Procedings of COBEM 2005
18th International Congress of Mechanical Engineering
November 6-11, 2005, Ouro Preto, MG

THREE-DIMENSIONAL NONLINEAR THEORY OF WATER IMPACT

A.A. Korobkin
Lavrentyev Institute of Hydrodynamics, Novosibirsk, Russia
kaa@hydro.nsc.ru

Abstract. *Original Wagner theory of water impact is generalized with the aim to account for both nonlinear effects and geometry of the impacting bodies to some extent. This has been done to improve predictions of the hydrodynamic forces acting on a body during its impact onto water surface and wave impact onto offshore structures. New model (MLM) of water impact has been developed. Within this model the wetted part of the structure and the distribution of the velocity potential in the wetted region are evaluated in the same way as in the classical Wagner theory but the pressure distribution in the wetted region is calculated by using the original nonlinear Bernoulli equation with approximate account for the impact geometry. Advantages of the present model are outlined for the ellipsoidal panels, which model yacht hulls, dropped down onto flat water surface. Experimental results for such truly 3D bodies were compared with the theoretical predictions of the body decelerations. It is shown that the new model essentially improves the classical Wagner approach and accurately predicts the hydrodynamic loads acting on structures during water impact. The new model does not require extensive numerical calculations and can be recommended for its practical use at the design stage.*

Keywords: *water impact, offshore structure, hydrodynamic loads, nonlinear effects, Wagner approach*

1. Introduction

In severe sea conditions the relative motion between a structure and waves can be very significant with high hydrodynamic loads localized in the impact regions. Such loads may lead to structure failure and should be taken into account in the equations of the structure motion. Impact loads acting on a moored floating structure may cause impulsive motion of the structure and breaking of the mooring lines.

Hydrodynamic loads acting on a structure during water impact are dependent on both the impact conditions and the structure shape. In some cases the loads can be obtained by using the strip method but in general case the truly three-dimensional calculations are required. It is well-known that impact loads obtained within the strip approach combined with the classical two-dimensional Wagner theory of water impact (see Wagner 1932) are greater than those predicted by three-dimensional Wagner theory. On the other hand, it is also well-known that the three-dimensional Wagner theory overpredicts the impact loads (Korobkin 2001) and account for nonlinear effects is necessary to obtain good agreement between the theoretical predictions and experimental data. Accurate model of slamming loads is required to formulate and to solve the problem on optimal design of the structure shape in a possible impact region. Optimal design of the offshore structure is still a complicated problem.

The Wagner theory of water impact is a main tool in engineering calculations. The two-dimensional Wagner theory is well developed at present for both rigid and elastic structures. The three-dimensional Wagner theory of water impact is still under development with several exact solutions and computer codes being available already. The Wagner theory is able to describe the main feature of the water impact, this is, the expansion of the wetted area at high rate during the initial stage of the structure-wave interaction and the high hydrodynamic loads on the structure associated with this high expansion rate. Wagner theory accounts for deformation of the water free surface during the impact and for the wetting correction due to such deformations. The wetted region of the impacted structure is unknown in advance and has to be determined as a part of the solution. The latter makes the Wagner model of impact to be nonlinear despite the fact that the boundary conditions within this model are linearised and imposed at a known surface, which is the initially undisturbed and flat water surface in the water entry problem.

The Wagner approach is formally valid during initial stage of a blunt body interaction with liquid, when the horizontal dimensions of the wetted region are much greater than the structure displacement (see Korobkin 1996). However, it has a capacity to be used also for bodies which are not blunt once the nonlinear effects are taken into account to some extent. The Wagner theory is also known as a "flat-disc approximation" because in water entry problems the wetted region of entering body is approximately substituted with an "equivalent" flat disc, which is the projection of the actual wetted region onto the initial flat position of the water surface. It is not easy to account for the shape of a structure in calculations of the impact loads but it is possible to do this approximately by modifying the Wagner approach. Also account for the nonlinear terms in the boundary conditions on the liquid free surface in the three-dimensional impact problem is a difficult task. We suggest to use the free-surface boundary conditions in the same form as in the original Wagner theory but calculate the hydrodynamic pressures by using the nonlinear Bernoulli equation.

The driving idea of the present approach is to keep the Wagner solution for the velocity potential but calculate the pressures, and the total impact force thereafter, with the help of the exact Bernoulli equation. This idea can be found in the paper by Wagner (1932), who also suggested an approximate method to account for the shape of entering body, but he did not recommend to use this complicated method in engineering problems. He specially designed a simplified approach known now as Original Wagner Theory (OWT), which correctly describes the main phenomena associated with water impact but overpredicts the hydrodynamic loads. Wagner argued that this overestimation is not a great problem for designers but simplicity of his formulae is very valuable in practical calculations.

Nowadays more accurate tools to predict the slamming loads are required by designers. CFD is considered as almost the only way to provide the required results. However, even now, when computer power is high enough to solve efficiently many complicated practical problems, three-dimensional calculations of slamming loads and pressure distributions are still a challenge for researchers. Moreover, it is getting more and more clear that computers cannot solve all problems and designers still need engineering tools, which are easy to handle and which provide reliable answers with appropriate accuracy as fast as possible.

This may well explain why about ten years ago approximate models of water impact started to be developed. Such models traditionally are based on the Wagner ideas, which have justified their reliability. Zhao et. al. (1996) developed so called Generalized Wagner Model, within which only the boundary conditions on the liquid free surface are simplified. This means that these boundary conditions are linearized and imposed on the horizontal line at the splash-up height, which is calculated by using the Wagner condition. The real shape of the body, the original body boundary conditions and the nonlinear Bernoulli equation are employed. This model was developed for two-dimensional case. Numerical results by Zhao et. al. (1996) agree well with experiments and numerical simulations of the impact problems for both the slamming force and the pressure distribution. Another model of impact was developed by Vorus (1996), who suggested to neglect the geometrical nonlinearity of the impact problem but account for the nonlinear terms in the boundary conditions. The non-linear boundary conditions are imposed on the initial level of the liquid. The nonlinear Bernoulli equation is used to evaluate the pressure on the entering body. Positions of the intersection points are determined from the condition that the hydrodynamic pressure is equal to zero at these points. Separation of the flow near the intersection points is taken into account. Numerical results obtained within the Vorus model by Xu (1998) are in good agreement with the experimental ones. Much simpler impact model was developed by Logvinovich (1969), who worked within the "flat-disc approximation" invented by Wagner (1932). Logvinovich recognized that the Wagner solution cannot be used close to the contact points, where the pressure takes its maximum value. The idea was to correct the velocity potential distribution given by Wagner, in order to account to some extent for some important features of the flow and pressure near the periphery of the wetted region. He argued that an additional term has to be added to the velocity potential distribution given by Wagner and the Wagner solution has to be used only in a part of the wetted area, where the nonlinear Bernoulli equation predicts positive hydrodynamic pressures. Logvinovich (1969) applied his model to both the wedge and cone entry problems, and showed good agreement of the theoretical predictions with experimental results. It is important to note that the Logvinovich model does not require extensive numerical calculations in contrast to the model by Zhao et. al. and the Vorus model. All three models can be used in two-dimensional and axisymmetric cases but their applicability to truly three-dimensional impact problems is not trivial.

The mentioned three models of water impact were analyzed by Korobkin (2005) in two-dimensional case. He stressed the point that these three models are very different but each of them essentially improves the prediction of the hydrodynamic loads. It was shown that, despite the fact that the Logvinovich model is the simplest one, it predicts the loads quite accurately compared with more complicated models and fully nonlinear calculations. In two-dimensional case Korobkin (2005) derived a new model, which is sightly more complex than that by Logvinovich (1969) but can be used for bodies with almost any deadrise angles. This new model was intensively tested against numerical results within the fully nonlinear potential flow theory, against the experimental results and was compared with commercial codes for slamming load calculations. Even for bodies with rather complex shapes the performance of this new model was found very acceptable for design purposes. This model is referred in the following as Modified Logvinovich Model (MLM) because in its final form it is close to the Logvinovich model but has much wider area of applicability, is based on different reasonings and has capacity to be extended to three-dimensional case.

In this paper the three-dimensional version of the MLM is presented and tested against the experimental results by Wraith (1998). In order to derive the model and to outline its main features, the problem of smooth three-dimensional body, which enters water free surface vertically at velocity varying with time, is considered. Moreover, the body is approximated by an elliptic paraboloid of finite weight and the body velocity is computed together with the hydrodynamic loads acting on it with the help of the second Newton's law. The new nonlinear model of water impact is applied to the case of drop tests performed by Wraith (1998). It is shown that the predictions by the new model are in a very good agreement with the experimental results. This agreement is rather impressive especially in comparison with the results obtained within the OWT by Korobkin (2001).

The three-dimensional unsteady problem of a smooth blunt rigid body entering an ideal and incompressible liquid is considered. Initially the liquid is at rest and occupies the lower half-space, $z < 0$. The body is initially above the liquid free surface, $z = 0$, with the lowest point of the body being at distance H_0 from the free surface. Then the body is released to fall down owing to the gravity. The body reaches the liquid surface at some instant of time, taken as initial one $(t = 0)$, and starts to penetrate the liquid vertically with its initial velocity being $V_0 = \sqrt{2gH_0}$, where g is acceleration due to gravity. Aerodynamic forces on the falling body are not taken into account. The body velocity drops due to the body interaction with the liquid. The body acceleration is maximal during an initial stage, when the penetration depth of the body is rather small. The hydrodynamic force on the body depends on both the body motion and the shape of its wetted part. At the initial stage the wetted part of the body is approximated by an elliptic paraboloid with r_x and r_y being the radii of the body curvature at the impact point. The axis Ox and Oy of the Cartesian coordinate system $Oxyz$ are chosen in such a way that $r_y \geq r_x$. It is assumed that external mass forces and surface tension give negligible contributions to the hydrodynamic force acting on the entering body. We shall determine the liquid flow and the body motion during the initial stage of penetration, to evaluate the hydrodynamic force acting on the entering body and to estimate the maximal deceleration of the body.

2. Hydrodynamic force acting on 3D body entering water

In this section we derive formula for the vertical component $F(t)$ of the hydrodynamic force acting on a three-dimensional body entering the lower half-space $z < 0$, which is occupied with ideal and incompressible liquid. The liquid flow caused by the body entry is assumed potential.

The position of the body is described by the equation $z = f(x, y) - h(t)$, where $h(t)$ is the penetration depth and the function $f(x, y)$ describes the body shape, $f(0, 0) = 0$ and $f(x, y) > 0$ as $x^2 + y^2 > 0$. The velocity potential $\varphi(x, y, z, t)$ of the flow caused by the entering contour satisfies the Laplace equation in the flow domain, the kinematic and dynamic boundary conditions on the liquid free surface and the body boundary condition on the entering contour. It is important to notice that the geometry of the flow region is unknown in advance and must be determined as a part of the solution. Only the initial stage of the impact is considered, when the penetration depth $h(t)$ is small compared with the horizontal dimension of the wetted part of the body $D(t)$. The latter is possible if and only if the deadrise angle of the body in the contact region is small.

The hydrodynamic force $F(t)$ acting on the entering body is given as

$$F(t) = \int\int_{D(t)} P(x, y, t)\mathrm{d}x\mathrm{d}y, \tag{1}$$

where $P(x, y, t) = p(x, y, f(x, y) - h(t), t)$ is the pressure distribution over the wetted part $D(t)$ of the blunt body during the initial stage, the hydrodynamic pressure $p(x, y, z, t)$ in the flow domain is given by the Cauchy-Lagrange integral

$$p(x, y, z, t) = -\rho\left(\varphi_t + \frac{1}{2}|\nabla\varphi|^2\right), \tag{2}$$

where ρ is the liquid density. Gravity and surface tension effects are not taken into account within the present theory. They are small during the initial stage of the impact, when the hydrodynamic force takes its maximum value. Note that the hydrodynamic force can be evaluated by using equation (1) if the pressure distribution $P(x, y, t)$ in the wetted region $D(t)$ and geometry of this region are known.

In order to evaluate the pressure distribution $P(x, y, t)$, we assume that both the velocity potential $\phi(x, y, t)$ in the wetted region, where $\phi(x, y, t) = \varphi(x, y, f(x, y) - h(t), t)$, and the geometry of this region are known. If so, equation (2) can be presented in the form

$$P(x, y, t) = -\rho\left(\phi_t + \frac{1}{2}|\nabla\phi|^2 - \frac{1}{2}\frac{(\nabla f \cdot \nabla\phi - \dot{h})^2}{1 + |\nabla f|^2}\right), \tag{3}$$

where the body boundary condition $\varphi_z = f_x\varphi_x + f_y\varphi_y - \dot{h}(t)$ has been taken into account, $\nabla\phi = (\phi_x, \phi_y)$ and $\nabla f = (f_x, f_y)$. Equation (3) is exact within the potential flow theory.

It should be noticed that the hydrodynamic force acting on the entering body is given by exact formulae (1) and (3), when the potential $\phi(x, y, t)$ in the contact region $D(t)$ and the geometry of this region are known. The idea of the present approach is to approximate the velocity potential $\varphi(x, y, z, t)$ and the contact region $D(t)$ with the help of those given by Wagner theory, which are $\varphi^{(w)}(x, y, z, t)$ and $D^{(w)}(t)$, respectively.

3. Velocity potential within Wagner theory

Formulation and analysis of the three-dimensional Wagner problem can be found in Scolan and Korobkin (2001). The Wagner formulation of the water entry problem is much simpler than the original formulation of fully nonlinear problem of potential flow with large deformations of the free surface. Within the Wagner theory the free-surface boundary conditions and the body boundary condition are linearized and imposed on the initial flat position of the water free surface, which is reasonable during the initial stage of the impact, when the penetration depth of the body is small. The velocity potential within the Wagner theory satisfies the following equations

$$\Delta\varphi^{(w)} = 0 \qquad (z < 0), \tag{4}$$

$$\varphi^{(w)} = 0 \qquad (z = 0, (x, y) \notin D^{(w)}(t)), \tag{5}$$

$$\varphi_z^{(w)} = -\dot{h}(t) \qquad (z = 0, (x, y) \in D^{(w)}(t)), \tag{6}$$

$$\varphi^{(w)} \to 0 \qquad (x^2 + y^2 + z^2 \to \infty), \tag{7}$$

where $D^{(w)}(t)$ is now a flat disc of unknown shape. The shape of the contact region has to be determined from an additional equation (Wagner equation), which implies that the shape of the entering body and the shape of the disturbed free surface match each other along the contact line $\partial D^{(w)}(t)$.

It is convenient to introduce a new unknown function $\psi^{(w)}(x, y, z, t)$ such that $\psi_t^{(w)} = \varphi^{(w)}$ and $\psi^{(w)} = 0$ at $t = 0$. According to its definition, the function $\psi^{(w)}(x, y, z, 0)$ is a displacement potential and its gradient $\nabla\psi^{(w)}$ is the displacement vector of liquid particles. The initial–value problem for the displacement potential

$$\Delta\psi^{(w)} = 0 \qquad (z < 0), \tag{8}$$

$$\psi^{(w)} = 0 \qquad (z = 0, (x, y) \notin D^{(w)}(t)), \tag{9}$$

$$\psi_z^{(w)} = f(x, y) - h(t) \qquad (z = 0, (x, y) \in D^{(w)}(t)), \tag{10}$$

$$\psi^{(w)} \to 0 \qquad (x^2 + y^2 + z^2 \to \infty) \tag{11}$$

is obtained by integrating equations (4) - (7) with respect to time (see Howison et al. (1991) for details) with account for the Wagner condition along the edge $\partial D^{(w)}(t)$ of the contact region. The Wagner condition leads to the requirement that the displacement potential $\psi^{(w)}(x, y, z, t)$ must be continuously differentiable up to the boundary, $z = 0$, of the flow region. In particular, $\psi^{(w)} \to 0$, $\psi_x^{(w)} \to 0$ and $\psi_y^{(w)} \to 0$ approaching the contact line $\partial D^{(w)}(t)$. In the latter form the Wagner condition is much easy to use in analysis than in its original form employed by Howison et al. (1991). Note that for any shape of the entering body the displacement potential $\psi^{(w)}(x, y, z, t)$ is dependent on the penetration depth h but not directly on time and it is independent on the previous history of the process. Correspondingly, equation $\psi_t^{(w)} = \varphi^{(w)}$ shows that the velocity potential $\varphi^{(w)}(x, y, 0, t)$ in the contact region can be presented as

$$\varphi^{(w)}(x, y, 0, t) = -\dot{h}(t)G(x, y, h), \tag{12}$$

where the function $G(x, y, h)$ is assumed known as the solution of the boundary-value problem (8) - (11).

The boundary-value problem (8) - (11) was solved analytically in the case of elliptic paraboloid entering water at a variable velocity (see Korobkin, 2002).

4. Impact by elliptic paraboloid

In the case of elliptic paraboloid entry the shape function $f(x, y)$ has the form

$$f(x, y) = \frac{x^2}{2r_x} + \frac{y^2}{2r_y}, \tag{13}$$

where r_x and r_y are radii of the body surface curvature at the impact point, $r_x \le r_y$. The eccentricity of the body cross sections, $\sqrt{1 - r_x/r_y}$, is denoted by ϵ. Elliptic paraboloid is a quite general shape, which can be used as an approximation of the actual shape of the impacting body.

In the paper by Korobkin (2002) it was shown that the contact line $\partial D^{(w)}(t)$ is the ellipse with minor semi-axis $a(t) = a_0\sqrt{r_x h(t)}$ and major semi-axis $b(t) = b_0\sqrt{r_x h(t)}$, where $a_0 = \sqrt{1 - e^2}b_0$, $b_0 = \sqrt{6/[2 - e^2 - \epsilon^2]}$ and e is the eccentricity of the ellipse $\partial D^{(w)}(t)$ defined as the root of the equation

$$\epsilon^2 = \frac{2(e^4 - e^2 + 1)E/K - (1 - e^2)(2 - e^2)}{(1 + e^2)E/K + e^2 - 1}, \tag{14}$$

$K(e)$ and $E(e)$ are elliptic integrals of first and second kind, respectively. Asymptotic formulae

$$e(\epsilon) \approx 0.2\epsilon(20 + 2\epsilon^2)^{\frac{1}{2}}, \qquad b_0(\epsilon) \approx 5\sqrt{6/(50 - 45\epsilon^2 - 2\epsilon^4)} \tag{15}$$

can be used for $0 \le \epsilon \le 0.5$ with the absolute errors less than 0.001.

The displacement potential obtained by Korobkin (2002) makes it possible to evaluate the function $G(x, y, h)$ in (12) as

$$G(x, y, h) = \frac{1}{E(e)}[a_0^2 r_x h - x^2 - (1 - e^2)y^2]^{\frac{1}{2}}. \tag{16}$$

Formula (16) will be used in approximate calculations of the pressure distribution.

5. Approximation of the pressure distribution in the wetted region

The basic idea of the present model is to approximate the velocity potential and the geometry of the wetted region as

$$\varphi(x,y,z,t) \approx \varphi^{(w)}(x,y,z,t), \qquad D(t) \approx D^{(w)}(t), \tag{17}$$

this is to assume that the water flow caused by the impact and the geometry of the wetted region are correctly described by the Wagner theory. By using the Taylor's expansion and (17), we obtain the velocity potential distribution in the wetted region $\phi(x,y,t)$ for small penetrations as

$$\phi(x,y,t) \approx \varphi^{(w)}(x,y,f(x,y)-h(t),t) \approx \varphi^{(w)}(x,y,0,t) + \varphi_z^{(w)}(x,y,0,t)[f(x,y)-h(t)]. \tag{18}$$

Equations (6) and (12) provide

$$\phi(x,y,t) \approx -\dot{h}(t)[G(x,y,h) + f(x,y) - h(t)], \tag{19}$$

which can be used for any three-dimensional blunt body entering water vertically. By substituting (19) into (3), we obtain the pressure distribution in the wetted region in the form

$$P(x,y,t) \approx \frac{1}{2}\rho\dot{h}^2\Big\{2G_h - |\nabla G|^2 + \frac{(\nabla G\cdot\nabla f)^2}{1+|\nabla f|^2} - 1\Big\} + \rho\ddot{h}[G(x,y,h) + f(x,y) - h(t)]. \tag{20}$$

In the case of elliptic paraboloid, it is convenient to introduce elliptical coordinates R and θ as

$$x = a_0\sqrt{r_x h}R\cos\theta, \quad y = b_0\sqrt{r_x h}R\sin\theta, \tag{21}$$

where $0 \le R \le 1$ and $0 \le \theta \le 2\pi$. Then the terms in (20) can be calculated by using the following formulae

$$G = \frac{a(t)}{E(e)}\sqrt{1-R^2}, \quad G_h = \frac{a(t)}{2h(t)}\frac{1}{E(e)\sqrt{1-R^2}}, \tag{22}$$

$$G_x = -\frac{1}{E(e)}\frac{R}{\sqrt{1-R^2}}\cos\theta, \quad G_y = -\frac{\sqrt{1-e^2}}{E(e)}\frac{R}{\sqrt{1-R^2}}\sin\theta. \tag{23}$$

By substituting (21)-(23) into (20), we obtain the formula for the nonlinear pressure distribution in the contact region

$$P(x,y,t) \approx \frac{1}{2}\rho\dot{h}^2\Big\{\frac{m_4(h)}{\sqrt{1-R^2}} - \frac{m_5(h,\theta)}{1-R^2} + \frac{m_6(h,\theta)}{m_7^2(h,\theta)+R^2} + m_8\Big\} + \rho\ddot{h}[m_9(h)\sqrt{1-R^2} + m_{10}R^2 - h\beta], \tag{24}$$

where

$$m_1(h,\theta) = \frac{a^2(t)}{r_x^2}\Big(\cos^2\theta + \frac{(1-\epsilon^2)^2}{1-e^2}\sin^2\theta\Big), \quad m_2(h,\theta) = \frac{a^2(t)}{E^2(e)r_x^2}(1-\epsilon^2\sin^2\theta)^2, \tag{25}$$

$$m_3(\theta) = \frac{1}{E^2(e)}(1-e^2\sin^2\theta), \quad m_{10}(h,\theta) = \frac{\beta a^2(t)}{2r_x}\Big(\cos^2\theta + \frac{1-\epsilon^2}{1-e^2}\sin^2\theta\Big), \tag{26}$$

$$m_4(h) = \frac{a(t)}{h(t)}, \quad m_5(h,\theta) = m_3(\theta) - \frac{\beta m_2(h,\theta)}{1+m_1(h,\theta)}, \quad m_6(h,\theta) = \frac{\beta m_2(h,\theta)}{m_1^2(1+m_1)}, \tag{27}$$

$$m_7(h,\theta) = \frac{1}{\sqrt{m_1}}, \quad m_8(h,\theta) = m_3 - \beta\frac{m_2}{m_1} - \alpha, \quad m_9(h) = \frac{a(t)}{E(e)}. \tag{28}$$

Note that in equations (24)-(28) two parameters, α and β, appear. These parameters are equal to unity, $\alpha = 1$ and $\beta = 1$, in the present model. If we limit ourselves with only the first term in (18), this is $\phi(x,y,t) \approx \varphi^{(w)}(x,y,0,t)$, then we arrive at a simplified model with $\alpha = -1$ and $\beta = 0$. The "flat-disc approximation" within the present model is obtained with $\alpha = 1$ and $\beta = 0$. In the following we consider only the so-called Modified Logvinovich Model with $\alpha = 1$ and $\beta = 1$.

It is important to notice that the first term in (24) is of the order of $O(h^{-\frac{1}{2}})$ and the second one is of the order $O(1)$ as $h \to 0$ and $R < 1$. However, the second term increases faster than the first one as $R \to 1$ and leads to negative pressure near the contact line because $m_5 > 0$ for small penetration depths h. This is a feature of all known nonlinear models of water impact (see Wagner (1932), Logvinovich (1969), Zhao et. al. (1996) and Vorus (1996)). Within these nonlinear models it is suggested to consider the hydrodynamic pressure only in that part of the contact region, where the pressure is positive. Correspondingly, in formula (1) for the hydrodynamic force the integration is suggested to perform only over a part of the contact region $D(t)$ with positive hydrodynamic pressure.

6. Numerical calculations of the hydrodynamic force

Equation (24) is convenient to find the region of positive pressure. This equation shows that the pressure is dependent on R, θ, h, $\dot{h}$ and $\ddot{h}$ but not on time t, this is $P(x, y, t) = \tilde{P}[R, \theta, h, \dot{h}, \ddot{h}]$. Analysis of equation (24) reveals that the pressure is negative where $R_0(\theta, h, \dot{h}, \ddot{h}) < R < 1$. The function R_0 is the root of the equation

$$\tilde{P}[R_0, \theta, h, \dot{h}, \ddot{h}] = 0, \tag{29}$$

which is solved numerically by the bisection method for each θ. Then equation (1) with account for equations (21) yields

$$F(t) = \frac{4a_0^2(e)r_x h}{\sqrt{1-e^2}} \int_0^{\frac{\pi}{2}} \mathrm{d}\theta \int_0^{R_0(\theta,h,\dot{h},\ddot{h})} \tilde{P}[R, \theta, h, \dot{h}, \ddot{h}]R\mathrm{d}R. \tag{30}$$

Equation (24) shows that the inner integral in (30) can be evaluated analytically and the integral with respect to θ can be evaluated numerically by using standard methods. However, having in mind generalization of the present model to arbitrary three-dimensional bodies, the calculations of the hydrodynamic force were also performed directly by using equation (1), where the pressure was calculated with the help of equation (20). In the latter equation we assumed that the functions $G(x, y, h)$, $f(x, y)$ and $h(t)$ are known together with their first derivatives. If so, we can calculate the pressure at any point of the contact region. In order to avoid integration of the negative pressures, equation (1) is suggested to be modified as

$$F(t) = \int\int_{D(t)} \max[0, P(x, y, t)]\mathrm{d}x\mathrm{d}y. \tag{31}$$

The double integral in (31) is calculated numerically by a standard routine. Formulae (30) and (31) provide identical results but formula (31) can be used for any shape of the entering body if the velocity potential in the contact region is known. This distribution of the velocity potential can be obtained numerically by using the formulation of the impact problem in terms of variational inequality (see Takagi, 2004 and Korobkin, 1982).

7. Deceleration of ellipsoidal panels in drop tests

Wraith (1998) performed experiments with ellipsoidal panels dropped onto flat water surface. The panel shape is described by equation (13), where $r_x = 0.375$m and $r_y = 0.5$m. Aspect ratio of the horizontal cross sections of the body is equal to 0.866. The panel mass M is equal to 13.75 kg and the drop height H_0 was equal to 11cm, 31cm, 61cm and 91cm. Impact velocity V_0 is calculated as $V_0 = \sqrt{2gH_0}$, where g is the gravity acceleration. In experiments deceleration of the panel entering water was measured.

In order to evaluate the panel deceleration, we use equation (30) or (31) for the hydrodynamic force and substitute it into the equation of the body motion after the impact

$$Mw = Mg - F(h, v, w), \tag{32}$$

where

$$\frac{dv}{dt} = w, \qquad v(0) = V_0, \tag{33}$$

$$\frac{dh}{dt} = v, \qquad h(0) = 0, \tag{34}$$

$v(t)$ is the velocity of the entering body and $w(t)$ is the body acceleration after the impact. Equations (33) and (34) are integrated numerically by the Runge-Kutta method of fourth order. Equation (32) is solved at each time step by iterations.

Comparisons of numerical and experimental results are shown in figures 1 and 2 for drop heights of 91cm and 61cm, when the classical Wagner theory essentially overpredicts the panel deceleration $|w(t)|$.

In the figures prediction of the deceleration by the present nonlinear model is marked with "mlm" and shown by red curved. Experimental results by Wraith (1998) are shown by green curves. Predictions by the classical Wagner theory are marked and shown by black curves. Note the time shift between experimental and numerical curves. The time shift is to "synchronize" the deceleration maxima. It is seen that the nonlinear model well simulates the body motion and the hydrodynamic loads acting on the entering body. Note the thin curve in figure 2 shown in blue. This curve has been obtained analytically with the help of equations (24)-(30), where $\alpha = 1$ and $\beta = 0$. Moreover, equation (29) was solved approximately assuming the penetration depth h small.

It is seen that the measured body accelerations just after the impact instant, $0 < t < 2$ms, are much smaller than the theoretical ones. This can be attributed either to air–cushion effect or acoustic one. New experiments at well–defined conditions and with transparent models are highly required to get clear ideas about the three-dimensional liquid flow and the body motion just after the impact.

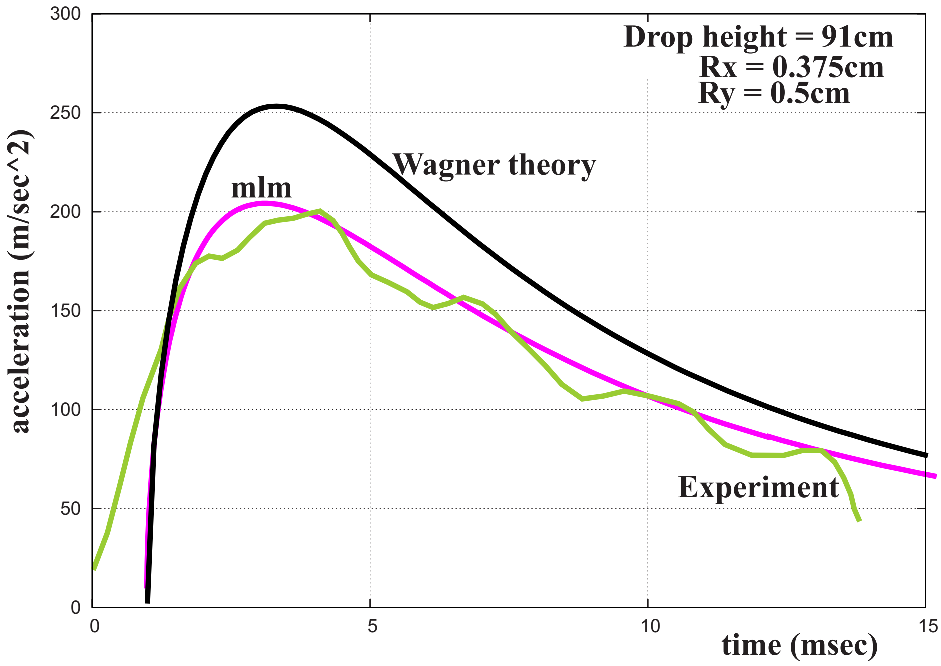

Figure 1. Deceleration of the ellipsoidal panel for the drop height of 91cm.

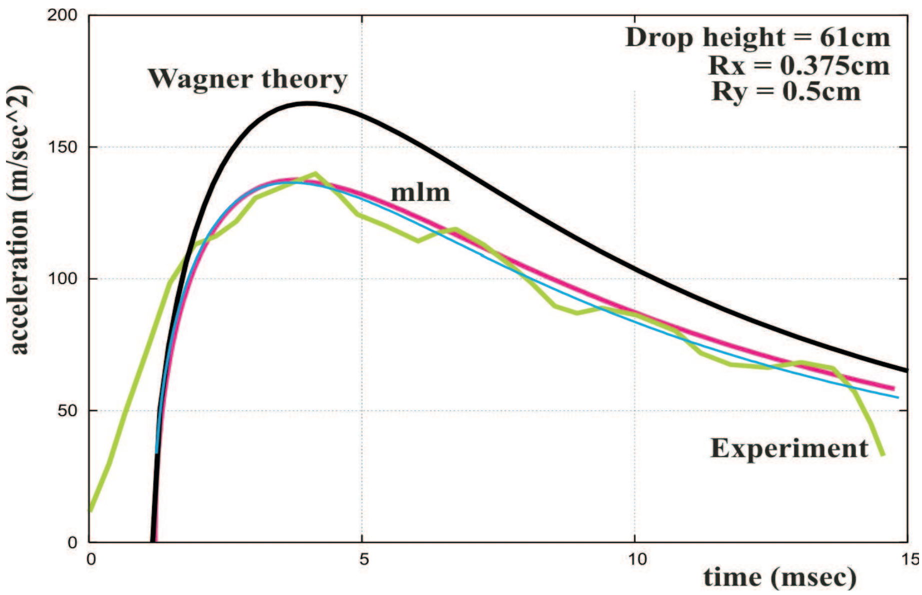

Figure 2. Deceleration of the ellipsoidal panel for the drop height of 61cm.

8. Conclusion

Calculations within the presented three-dimensional nonlinear model are organized as follows: (1) The Wagner problem of water impact is solved within the flat-disc approximation; (2) Taylor expansion is used to find the velocity potential distribution over the wetted part of the body surface; (3) Nonlinear Bernoulli equation is used to find the pressure distribution in the contact region; (4) The hydrodynamic pressure is integrated over the part of the contact region, where the pressure is positive, to evaluate the hydrodynamic force acting on the body.

If the body velocity after the impact must be evaluated as a part of the solution together with the water flow and the pressure distribution, iteration procedure is employed to find the body acceleration. To evaluate the body velocity and the penetration depth, time integration is performed by using the Runge-Kutta scheme of the forth order. Results of numerical calculations are compared with the experimental results by Wraith (1998), who performed drop tests with ellipsoidal panels. Wagner solution within the flat-disc approximation for such panels can be found in Korobkin (2001, 2002). It has been shown that MLM much better predicts the evolutions of the hydrodynamic force and the body acceleration than the original Wagner approach.

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10. Responsibility notice

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