# DESIGN AND SIMULATION OF A PIEZORESISTIVE PRESSURE MICROSENSOR

# Mariana A. Fraga

Universidade de São Paulo – USP Instituto Tecnológico de Aeronáutica - ITA maf@lsi.usp.br , mafraga@ita.br

#### Leandro L. Koberstein

Universidade de São Paulo - USP kobers@lsi.usp.br

Sidney F. da Luz Universidade de São Paulo - USP sfluz@lsi.usp.br

Luiz A. Rasia Universidade Regional do Noroeste do Rio Grande do Sul – UNIJUI rasia@tche.unijui.br

#### Humber Furlan

Faculdade de Tecnologia de São Paulo – FATEC-SP humber@lsi.usp.br

**Abstract.** This paper describes a method for design and simulation of a piezoresistive pressure sensor. The main feature involved in the design of a pressure sensor is the sensitivity. In this work, the sensitivity is related to the ratio between width and thickness of the diaphragm. The sensor was designed by an analytical solution The diaphragm was considered a thin plate and the piezoresistors are arranged in the Wheatstone bridge configuration. The device designed was analyzed by Finite Element Analysis (FEA) using the software ANSYS. Comparison of FEA and analytical results show a small variation. The paper also discuss the linearity of the sensor.

Keywords: piezoresistive effect, thin plate, ANSYS, linearity, sensitivity

# 1. Introduction

Piezoresistivity is a material property where the bulk resistivity is influenced by the mechanical stresses applied to the material (Sze, 1994). Monocrystaline silicon has a high piezoresistivity, combined with excellent mechanical properties, which makes it particularly suited for the conversion of mechanical deformation to an electrical signal. A piezoresistive pressure sensor consists of a thin monocrystaline silicon diaphragm supported by a thick silicon rim. The diaphragm is fabricated by anisotropic etching. The piezoresistive pressure sensor is the calculation of stress in a diaphragm as a function of applied pressure (Sze, 1994) (Fraga *et al*, 2004). The use of a thin rectangular diaphragm solves this because acts like a plate with small deflection.

The piezoresistive pressure sensor is a device, which can be used to measure static, pressure, or a pressure in moving fluids. There are three types of pressure measurements: (1) Absolute pressure, (2) Differential pressure and (3) Gauge pressure. The absolute pressure is measured relative to perfect vacuum. An example is atmospheric pressure. The differential is the difference in pressure between two points of measurement. Blood pressure is one example. Intake manifold vacuum in an automobile engine is an example of vacuum gauge measurement. Note that the same sensor can be used to measure all types of pressure (Singh *et al*, 2001).

# 2. Piezoresistive Effect

The piezoresistive effect is the change in resistivity of a material caused by the application of a stress. Therefore, can be described by relating each of the six fractional resistivity changes  $\Delta \rho_i / \rho$  to each of the six stress components (three normal stresses:  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$  and three shear stresses:  $\tau_1$ ,  $\tau_2$ ,  $\tau_3$ ). Mathematically this yields a matrix of 36 coefficients. By definition, the elements of this matrix are called piezoresistance coefficients,  $\pi_{ij}$ , expressed in Pa<sup>-1</sup>. For the cubic crystal structure of silicon, three different coefficients remain,  $\pi_{11}$ ,  $\pi_{12}$  and  $\pi_{44}$ , and the matrix takes the following form (Dally, 1978):

	$\Delta \rho_1$		$\pi_{11}$	$\pi_{12}$	$\pi_{12}$	0	0	0	$\left\lceil \sigma_{1} \right\rceil$
	$\Delta  ho_2$		$\pi_{12}$	$\pi_{11}$	$\pi_{12}$	0	0	0	$\sigma_2$
1	$\Delta  ho_3$	_	$\pi_{12}$	$\pi_{12}$	$\pi_{11}$	0	0	0	$\sigma_3$
ρ	$\Delta  ho_4$	-	0	0	0	$\pi_{44}$	0	0	$\tau_1$
	$\Delta  ho_5$		0	0	0	0	$\pi_{44}$	0	$ au_2$
	$\Delta  ho_6$		0	0	0	0	0	$\pi_{44}$	$\tau_3$

#### 3. Design of the diaphragm

The use of plate theory is appropriate for the design of thin diaphragms. Thin plate or small deflection theory is often used, and is appropriate for deflections less than 1/5 of the diaphragm thickness (Timoschenko and Woinoswky-Krieger, 2001). Maximum deflection, y, of a rectangular plate under a uniform applied pressure p is given by:

(1)

$$y = \frac{\alpha p b^4}{E t^3} \tag{2}$$

Where b and t are, respectively, the width and thickness of the plate, E is Young's modulus. The parameter  $\alpha$  is function of the boundary conditions applied at the edges of the plate and of the relation between length and width (a/b). The rectangular diaphragm acts like a plate with all edges fixed. Therefore,  $\alpha$  is given in Tab. 1(Roark).

Table 1. The parameter  $\alpha$  for rectangular plates with all edges fixed.

a/b	1.0	1.2	1.4	1.6	1.8	2.0	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
α	0.0138	0.0188	0.0226	0.0251	0.0267	0.0277	0.0284
β1	0.3078	0.3834	0.4356	0.4680	0.4872	0.4974	0.500

The objective is design a piezoresistive pressure sensor with square diaphragm, therefore  $\alpha$ =0.0138. The material was considered to be monocrystaline silicon with E=180GPa and v=0.28. The small deflection theory determine that:

$$y < \frac{t}{5} \tag{3}$$

Substituting the equation (2) in (3):

$$\left(\frac{b}{t}\right)^4 < \frac{E}{5\alpha p} \tag{4}$$

For  $p=1.01 \text{ X } 10^5 \text{ N/m}^2 (1 \text{ atm})$ :

$$\frac{b}{t} < 71\tag{5}$$

The maximum longitudinal stress is given by:

$$\sigma_l = \frac{\beta_1 p b^2}{t^2} \tag{6}$$

The transversal stress is given by:

$$\sigma_t = v\sigma_l \tag{7}$$

The Piezoresistive effect in p-type piezoresistor is defined by:

$$\frac{\Delta R}{R} = \frac{\pi_{44}}{2} \left( \sigma_l - \sigma_t \right) \tag{8}$$

Substituting (6) and (7) in (8):

$$\frac{\Delta R}{R} = \frac{\pi_{44}}{2} (1 - \upsilon) \sigma_l \tag{9}$$

For  $\pi_{44}$ =8 x 10<sup>-10</sup>m<sup>2</sup>/N (Rasia, 1997):

$$\frac{b}{t} = \sqrt{\frac{\frac{\Delta R}{R}}{8.9533 \times 10^{-6}}} \tag{10}$$

Substituting (10) in (5):

$$\frac{\Delta R}{R} < 0.045 \tag{11}$$

As can be noted, the expressions (5) and (11) define the maximum values for  $\left(\frac{b}{t}\right)$  and  $\frac{\Delta R}{R}$  respectively. Figure 1 was obtained by the expression (10) that shows  $\frac{\Delta R}{R}$  as function of  $\left(\frac{b}{t}\right)$ .



Figure 1. The change of resistance as a function of (b/t).

The relation  $\frac{\Delta R}{R}$  should be on the order of 0.01 (1%). For this reason, was selected  $\left(\frac{b}{t}\right) = 40$  that presents  $\frac{\Delta R}{R} = 0.0143$  (Fig. 1).

The sensor designed presents square diaphragm with  $a=400\mu m$ ,  $b=400\mu m$  and  $t=10\mu m$ , respectively, length, width and thickness. Figure 2 shows the dimensions of the sensor.



Figure 2. Dimensions of the Piezoresistive Pressure Microsensor designed.

The anisotropic wet chemical etching will be used for fabrication of the sensor. This process determine that the angle between the planes (111) and (001) is 54.73° (Elwenspoek and Jansen, 1998).

#### 3.1 Modeling

The design of the Piezoresistive Pressure Sensor was analyzed by Finite Element Analysis (FEA) using the ANSYS software. Shell63, a structural shell element with isotropic properties, was used to simulate the diaphragm. Solid45, a brick element with 8 nodes, was used to model the support structure of the sensor. Figure 3 shows the modeling in ANSYS.



Figure 3. Modeling of the sensor.

Von Misses stress distribution was calculated by the FEA as shows Figure 4. Note that stress is maximum at the edge and minimum at the center of the diaphragm. The knowledge of this distribution is a design guideline for location of the piezoresistors. In the Wheatstone bridge configuration this is achieved placing two piezoresistors parallel to opposite edges of the diaphragm and the other two perpendicular to the other two edges. The analytical solution was compared to finite element results (Tab. 2).



Figure 4. Von Misses Stress Distribution.

Table 2- Comparison Analytical and FEA results.

Results	Analytical Solution	ANSYS	Error (%)
Maximum Deflection, y	0.1982 µm	0.201 µm	1.42
Maximum Longitudinal stress, $\sigma_1$	0.49740 x 10 <sup>8</sup> N/m <sup>2</sup>	0.49391 x 10 <sup>8</sup> N/m <sup>2</sup>	0.7
$\Delta R/R$	0.015	0.014	6

# 4. Design of the Piezoresistors

The expression that defines electrical resistance is (Sze, 1994):

$$R = R_s \frac{l}{w}$$
(12)

Where l and w are, respectively, length and width of the resistor. The sheet resistance ( $R_s$ ) is given by:

$$R_s = \frac{\rho}{d_p} \tag{13}$$

Where  $\rho$  is the a resistivity of the material and  $d_p$  is the deep of the piezoresistor determined by the designer. P-type piezoresistors, with piezoresistance coefficient ( $\pi_{44}$ ) of  $8 \times 10^{-10} \text{m}^2/\text{N}$ , present a resistivity of 0.01  $\Omega$  –cm. For  $d_p=1 \mu \text{m}$  has that  $\text{R}_{\text{s}}=100 \Omega/\text{square}$ . Figure 5 shows the dimensions and position of the piezoresistors.



Figure 5. Position and dimensions of the piezoresistors on the diaphragm.



Figure 6. Longitudinal Stress along the length of the piezoresistor.

Figure 6 shows that yhe longitudinal stress change along the length of the piezoresistor. Therefore, the change of the effective resistance due outside pressure is given by (Fraga *et al*, 2005):

$$\frac{\Delta R}{R} = \frac{\pi_{44}}{2} (1 - \upsilon) \sigma_{lm} \tag{14}$$

Were  $\sigma_{lm}$  medium stress along the length of the piezoresistor.

#### 4.1. Sensitivity and Linearity

The sensitivity of a piezoresistive pressure sensor is defined as:

$$S = \frac{FSO}{\Delta P}$$
(15)

Where FSO (Full Scale Output) is given by:

$$FSO = \frac{\Delta V}{V_s} \tag{16}$$

 $V_s$  is supply voltage,  $\Delta V$  is output voltage and  $\Delta P$  is the maximum applied pressure. For a four-arm Wheatstone bridge,  $\Delta V/V_s$  is related to the fractional change in resistance as follows:

$$\frac{\Delta V}{V_s} = \frac{\Delta R}{R} \tag{17}$$

For  $V_s=5V$  the FSO is showed in Fig. 7.



Figure 7. FSO vs. applied pressure

The Nonlinearity can be calculated by (Liwei and Chu, 1999):

$$NL = \frac{R(P) - \{ \left[ R(P_{ref}) - R(0) \right] P / P_{ref} + R(0) \}}{R(P_{ref}) - R(0)}$$
(18)

Where R(0) is the resistance at P=0,  $R(P) = R(0) + \Delta R(P)$ ,  $R(P_{ref})$  is the resistance when P=101KPa (design pressure). Figure 8 shows the nonlinearity of the designed sensor as function of the applied pressure.



Figure 8 .Nonlinearity vs. applied pressure.

# 5. Conclusions

In this work was presented a methodology for design and simulation of piezoresistive sensor. The sensor contains only the transduction part. It does not include the processing circuitry. Comparison of FEA and analytical results show a small variation. The sensor designed present good sensitivity and linearity. In future works we will present experimental results of the sensor.

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