considering the two-dimensional and axisymmetric cases or even specific three-dimensional cases whith elliptic contact lines between the body and the liquid (an elliptic paraboloid, for example), it has been proved that the kinetic energy of the entering body is equally transferred between the bulk of the liquid and the expelling jets. However, untill the present, even for the case of a constant velocity impact, there is no extension of this result to a generic form of the body. The aim of this work is to re-address the problem and to provide a formal extension for generic convex shapes, for the constant velocity case.*

Keywords: *water entry, energy distribution, free surface flow, inviscid hydrodynamics.*

1. Introduction

The initial stage of the water entry problem of blunt-bodies onto the water free surface is considered herein. This problem is a classical matter in fluid mechanics and also responsible for an important core of relevant problems in engineering. Interests in this field arose from the seaplane landing problem in the early thirties. Now the problem has received a special attention from naval architects and ocean engineers, concerned with the violent interaction between the water and offshore structures and ships. First theories of solid-body impact with a liquid were the penetration theory of Von Kármán (1929) and the wetted-surface correction of Wagner (1931). Fundamentals of these works have been largely used in the seeking of solutions of many applied problems. A good and comprehensive review of the water entry problem can be found in Korobkin and Pukhnachov (1988).

The simplest problem is the 'zero-gravity' initial stage of the impact of a rigid-body against the water free surface, in which the liquid is considered to be ideal and incompressible and the corresponding flow potential. We then have a nonlinear boundary-value problem for the velocity potential. However, even in this form, the problem is still too complicated for theoretical analysis. This is the reason why the application of different approximate models, using asymptotic methods and simplified assumptions, on both the flow pattern and the free surface elevation, have been thoroughly studied. A possible approach is to use the pressure-impulse concept, where the time-scale is supposed to be so small that both the free surface and the wetted portion of the body are supposed to be collapsed onto the horizontal plane. Besides that, convective terms are neglected in the Bernoulli equation. Although the problem turns then out to be linear, the boundary condition in the contact region changes from the Dirichlet's to the Neuman's type, provoking a singularity, in both pressure and velocity fields, to appear. These singularities are integrable and one can find, at first order, the impact force and the free surface elevation from the kinematic equation. Nevertheless, as pointed out by Korobkin and Peregrine (2000), "…the kinetic energy of the flow after the impact and the work done to oppose the pressure force calculated within the pressure-impulse approach do not correspond to each other". This suggests that, for modeling consistency under the incompressibility hypothesis, that the kinetic energy is, somehow, drained out of the bulk of the fluid.

This very interesting question has been extensively studied. According to Korobkin and Peregrine (2000), "it is expected that a correct description of nonlinear free surface motion may properly explain where the 'lost' energy goes." Pesce (2005) points out that "...the spray can be viewed as a *local relief* of a very large pressure field, developed in the neighborhood of the impacting body. This is the way to relieve energy, since the body is supposed rigid and the fluid compressibility has been taken as null". In fact, when carrying out an asymptotic analysis in the surroundings of the contact region, it has been shown that although the jets do not contribute at first order to the impact force calculation, they do contribute to an energy balance. This conclusion is enforced when it is shown for the two-dimensional case or for particular three-dimensional geometries that, for a constant downward velocity of the body, the kinetic energy is equally split between the bulk of the fluid and the jets. The works of Molin, Cointe and Fontaine (1996), Scolan and Korobkin (2003) and Cointe *et al*. (2004) show this result for the two-dimensional, three-dimensional and asymmetric cases, respectively. However, there is not yet any formal procedure that extends this result to an arbitrary and generic shape of the impacting body.

This work intends to show how this result can be extended to arbitrary and generic body shapes, though still restricted to the constant downward velocity case.

2. Mathematical formulation

For the sake of simplicity, consider first the normal impact of a rigid, strictly convex, smooth and symmetric body traveling vertically at varying speed *U*. Initially the liquid is at rest and occupies the lower half-space, $z < 0$. As shown in Fig.1, at $t = 0$ the solid-body touches the free surface at a single point, taken as the origin of the Cartesian coordinate system *Oxyz*. The liquid is assumed to be ideal and incompressible, and the corresponding flow to be potential. External mass forces and capillary effects are negligible and not taken into account. The body shape can be described by the function $f(\varepsilon\mathbf{x}) = 0$, where $f(0) = 0$ and $\varepsilon > 0$. The position of the body is given by $z = f(\varepsilon\mathbf{x}) - H$, where *H* is the penetration depth. The free surface elevation is given by the function $z = h(\mathbf{x}, t)$. At $t > 0$ the body starts to penetrate the liquid vertically as sketched in Fig. 2.

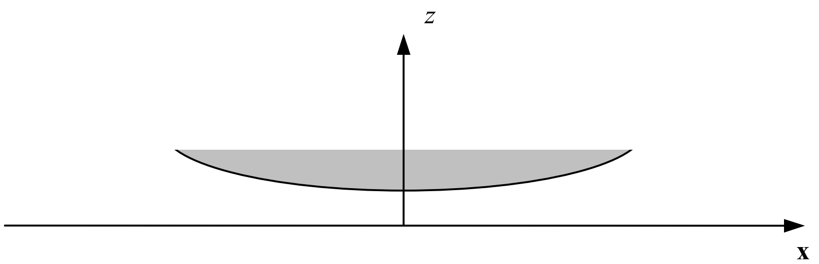

Figure 1: Illustration of the Cartesian coordinate system *Oxyz* and the impacting body at $t = 0$.

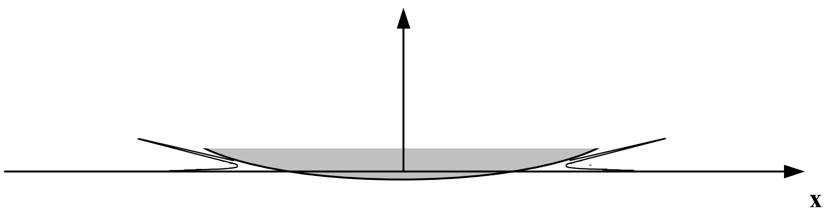

Figure 2: Simplified scheme of the flow at $t > 0$,for small values of penetration.

The formulation of the problem follows from the application of usual kinematic and dynamic conditions on the free surface, an evanescence condition for the velocity potential, impermeability of the body and incompressibility of the fluid. Using Euler variables, the problem can be formulated for the velocity potential $\phi(x, y, z, t)$ as follows

$$\Delta\phi = 0 \quad \text{in the fluid} \tag{1}$$

$$(\nabla\phi - \mathbf{U}) \cdot \mathbf{n} = 0 \quad \text{on} \quad z = f(\varepsilon\mathbf{x}) - H \tag{2}$$

$$\frac{\partial h}{\partial t} + \frac{\partial \phi}{\partial \mathbf{x}}\frac{\partial h}{\partial \mathbf{x}} - \frac{\partial \phi}{\partial z} = 0 \quad \text{on} \quad z = h(\mathbf{x}, t) \tag{3}$$

$$\frac{\partial \phi}{\partial t} + \tfrac{1}{2}\nabla\phi \cdot \nabla\phi = 0 \quad \text{on} \quad z = h(\mathbf{x}, t) \quad , \tag{4}$$

together with initial and far field conditions,

$$\phi(\mathbf{x}, z, 0) = 0, \; h(\mathbf{x}, 0) = 0 \tag{5}$$

$$\phi,\; h \to 0 \quad \text{as} \quad \|\mathbf{x}\| \to \infty \quad . \tag{6}$$

Here, **n** is the positive, outward, normal unit vector, external to the submerged body surface, and $\mathbf{U} = (0,0,-U)$ is the vertical velocity of the body.

2.1. The first order problem

The boundary-value problem, given by Eqs. (1) to (6), is quite difficult to be analyzed in the present form. Asymptotic solutions for small dead-rise angles, i.e. $\varepsilon \ll 1$, may be derived; see, e.g., Cointe and Armand (1987) or Faltinsen and Zhao (1997). For smooth and strictly convex geometries, taking a small dead-rise angle is equivalent to assume that the penetration depth L is much smaller than the typical radius of the body, at the point of impact.

At leading order, the flow field can then be written as that corresponding to the flow pasting an expanding flat-disc of radius $d(t)$ in an unbounded fluid as shown in Fig. 3.

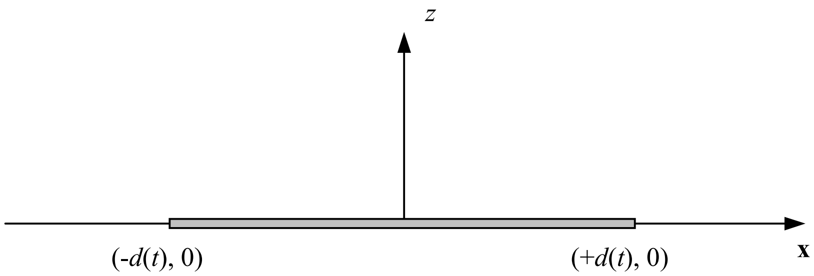

Figure 3: Corresponding flow over an expanding flat-disc.

The first order problem turns out to be

$$\Delta\phi = 0 \quad \text{in the fluid} \tag{7}$$

$$\frac{\partial\phi}{\partial z} = -U \quad \text{on} \quad z = 0,\ \|\mathbf{x}\| < d(t) \tag{8}$$

$$\frac{\partial h}{\partial t} = \frac{\partial\phi}{\partial z} \quad \text{on} \quad z = 0,\ \|\mathbf{x}\| > d(t) \tag{9}$$

$$\phi = 0 \quad \text{on} \quad z = 0,\ \|\mathbf{x}\| > d(t) \tag{10}$$

$$\phi(\mathbf{x}, z, 0) = 0,\ h(\mathbf{x}, 0) = 0 \tag{11}$$

$$\phi,\ h \to 0 \quad \text{as} \quad \|\mathbf{x}\| \to \infty \quad . \tag{12}$$

The free surface elevation can then be constructed by using the kinematic boundary condition (9) and the pressure field p determined on the wetted-surface of the body, via a proper application of Bernoulli equation. The point that should be emphasized is that, regarding the jets, no consideration has been done to obtain the pressure field on the body surface. In fact, the integration of the leading order pressure on the wetted-portion of the body gives the first-order impact force as shown, for example, in Cointe and Armand (1987).

3. On energy balance

In order to proceed with a global energy balance, it is interesting to consider a fluid domain partition according to Figs. 4 and 5. The whole domain, $\Omega(t)$, occupied by the liquid changes with time at any instant $t > 0$. The bulk of the fluid $\Omega_{bulk}(t)$ is defined, by excluding the jets, as enclosed by its boundaries: the 'free surface' S_F, the wetted-portion of the body S_B, the jet-root surface S_{JR} and the distant control surface S_∞. The jet Ω_J is enclosed by S_{JR} and by the jet external surface S_J. The whole free surface is therefore given by $S_F \cup S_J$. S_F and S_∞ are fixed-surfaces. S_{JR} is a material and permeable surface. S_B and S_J are material and non-permeable surfaces. S_F, given by $z = 0$, is not strictly speaking a free surface, but just a leading order approximation for the free surface, defined far enough from the intersection with the body, at the very initial stage of the fluid-body interaction.

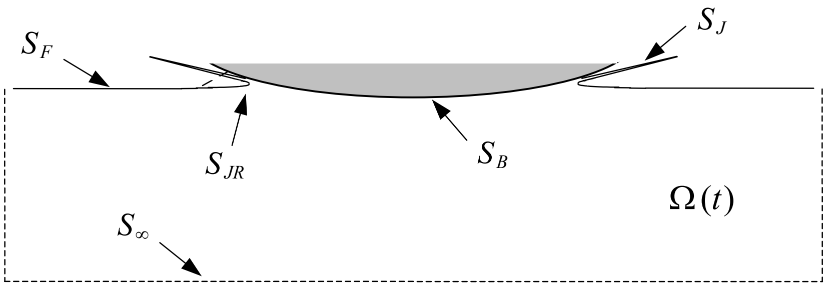

Figure 4: The whole volume of fluid in an arbitrary instant of time and its enclosure surfaces.

Figure 5: Jet root in detail.

According to Oliver (2003), the leading order pressure on the body, in the neighborhood of the jet-root, may be sketched as in Fig. 6.

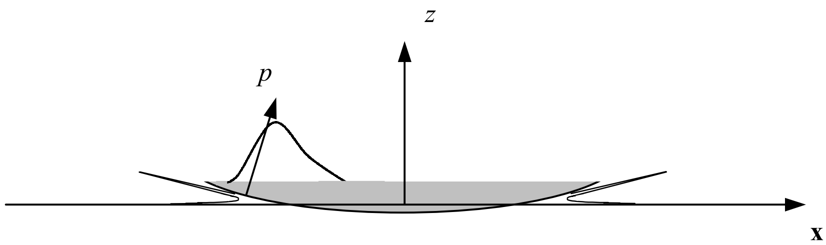

Figure 6: A scheme of the leading order pressure in the vicinity of the jet root.

Both, S_F and S_J are supposed to be surfaces at zero pressure. If the variation of pressure across (transversally) the jet is taken to be negligible, then S_{JR} may be said to be at zero pressure on the jet side, what means the whole jet volume Ω_J would be itself at zero pressure. However, approaching S_{JR} from the bulk of the fluid side, a very high pressure exists. As the flow has been taken as incompressible and, accordingly, pressure discontinuities have not been considered, the abrupt variation actually provokes a very high pressure-gradient through the jet-root, as shown in Fig. 6, explaining the jet formation. In this sense, the jet roots may be defined at the first point of zero pressure on the body.

Let u_J be the average normal velocity at the jet root, not to be confused with u_{JR}, defined as the normal velocity of the jet root surface, S_{JR}, itself. For a generic and arbitrary body shape, the problem formulated by Eqs. (7) to (12), now written only for the bulk of fluid, turns out to be

$$\Delta\phi = 0 \quad \text{in} \quad \Omega_{bulk}(t) \tag{13}$$

$$\frac{\partial\phi}{\partial z} = -U \quad \text{on} \quad S_B \tag{14}$$

$$\nabla\phi \cdot \mathbf{n} = u_J \quad \text{on} \quad S_{JR} \tag{15}$$

$$\frac{\partial h}{\partial t} = \frac{\partial \phi}{\partial z} \quad \text{on} \quad S_F \tag{16}$$

$$\phi = \frac{\partial \phi}{\partial t} = 0 \quad \text{on} \quad S_F \tag{17}$$

$$\phi(\mathbf{x}, z, 0) = 0,\ h(x, 0) = 0 \tag{18}$$

$$\phi,\ h \to 0 \quad \text{in the far field.} \tag{19}$$

The boundary condition (17) is correct to leading order on the 'free surface' S_F. On the other hand, the kinetic energy in the bulk of the fluid may be written as,

$$T_{bulk} = \frac{1}{2}\rho \iiint_{\Omega_{bulk}} (\nabla\phi)^2 \mathrm{d}v = \frac{1}{2} M_a U^2 \ , \tag{20}$$

where $M_a(z)$ is the added mass of the body, in the vertical direction, properly defined in the bulk of the fluid by just considering the wetted part of the body surface, up to the jet-root. Actually, the added mass is an explicit function of the penetration, $z(t)$ – which, in the Wagner approximation, turns to be represented by $d(t)$. This explicit dependence makes the classical impact problem a particular case of a more general class of problems in mechanics, treated under the Lagrangean formalism, via an extended Lagrange equation; see Pesce (2003).

The kinetic energy time rate is, therefore,

$$\frac{\mathrm{d}T_{bulk}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}\rho \iiint_{\Omega_{bulk}} (\nabla\phi)^2 \mathrm{d}V\right) \quad . \tag{21}$$

Using the transport theorem we find

$$\frac{\mathrm{d}T_{bulk}}{\mathrm{d}t} = \rho \iint_{S_B \cup S_{JR} \cup S_F \cup S_\infty} \frac{\partial \phi}{\partial t} \nabla\phi \cdot \mathbf{n} \mathrm{d}S + \frac{1}{2}\rho \iint_{S_B \cup S_{JR} \cup S_F \cup S_\infty} (\nabla\phi)^2 \mathbf{u} \cdot \mathbf{n} \mathrm{d}S \ . \tag{22}$$

Using the boundary conditions, Eq. (22) may be equivalently written as

$$\frac{\mathrm{d}T_{bulk}}{\mathrm{d}t} = -\iint_{S_B} p\mathbf{U} \cdot \mathbf{n} \mathrm{d}s + \iint_{S_{JR}} \frac{\partial \phi}{\partial t} u_J \mathrm{d}S + \frac{1}{2}\rho \iint_{S_{JR}} (\nabla\phi)^2 u_{JR} \mathrm{dS} \ . \tag{23}$$

By a simple algebraic manipulation

$$\frac{\mathrm{d}T_{bulk}}{\mathrm{d}t} = -\iint_{S_B} p\mathbf{U} \cdot \mathbf{n} \mathrm{d}S + \rho \iint_{S_{JR}} \left(\frac{\partial \phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2\right) u_{JR} \mathrm{d}S + \rho \iint_{S_{JR}} \frac{\partial \phi}{\partial t} (u_J - u_{JR}) \mathrm{d}S \ . \tag{24}$$

However, as pointed out before, the pressure has been considered to be zero on S_{RJ}. Therefore, from Bernoulli equation, Eq. (24) reads

$$\frac{\mathrm{d}T_{bulk}}{\mathrm{d}t} = -\iint_{S_B} p\mathbf{U} \cdot \mathbf{n} \mathrm{d}S - \rho \iint_{S_{JR}} \frac{1}{2} (\nabla\phi)^2 (u_J - u_{JR}) \mathrm{d}S \ . \tag{25}$$

The first term on the right hand side of Eq. (25) may be identified as the power of the pressure forces acted by the body on the bulk of the fluid. It may be written as; see Pesce (2003) for a discussion from the point of view of analytic mechanics,

$$-\iint_{S_B} p\mathbf{U}\cdot\mathbf{n}\mathrm{d}S = U\frac{\mathrm{d}}{\mathrm{d}t}(M_a U) \quad . \tag{26}$$

The opposite of the second term represents the flux of kinetic energy through the jets. From Eq. (25) and (20), we obtain

$$\rho\iint_{S_{JR}} \frac{1}{2}(\nabla\phi)^2(u_J - u_{JR})\mathrm{d}S = \frac{1}{2}U^2\frac{\mathrm{d}M_a}{\mathrm{d}t} . \tag{27}$$

Note that Eq. (27) is a general result – except for the fact that the impact has been considered vertical – since no restriction on the form of the body has been assumed. It also follows that,

$$\frac{\mathrm{d}T_{bulk}}{\mathrm{d}t} = \frac{1}{2}U^2\frac{\mathrm{d}M_a}{\mathrm{d}t} - UM_a\frac{dU}{dt} . \tag{28}$$

Nevertheless, if the downward velocity is considered to be constant, $U = U_0$,

$$\frac{\mathrm{d}T_{bulk}}{\mathrm{d}t} = \frac{1}{2}{U_0}^2\frac{\mathrm{d}M_a}{\mathrm{d}t} \quad . \tag{29}$$

Therefore, from Eq. (27), in the *constant downward velocity case*,

$$\frac{\mathrm{d}T_{bulk}}{\mathrm{d}t} = \rho\iint_{S_{JR}} \frac{1}{2}(\nabla\phi)^2(u_J - u_{JR})\mathrm{d}S . \tag{30}$$

In words, it can then be said that, in the case of constant velocity, the amount of kinetic energy drained through the jets becomes equal to that transmitted to the bulk of fluid, irrespective the shape of the body.

4. Conclusions

This work addressed the classic water entry problem. The first order problem formulation was derived under the hypothesis of incompressibility. To proceed with a global energy balance, the kinetic energy entering the jets has to be properly considered. In fact, according to Scolan and Korobkin (2003), a substantial part of the kinetic energy is transmitted to the jets. It has been shown that, at least for a constant velocity impact, but for an arbitrary and generic line of contact between the body and the liquid, the kinetic energy is equally transmitted between the jets and the bulk of the fluid.

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