Looking for optimal topologies in 2D potential problems with genetic algorithms and BEM

Anflor, C.T.M.
Universidade Federal do Rio Grande do Sul
Mechanical Engineer Department - GMAP-PROMEC
Rua Sarmento Leite 425
Porto Alegre – RS – Brasil – 90430-131
anflorgoulart@yahoo.com.br

Cisilino, A.P.
Universidad Nacional de Mar del Plata
Institution Welding and Fracture Division – INTEMA - CONICET
Av. Juan B. Justo 4302
(7600) Mar del Plata, Argentina
cisilino@fi.mdp.edu.ar

Marczak, R.J.
Universidade Federal do Rio Grande do Sul
Mechanical Engineer Department - GMAP-PROMEC
Rua Sarmento Leite 425
Porto Alegre – RS – Brasil – 90430-131
anflorgoulart@yahoo.com.br

Abstract. *This work is concerned with the use of genetic algorithms (GA) applied to the search of optimal topologies in potential problems using boundary element methods (BEM). The main objective is to investigate how a domain initially filled with holes evolve during an optimization process and, more importantly, to verify how the final shapes compare with other available solutions. The paper summarizes the basis of GA applied to shape optimization along with the BEM. The external boundary of the problem remains fixed while the positions and dimensions of the cavities within the domain are optimized in order obtain prescribed values of temperature and/ or flux on selected portions of the boundary. Two approaches are followed for the localization of the cavities: Circular holes of arbitrary radii and general convex cavities defined by straight segments. The GA parameters used in the former one are the center coordinates and the radii of the holes, while the vertex coordinates of the cavities are used as optimization parameters in the second one. The performance of the proposed algorithm is discussed trough a member of elementary numerical examples.*

Keywords: *genetic algorithms, shape optimization, potential problems, boundary elements*

1. Introduction

Studies have been published in last years on boundary methods elements (Brebbia and Dominguez, 1977; Wessel and Cisilino, 2003; Comino and Gallego, 2003) and genetic algorithms (Goldberg, 1989; Cerrolaza and Annicchiarico, 2000; Katsifarakis *et al.*, 1999; Mera *et al.* 2004) all these investigations were applied under different combinations to solve a proposed problem.

The main objective of this work is to investigate an optimal topology search procedure based on genetic algorithms as an alternative to the well known homogenization techniques. Since GA sample the whole objective function, it minimizes the chance of selecting local minimum solutions. The choice of the BEM as the solution method is related to its excellent accuracy and boundary-only discretization. Therefore, the BEM+GA combination has potential to solve optimization problems accurately.

The basic idea is to start with a domain initially filled with a number of holes, and to investigate how the holes change their shape and position during an optimization process. In situations where the initial number of holes coincides with the one of the optimal topology, the procedure becomes a simple shape optimization process. However, when a large number of holes pre-exist, it is believed that they should iteratively concentrate around those empty areas predicted by the topology optimization. Then a proper algorithm could be used to remove the unnecessary material.

These ideas are preliminarily explored herein for 2D Poisson equation problems. The external boundary of the problem remains fixed during the process, while the cavities within domain are optimized in order to minimize the error to the analytical solution.

2.The Boundary Element Method

The BEM, for two-dimensional potential problems is very well established. In what follows only a brief description of the method is given. Further details can be found in Brebbia, C. A. and Dominguez (1992) and Banerjee (1994). Equation (1.1) presents the boundary integral equation which relates the potential u and flux q over the boundary Γ , in absence of body sources,

$$\frac{1}{2}u^i(x) + \int_\Gamma u(x)q^*(x,x')d\Gamma = \int_\Gamma q(x)u^*(x,x')d\Gamma \tag{1.1}$$

The functions u* and q* are the so-called potential and flux fundamental solutions at x due to a unit source applied at x'.

$$u^* = \frac{1}{2p}\int_\Gamma \ln\left(\frac{1}{r}\right)d\Gamma \qquad r = \| x - x` \| \tag{1.2}$$

The next step consists in discretizating the problem boundary Γ using N linear boundary elements, see Figure 1.

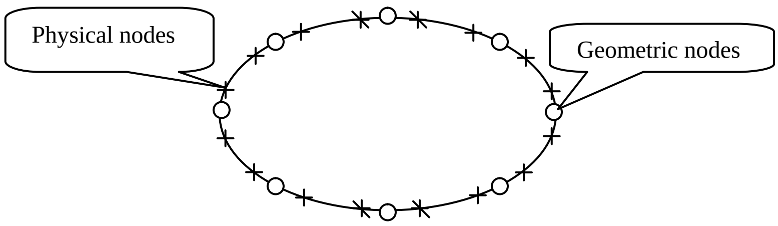

Figure 1 – Boundary element discretization

The values of u and q at any point on a element can be written in terms of the nodal values and the two interpolation functions ϕ_1 and ϕ_2:

$$u(z) = f_1 u_1 + f_2 u_2 = [f_1\ f_2]\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$
$$q(z) = f_1 q_1 + f_2 q_2 = [f_1\ f_2]\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \tag{1.3}$$

where ξ is a local intrinsic coordinate defined in the range [–1,+1] and ϕ_1 and ϕ_2 are the standard discontinuous linear shape functions (Brebbia and Dominguez, 1992). Considering the discretized version of equation (1.1), the integral in the left hand side over the element j can be written as,

$$\int_{\Gamma j} uq^* d\Gamma = \int_{\Gamma j} [f_1\ f_2]\, q^* d\Gamma \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = [h_1^{ij}\ h_2^{ij}]\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \tag{1.4}$$

where for each element j we have the two terms,

$$h_1^{ij} = \int_{\Gamma j} f_1 q^* d\Gamma$$
$$h_2^{ij} = \int_{\Gamma j} f_2 q^* d\Gamma \tag{1.5}$$

Similarly, the integral on the right hand side results:

$$\int_{\Gamma j} qu^* d\Gamma = \int_{\Gamma j} [f_1\ f_2]\, u^* d\Gamma \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = [g_1^{ij}\ g_2^{ij}]\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \tag{1.6}$$

where

$$g_1^{ij} = \int_{\Gamma j} f_1 u^* d\Gamma$$
$$g_2^{ij} = \int_{\Gamma j} f_2 u^* d\Gamma \tag{1.7}$$

After the substitution of equation 1.7 and Equation 1.5 for all the "j" elements in the discretized counterpart of Equation 1.1 results:

$$c^i u^i + \sum_{j=1}^{N} H^{ij} u^j = \sum_{j=1}^{2N} G^{ij} q^j \tag{1.8}$$

After the imposition of all boundary conditions, the system in Eq.(1.8) can be reordered in such a way that all the unknowns are taken to the left hand side, resulting the following system of equations:

$$[A]\{X\} = \{F\} \tag{1.9}$$

3. Implementation

Genetic algorithms are an efficient optimization tool (Katsifarakis *et al.*, 1999), especially when the objective function has many local minimums. The method aims to imitate a biological process based on the evolution of species. The concepts of genetic evolution can be easily applied in topology optimization. GA begins with an initial, randomly chosen population of chromosomes. In the present context, each chromosome contains the information about the problem topology (discrete coordinates and size of the cavities), and it is numerically represented by a string of bits. The fitness of each member of the population (i.e. their aptitude to satisfy the prescribed boundary conditions) is evaluated in this work by means of BEM models.

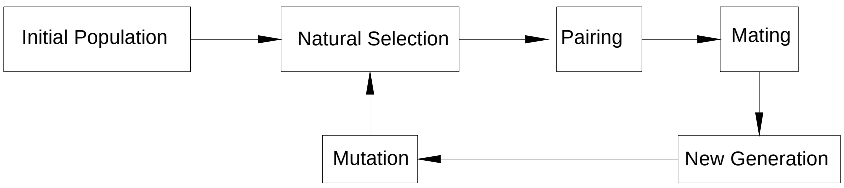

Figure 2. Generic GA Scheme

Next, a new population is produced by operators which imitate biological processes, and are applied to the chromosomes of the previous generation. The principal genetic operators (Fig. 2) are (Goldberg, 1989): a) natural selection; b) pairing; c) mating; and d) mutation. Other operators have been proposed in the literature (Mera *et al.*, 2004). In the present work, the selection operator will discard those topologies with the highest cost function values (i.e. those topologies that are worst fitted to fulfill the objective function). The paring operator defines pairs using the best fitted individuals (ie those topologies best fitted to fulfill the objective function). Mating is the next operator, and it consists in creating one or more "offsprings" (new topologies) from the "parents" that were previously selected during the pairing process. This work adopted a single crossover mating operator (Cerrolaza et al., 2000). This means that every pair of parents produces two offsprings, and both will be members of the next generation. The crossover point is determined by a percentage of parent chromosomes that the offprint inherits. Table 1 presents a good demonstration of a pairing. Note that the first offsprings will receive a part (underline) of father's chromosome and another part of mother's chromosome (not underline). The same procedures occurs for the second offspring, but with that portions that was not used to form the first offspring .

Genetic code	Individuals
0 0 1 0 0 1 1 0 1 0	Dad – topological structure 1
1 1 0 1 0 1 0 0 1 1	Mom – topological structure 2
0 0 1 0 0 1 1 0 1 1	Child 1 – new topological structure 3
1 0 0 1 0 1 0 0 1 1	Child 2 – new topological structure 4

Table 1. Crossover Point

The last operator, the stochastic mutations, randomly alters a small portion of the chromosomes. This operator avoids the algorithm of getting trapped in a local minimum. The above described process is repeated until the maximum number of (prescribed) iterations is attained or the convergence is achieved. Figure 2 presents a scheme of the complete algorithm including the BEM and GA subroutines. In many applications, the optimization process is subjected to constraints. In this port two types of constrains are applied: the cavities can not intersect neither each other nor the external boundary. Those chromosomes (topologies) that do not satisfy the constraints are excluded from the analysis. This is done by penalization, i.e. by assigning to the excluded chromosomes a high cost value. In this way, the chromosome will be automatically eliminated by the selection operator.

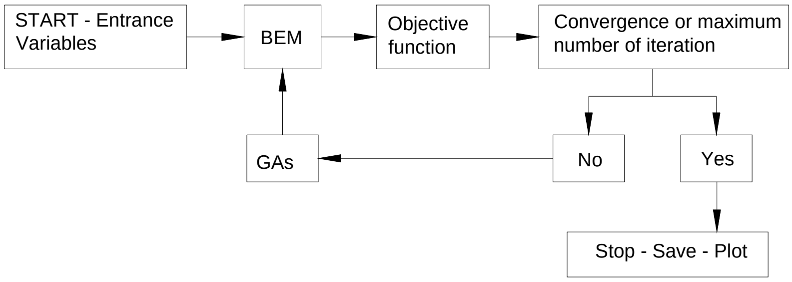

Figure 2. Routine Scheme, AG plus BEM.

4. Numerical examples

In this section simple examples are presented showing applications of the proposed formulation. Since these are preliminary tests, all cases refer to steady state heat transfer. The objective function for the first two examples is to maximize the flux in the internal points previously defined. For the third and fourth examples, the obvious solution for the optimal topology corresponds to the geometry of a thick pipe, whose analytical solution is well known. Therefore, the solution error along the boundary can be used as the cost function to be minimized.

4.1 Example 1

In this example a square plate (6mx6m) with conductivity $k=236W/m^2$ and boundary conditions as illustrated in Figure 4 is considered. Two temperatures are prescribed in opposite corners of the plate, one with 100°C and the other with 0°C. The goal of the optimization procedure is to find the optimum position for six cavities in the plate in order to maximize the flux in the center of the plate. Parameters for the GA algorithm are given in Tab. 3.

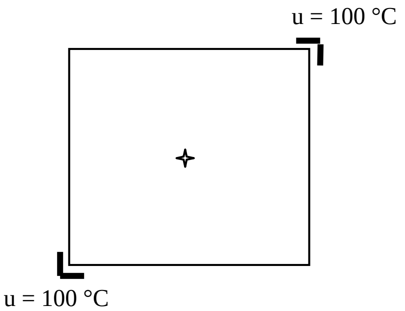

Figure 4. Boundary conditions for example 2

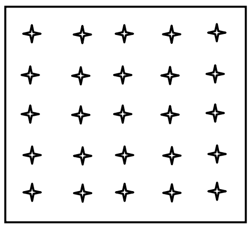

Figure 4b. Admissible hole locations

Table 2 - Genetic parameters for example 2

Genetic parameters	
Population size	16
Number of generations	50
Crossover probability	0.3
Mutation probability	0.03

Figure 4b for examples 1 and 2 and shows all possible positions to locate the cavities inside the domain. Figure 5 shows the evolution of the cost function while Fig. 6 shows the maps of temperature and gradient for the final result (obtained after 49 generations). To calculate this maps was used internal points inside the domain, what explains square holes depicted in the Fig. 6. It is clear to see that the optimum distribution of the cavities is symmetrical with respect to the plate diagonal with the prescribed temperatures. In this way, the effect of the cavities on the heat transfer along in the diagonal direction is maximized.

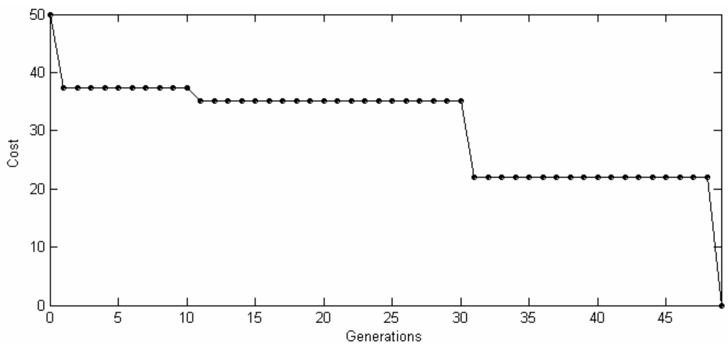

Figure 5. Cost function convergence to example 1.

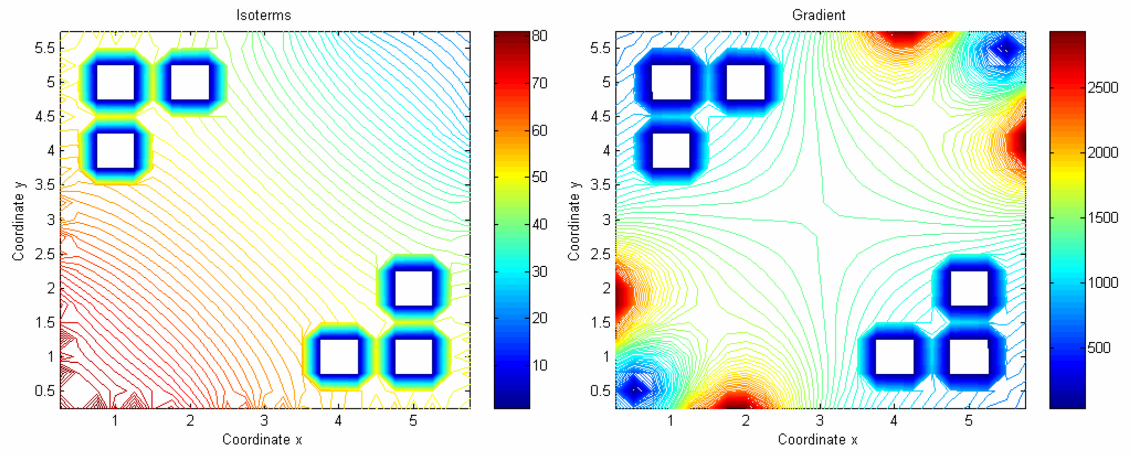

Figure 6. Temperature and gradient maps for example 1.

4.2 Example 2

The geometry of this example is the same to the previous one, but using the boundary conditions illustrated in Fig. 7. The objective is to maximize flux in the three points close to the top edge of the plate (see Fig. 7) by optimizing the position of six circular cavities. Parameters for the GA algorithm are given in Tab. 3.

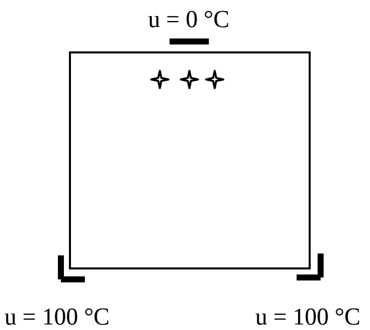

Figure 7. Boundary conditions to example 1.

Table 3. Genetic parameters for example 3.

Genetic parameters	
Population size	32
Number of generations	50
Crossover	0.3
Mutation probability	0.03

The evolution of the normalized cost function are presented in Fig. 8. It can bee seen that the cost function does not longer improve after approximately 60 generations. The resulting topology is illustrated in Fig. 9. Also shown in Fig. 9 are the maps with the distribution for the temperature and the gradient, obtained at internal points, which explains square holes depicted in the figure. The cavities have adopted a nearly symmetric configuration and they are located in zones with the minimum gradient.

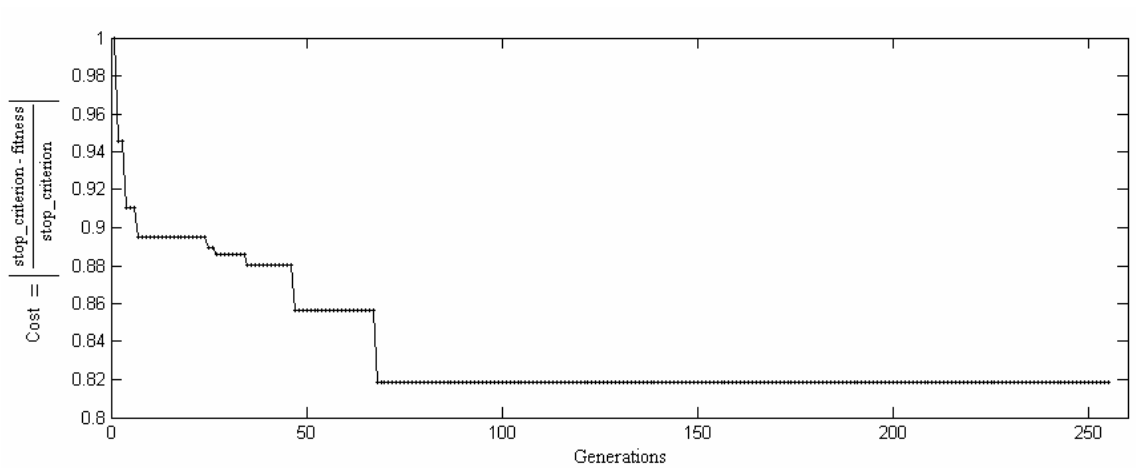

Figure 8. Cost Function convergence for example 3.

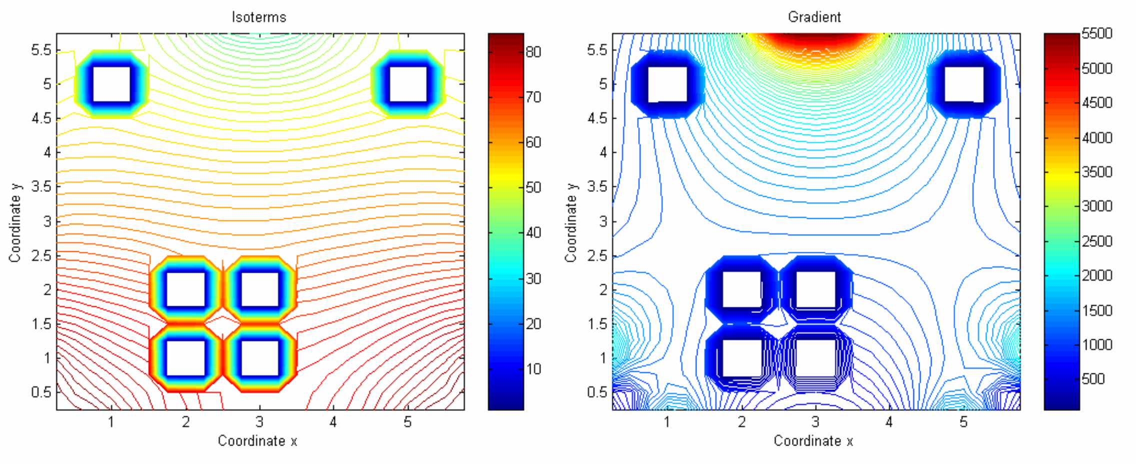

Figure 9 – Temperature and gradient maps for example 3.

4.3 Example 3:

This example consists of a circular domain with r=32 m and three circular holes. The external temperature of the circular domain is set to 0 °C, while the temperature in the holes is 100 °C. The design variables are the position and the radii of the holes. The conductivity is set to k=1 W/m^2. The objective of the optimization is to find the position of the holes in order to generating a resulting flux qa = 14.0817 W/m2, which corresponds to the analytical solution of a hollow cylinder with internal radius ri=3 m. Parameters for the GA algorithm are given in Tab. 4. In addition, a restriction was imposed to the minimum size of the holes in order to avoid numerical problems. Figure 11 illustrates the resulting topology for some generations. The evolution of the cost function is depicted in Fig. 12. The solution for the generation 50 is given by a main hole of radius 3m and centered at (35,26). This hole is almost coincident to that used to compute the objective function. The second hole has radius 1m (the minimum size allowed) and it is located at (28,20). The third hole with the same radius is located at (25,26). The flux attained for the generation 104 is 13.37902 W/m^2, being very close to that calculated in the analytical solution.

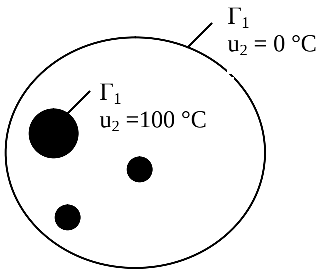

Table 4. Genetic parameters for example 4.

Genetic parameters	
Population size	16
Number of generations	50
Crossover probability	0.2
Mutation probability	0.05

Figure 10. Illustration of example 4.

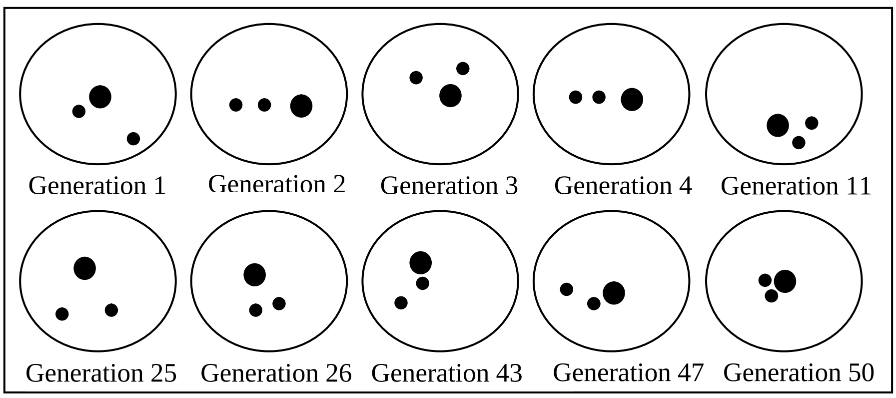

Figure 11. Intermediary solutions of example 3.

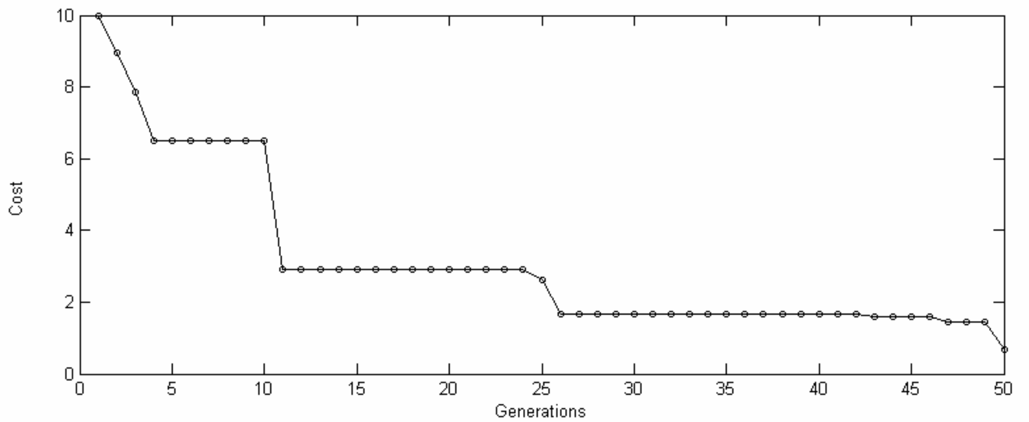

Figure 12. Cost Function evolution for example 3.

4.4 Example 4

This example deals with the optimal shape and location of convex cavities defined by straight segments inside the domain (Fig. 8). Again, the reference solution used was the heat transfer in a thick pipe, but the internal and external radii were set to 15 and 32 cm, respectively. The conductivity was set to k=1 W/m^2. Sixteen linear elements were used in the external mesh. The internal mesh was automatically generated, by dividing each edge in a suitable number of elements. The objective function was the same of the previous examples, and an initial population of 64 individuals was used. Other GA variables are shown in Tab. 4.

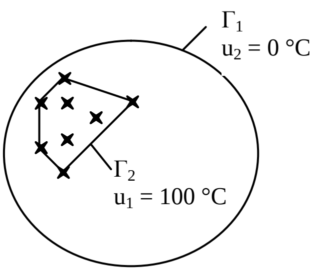

Figure 13. Illustration of example 4.

Table 5. Genetic parameters for example 4.

Population size	64
Number of generations	100
Crossover probability	0.4
Mutation probability	0.05

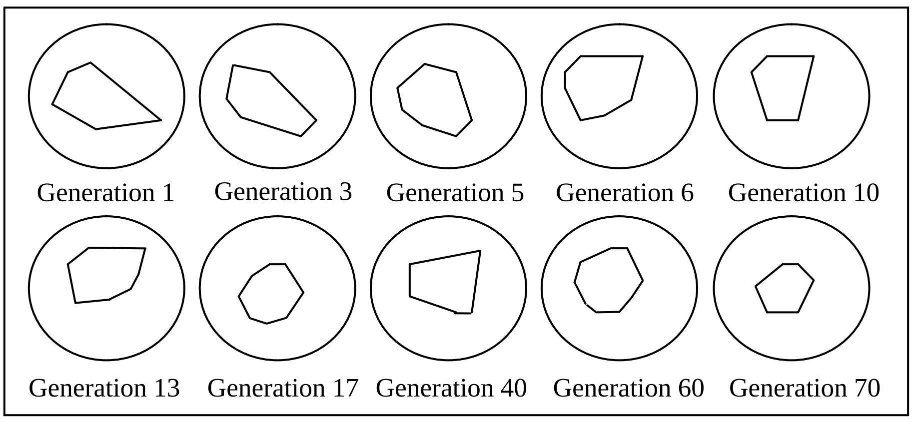

Figure 14 – Intermediary solution of example 4.

Figure 14 show the evolution of the optimization process. The shape generated when the process was halted (generation 70) is not an exact circle, in spite of the good cost function convergence showed (Fig.15).

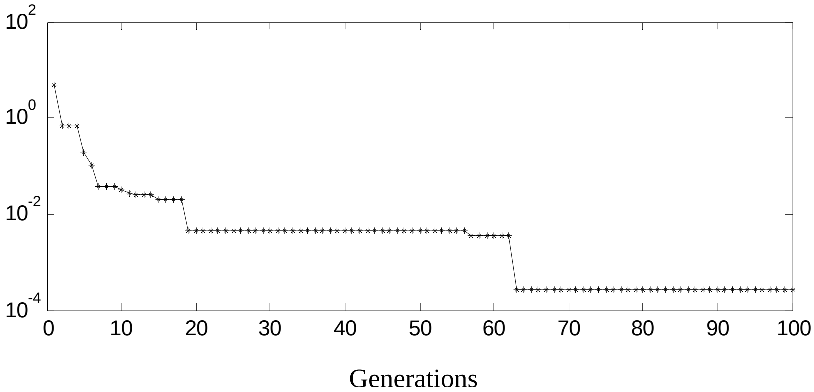

Figure 15. Cost Function evolution for example 4.

5.Concluding Remarks

One of the goals of the present BEM+GA approach consists in finding the optimum positions and dimensions of circular cavities in order to extremize flux based objective functions at selected positions in the domain or portions of the boundary. The other goal is to employ the use of the convex coordinates inside the domain to generate the best possible set of cavities. This technique showed satisfactory results regarding topology optimization indicating that it has potential to be further developed. It is important to point out that GA is not a deterministic routine, and therefore it enables to find different solution configurations for the same problem. The use of the BEM presents an attractive advantage over other numerical techniques, since it does not require the discretization of the domain. It is also worth to point that the computational cost is an important issue in any optimization procedure. All examples presented in this paper showed good performance and accuracy, but the computational cost depends directly on the chromosome's size, suggesting a serious drawback of the present approach, particularly for problems with a large number of design variables. In these cases, the use of parallel computing techniques seems to be mandatory.

6. References

Banerjee, P. K., 1994, "The Boundary Element Methods in Engineering", Ed. McGraw-Hill, New York, United States.

Brebbia, C. and Dominguez, J., 1977, "Boundary Elements for Potential Problems", AMM - Appl. Math. Modelling Vol.1, pp.372-378.

Brebbia, C. A. and Dominguez, J., 1992, "Boundary Elements an Introductory Course", Ed. McGraw-Hill, New York, United States.

Cerrolaza, W. , Annicchiarico, W., Martinez, M., 2000, "Optimization of 2D boundary element models using b-splines and genetic algorithms", EABM - Engineering Analyses with boundary Elements", Vol. 24, pp.427-440.

Comino, L., Gallego, R., 2003, "Shape Sensitivity of the Anisotropic Elastic Response", The 4th International Conference on Boundary Element Techniques, Granada, pp. 165-170.

Goldberg, D.E., 1989, "Genetic Algorithms in search optimization and machine learning Reading", Ed. Addison-Wesley, Massachusetts, United States.

Katsifarakis, K.L., Karpouzos, D.K, Theodossiou, N., 1999, "Combined use of BEM and genetic algorithms in groundwater flow and mass transport problems", EABE - Engineering Analysis with Boundary Elements, Vol. 23, pp. 555–565.

Mera, N.S., Elliott, L., Ingham, D.B, 2004. "Numerical solution of a boundary detection problem using genetic algorithms", EABM - Engineering Analysis with Boundary Elements, Vol. 28, pp.405–411.

C. Wessel, A. Cisilino, B. Sensale, 2003, "BEM Implementation of The Biological Growth Method For Structural Shape Optimisation", The 4th International Conference on Boundary Element Techniques, pp.170-176.

8. Responsibility notice

The authors are the only responsible for the printed material included in this paper.