NONLINEAR DYNAMICS OF A SHAPE MEMORY OSCILLATOR USING A CONSTITUTIVE MODEL WITH INTERNAL CONSTRAINTS

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Abstract *The dynamical response of systems with shape memory actuators presents intrinsically nonlinear characteristics and a rich behavior. Different applications are exploiting the characteristics of the SMA dynamical response. In general, one can summarize these characteristics into two groups: Dissipation characteristics related to hysteresis loops and property changes caused by phase transformations. This contribution deals with the nonlinear dynamics of shape memory systems where the restitution force is described by a constitutive model with internal constraints. An iterative numerical procedure based on the operator split technique, the orthogonal projection algorithm and the fourth order Runge-Kutta method is developed to deal with nonlinearities in formulation. Numerical investigation is carried out showing some characteristics of SMA dynamical response.*

Keywords: *Shape memory alloys, chaos, nonlinear dynamics***.**

1. Introduction

The remarkable properties of shape memory alloys (SMAs) are attracting much technological interest in several fields of sciences and engineering, varying from medical to aerospace applications. Machado & Savi (2002, 2003) make a review on the most relevant SMA applications within orthodontics and biomedical areas. Engineering applications are also extensive. They are ideally suited for use as fastener, seals, connectors and clamps (van Humbeeck, 1999). Self-actuating fastener, thermally actuator switches. Moreover, aerospace technology is also exploiting SMA properties in order to buit self-erectable structures, stabilizing mechanisms, solar batteries, non-explosive release devices and other possibilities (Denoyer *et al.*, 2000). Micromanipulators and robotics actuators have been built employing SMAs properties to mimic the smooth motions of human muscles (Garner *et al.*, 2001; Webb *et al.*, 1999; Rogers, 1995). Furthermore, SMAs are being used as actuators for vibration and buckling control of flexible structures (Birman, 1997; Rogers, 1995).

The dynamical response of systems with shape memory actuators presents intrinsically nonlinear characteristics and a rich behavior, being previously addressed in different references (Seelecke, 2002; Ghandi & Chapuis, 2002; Collet *et al.*, 2001; Salichs *et al.*, 2001; Saadat *et al.*, 2002; Schmidt & Lammering, 2004; Williams *et al.*, 2002; Feng & Li, 1996; Mosley & Mavroidis, 2001; Lagoudas *et al.*, 2004). The main drawback of SMAs is their slow rate of change, related to temperature variation. Different applications are exploiting the SMA dynamical response characteristics. In general, one can summarize these characteristics into two groups: Dissipation characteristics related to hysteresis loops and property changes caused by phase transformations. The dissipation characteristics may be exploited as an adaptive passive control employed in bridges and civil structures subjected to earthquakes, for example (Williams *et al.*, 2002; Salichs *et al.*, 2001; Saadat *et al.*, 2002). The property changes due to phase transformations, on the other hand, may exploit either forces or displacements generated by this phenomenon. On the basis of dynamical responses, Savi & Braga (1993a,b) had discussed the chaotic behavior in shape memory oscillators where the restitution force is provided by shape memory helical springs. Machado *et al.* (2004) discuss bifurcation and crises in a shape memory oscillator. Savi & Pacheco (2002) had studied some characteristic of shape memory oscillators with one and two-degree of freedom, showing the existence of chaos and hyperchaos in these systems. Machado *et al.* (2003) revisited the analysis of coupled shape memory oscillators, considering two-degree of freedom oscillators. Savi *et al.* (2002a) analyze a shape memory two-bar truss, showing a very rich response. Recently, some experimental analysis confirms the chaotic behavior of shape memory systems (Mosley & Mavroidis, 2001).

This contribution deals with the nonlinear dynamics of shape memory systems where the restitution force is described by a constitutive model with internal constraints (Paiva *et al.*, 2005). The proposed model is based on the Fremond's theory (Fremond, 1987, 1996), later modified by Savi *et al.* (2002) and Baêta-Neves *et al.* (2004). The proposed model includes four macroscopic phases in the formulation (three variants of martensite and an austenitic phase) considering different properties to each phase. An iterative numerical procedure based on the operator split technique (Ortiz *et al.*, 1983), the orthogonal projection algorithm (Savi *et al.*, 2002) and the fourth order Runge-Kutta method is developed to deal with nonlinearities in the formulation. Numerical investigation is carried out showing some characteristics of SMA dynamical response.

2. Constitutive Model

There are different ways to describe the thermomechanical behavior of SMAs. Here, a constitutive model that is built upon the Fremond's model and previously presented in different references (Savi *et al.*, 2002, Baêta-Neves *et al.*, 2004, Paiva *et al.*, 2005) is employed. This model considers different material properties and four macroscopic phases for the description of the SMA behavior. The model also considers plastic strains and plastic-phase transformation coupling, which turns possible the two-way shape memory effect description. Moreover, tension-compression asymmetry is taken into account.

Besides elastic strain (ε_e) and temperature (T), the model considers four more state variables associated with the volumetric fraction of each phase: β_1 is associated with tensile detwinned martensite, β_2 is related to compressive detwinned martensite, β_3 represents austenite and β_4 corresponds to twinned martensite. A free energy potential is proposed concerning each isolated phase. After this definition, a free energy of the mixture can be written weighting each energy function with its volumetric fraction. With this assumption, it is possible to obtain a complete set of constitutive equations that describes the thermomechanical behavior of SMAs as presented bellow:

$$\sigma = E\varepsilon + \left(\alpha^C + E\alpha_h^C\right)\beta_2 - \left(\alpha^T + E\alpha_h^T{}_1\right)\beta_1 - \Omega\left(T - T_0\right) \tag{1}$$

$$\begin{aligned}\dot{\beta}_1 = \frac{1}{\eta_1}\Big\{ & \alpha^T\varepsilon + \Lambda_1 + \beta_2\left(\alpha_h^C\alpha^T + \alpha_h^T\alpha^C + E\alpha_h^T\alpha_h^C\right) - \beta_1\left(2\alpha_h^T\alpha^T + E\alpha_h^{T^2}\right) + \\ & + \alpha_h^T\left[E\,\varepsilon - \Omega\left(T - T_0\right)\right] - \partial_1 J_\pi \Big\} + \partial_1 J_\chi\end{aligned} \tag{2}$$

$$\begin{aligned}\dot{\beta}_2 = \frac{1}{\eta_2}\Big\{ & -\alpha^C\varepsilon + \Lambda_2 + \beta_1\left(\alpha_h^T\alpha^C + \alpha_h^C\alpha^T + E\alpha_h^C\alpha_h^T\right) - \\ & - \beta_2\left(2\alpha_h^C\alpha^C + E\alpha_h^{C^2}\right) - \alpha_h^C\left[E\,\varepsilon - \Omega\left(T - T_0\right)\right] - \partial_2 J_\pi \Big\} + \partial_2 J_\chi\end{aligned} \tag{3}$$

$$\begin{aligned}\dot{\beta}_3 = \frac{1}{\eta_3}\Big\{ & -\frac{1}{2}\left(E_A - E_M\right)\left(\varepsilon + \alpha_h^C\beta_2 - \alpha_h^T\beta_1\right)^2 + \Lambda_3 + \\ & + \left(\Omega_A - \Omega_M\right)\left(T - T_0\right)\left(\varepsilon + \alpha_h^C\beta_2 - \alpha_h^T\beta_1\right) - \partial_3 J_\pi \Big\} + \partial_3 J_\chi\end{aligned} \tag{4}$$

where $E = E_M + \beta_3\left(E_A - E_M\right)$ is the elastic modulus while $\Omega = \Omega_M + \beta_3\left(\Omega_A - \Omega_M\right)$ is related to the thermal expansion coefficient. Notice that subscript "*A*" refers to austenitic phase, while "*M*" refers to martensite. Besides, different properties are assumed to consider tension-compression asymmetry, where the superscript "*T*" refers to tensile while "*C*" is related to compressive properties. Moreover, parameters $\Lambda_1 = \Lambda_1(T)$, $\Lambda_2 = \Lambda_2(T)$ and $\Lambda_3 = \Lambda_3(T)$ are associated with phase transformations stress levels. Parameter α_h is introduced in order to define the horizontal width of the stress-strain hysteresis loop, while α helps vertical hysteresis loop control on stress-strain diagrams.

The terms $\partial_n J_\pi$ (n = 1,2,3) are sub-differentials of the indicator function J_π with respect to β_n (Rockafellar, 1970). The indicator function $J_\pi\left(\beta_1, \beta_2, \beta_3\right)$ is related to a convex set π, which provides the internal constraints related to the phases' coexistence. With respect to evolution equations of volumetric fractions, η_1, η_2 and η_3 represent the internal dissipation related to phase transformations. Moreover $\partial_n J_\chi$ (n = 1,2,3) are sub-differentials of the indicator function J_χ with respect to $\dot{\beta}_n$ (Rockafellar, 1970). This indicator function is associated with the convex set χ, which establishes conditions for the correct description of internal sub-loops due to incomplete phase transformations and also avoids phase transformations $M+ \Rightarrow M$ or $M- \Rightarrow M$.

Concerning the parameters definition, linear temperature dependent relations are adopted for Λ_1, Λ_2 and Λ_3 as follows:

$$\Lambda_1 = -L_0^T + \frac{L^T}{T_M}(T - T_M) \qquad \Lambda_2 = -L_0^C + \frac{L^C}{T_M}(T - T_M) \qquad \Lambda_3 = -L_0^A + \frac{L^A}{T_M}(T - T_M) \tag{5}$$

Here, T_M is the temperature below which the martensitic phase becomes stable. Besides, L_0^T, L^T, L_0^C, L^C, L_0^A and L^A are parameters related to critical stress for phase transformation, remembering that the indexes "*T*" refers to tensile, "*C*" to compression and "*A*" to austenite.

In order to contemplate different characteristics of the kinetics of phase transformation for loading and unloading processes, it is possible to consider different values to the parameter η_n (n = 1,2,3), which is related to internal dissipation: η_n^L and η_n^U during loading and unloading process, respectively. For more details about the constitutive model, see Paiva *et al.* (2005).

3. Shape Memory Oscillator

Consider a single-degree of freedom oscillator, which consists of a mass m attached to a shape memory element of length L and cross-section area A. A linear viscous damper, associated with a parameter c, is also considered (Figure 1). The system is harmonically excited by a force $F=F_0 \sin(\omega t)$.

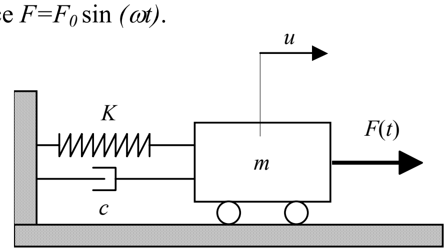

Figure 1. Shape Memory Oscillator.

With these assumptions, equation of motion may be formulated by considering the balance of linear momentum, assuming that the restitution force is provided by a SMA element described by the constitutive equation presented in the previous section. Therefore, the following equation of motion is obtained,

$$m\ddot{u} + c\dot{u} + K(u) = F_0 \sin(\omega t) \tag{6}$$

Notice that the restitution force may be expressed as $K(u) = \sigma A$. Using the constitutive equation for SMAs, one writes,

$$m\ddot{u} + c\dot{u} + EA\varepsilon + (A\alpha^C + EA\alpha_h^C)\beta_2 - (A\alpha^T + EA\alpha_h^T)\beta_1 - \Omega A(T - T_0) = F_0 \sin(\omega t) \tag{7}$$

In order to obtain a dimensionless equation of motion, system's parameters are defined as follows,

$$\omega_0^2 = \frac{E_R A}{mL};\ \bar{\xi} = \frac{c}{m\omega_0}\ ;\quad \bar{\alpha}^{C,T} = \frac{\alpha^{C,T} A}{mL\omega_0^2} = \frac{\alpha^{C,T}}{E_R};\quad \bar{\alpha}_h^{C,T} = \frac{\alpha_h^{C,T} E_R A}{mL\omega_0^2} = \alpha_h^{C,T}\ ;$$
$$\delta = \frac{F_0}{mL\omega_0^2} = \frac{F_0}{E_R A};\quad \bar{\Omega} = \frac{\Omega_R A T_R}{mL\omega_0^2} = \frac{\Omega_R T_R}{E_R}\ ;\quad \mu_E = \frac{E}{E_R};\quad \mu_\Omega = \frac{\Omega}{\Omega_R};\quad \varpi = \frac{\omega}{\omega_0} \tag{8}$$

These definitions allow one to define the following dimensionless variables, respectively related to mass displacement (U) and time (τ).

$$U = \frac{u}{L}\,; \quad \theta = \frac{T}{T_R}\,; \quad \tau = \omega_0 t \tag{9}$$

Therefore, the dimensionless equation of motion has the form:

$$U'' + \xi U' + \mu_E U + (\overline{\alpha}^C + \mu_E \overline{\alpha}_h^C)\beta_2 - (\overline{\alpha}^T + \mu_E \overline{\alpha}_h^T)\beta_1 - \mu_\Omega \overline{\Omega}(\theta - \theta_0) = \delta \sin(\varpi\tau) \tag{10}$$

where derivatives with respect to dimensionless time are represented by $(\,)' = d(\,)/d\tau$. This equation of motion can be written in terms of a system of first order differential equations as follows,

$$\begin{aligned} x' &= y \\ y' &= \delta \sin(\varpi\tau) - \xi y - \mu_E x - (\overline{\alpha}^C + \mu_E \overline{\alpha}_h^C)\beta_2 + (\overline{\alpha}^T + \mu_E \overline{\alpha}_h^T)\beta_1 + \mu_\Omega \overline{\Omega}(\theta - \theta_0) \end{aligned} \tag{11}$$

In order to deal with non-linearities of these equations of motion, an iterative procedure based on the operator split technique (Ortiz *et al.*, 1983) is employed. With this assumption, the fourth order Runge-Kutta method is used together with the projection algorithm proposed in Savi *et al.* (2002) to solve the constitutive equations. The solution of the constitutive equations also employs the operator split technique together an *implicit Euler* method. For β_n (n = 1,2,3) calculation, the evolution equations are solved in a decoupled way. At first, the equations (except for the sub-differentials) are solved using an iterative *implicit Euler* method. If the estimated results obtained for β_n does not fit the imposed constraints, an *orthogonal projection algorithm* pulls their value to the nearest point on the domain's surface (Paiva *et al.*, 2005).

4. Numerical Simulations

This section presents some numerical simulations developed in order to show the qualitative behavior of SMA dynamical responses. In all simulations, it is considered parameters presented in Table 1, experimentally adjusted for NiTi wires (Paiva *et al.*, 2005). Moreover, it is assumed a SMA element with A = 1.96 x 10^{-5} m and L = 50 x 10^{-5} m, and also a unitary mass. It is also assumed that references variables (E_R, Ω_R, T_R) are estimated in the temperature T_M, that is, E_M, Ω_M and T_M.

Table 1. SMA parameters.

E_A (GPa)	E_M (GPa)	α^T (MPa)	α^C (MPa)	ε_R^T	ε_R^C
54	42	330	330	0.0555	–0.0555
L_0^T (MPa)	L^T (MPa)	L_0^C (MPa)	L^C (MPa)	L_0^A (MPa)	L^A (MPa)
0.15	41.5	0.15	41.5	0.63	185
	Ω_A (MPa/K)	Ω_M (MPa/K)	T_M (K)	T_A (K)	
	0.74	0.17	291.4	307.5	
η_1^L (KPa.s)	η_1^U (KPa.s)	η_2^L (KPa.s)	η_2^U (KPa.s)	η_3^L (KPa.s)	η_3^U (KPa.s)
0.246	0.665	0.246	0.665	0.246	0.665

4.1. Free Vibration

At first, free vibration is focused on. This is done by letting δ vanish in the non-dimensional equations of motion. The system has different equilibrium points depending on temperature. In order to illustrate the free response of the oscillator, a system without viscous damping (ξ = 0) is considered. Results from simulations are presented in the form of phase portraits. Figure 2 presents the free response of the system at different temperatures: θ = 1.20, representing a high temperature where austenite is stable with free-stress; and θ = 0.99, a low temperature where martensite is stable with free-stress. Notice that, for high temperature one has only a single equilibrium point. On the other hand, for low temperatures, there are five equilibrium points. From these, three are stable while the other two are unstable. Here, one denotes as positive equilibrium point as the stable point which has a positive displacement, while negative equilibrium point as the stable point which has the negative displacement.

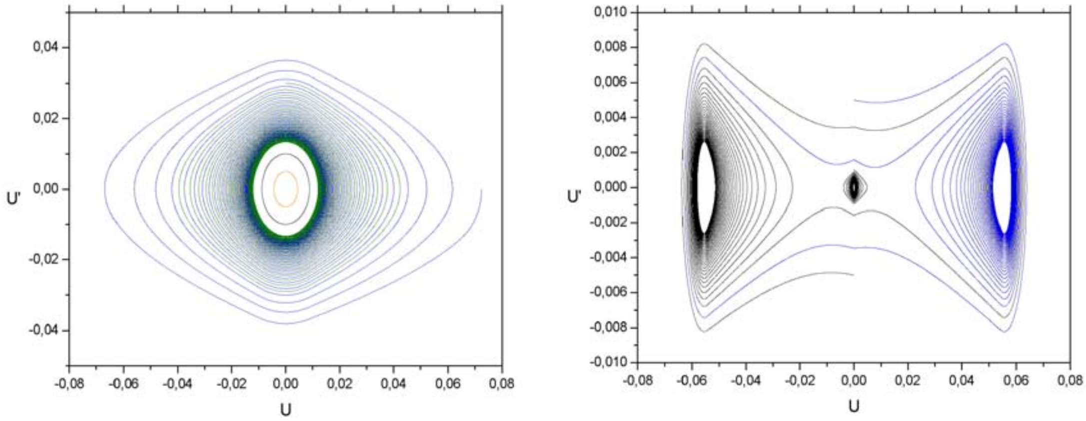

Figure 2. Phase portrait for different temperatures.

Figure 3 and 4 presents the free response for two situations (high and low temperatures). Notice that the system dissipates energy while it passes through the hysteresis loop. After that it tends to stabilize in an orbit where there is no phase transformation. Since for low temperatures, the system has different stable equilibrium points, it can stabilize in different positions, depending on initial conditions.

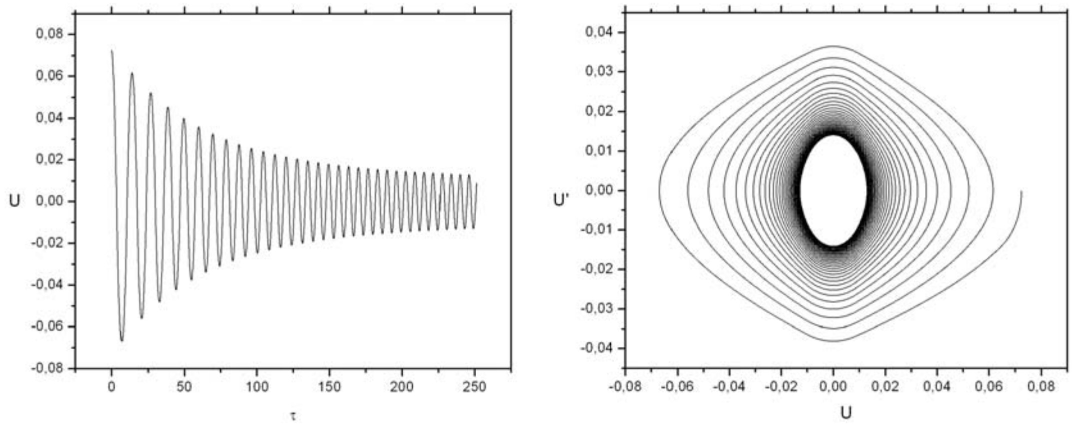

Figure 3. Free vibration response for high temperature.

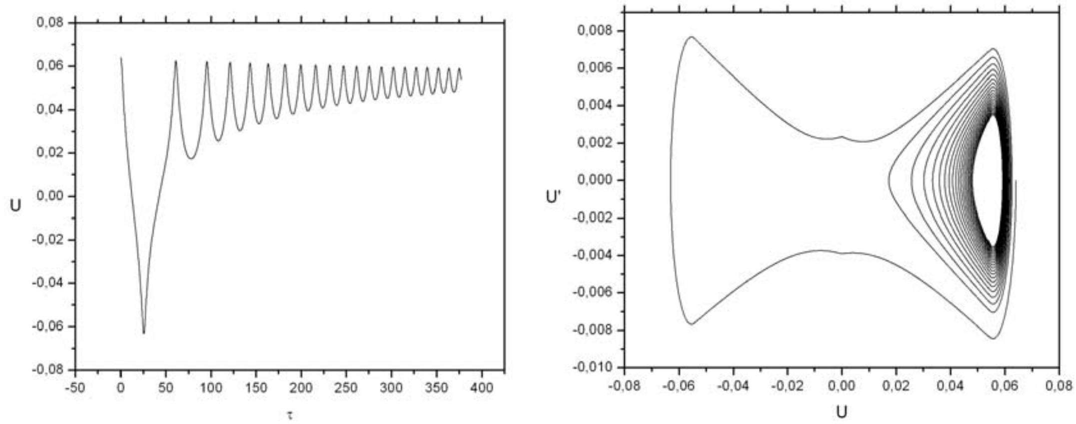

Figure 4. Free vibration response for low temperature.

This characteristic of the system may be exploited in different applications. One could cite a situation where temperature variations cause changes in the system position. In order to illustrate this behavior, it is shown a simulation where temperature varies as indicated in Figure 5. Figure 5 also shows the system response, presenting time history and phase space. Notice that the system oscillates around one point when temperature is high, changing this point when temperature decreases.

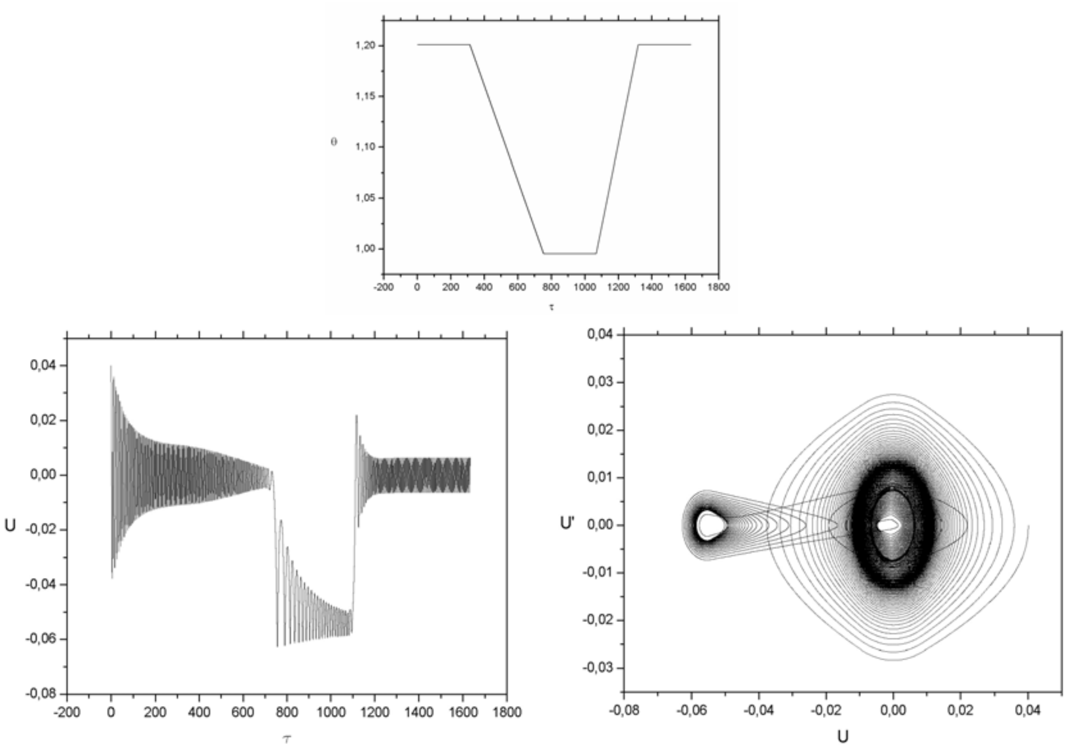

Figure 5. Free vibration response due to temperature variations.

4.2. Forced Vibration

The behavior of the forced system is far more complex. In this section, different kinds of shape memory oscillator responses are shown assuming a system with $\xi = 5\times10^{-4}$ and $\varpi = 1$. By assuming $\delta = 2\times10^{-3}$, a period-1 motion occurs. Figure 6 shows the phase state and also the Poincaré map related to this motion. By changing the forcing characteristics, it is possible to observe different kinds of response. Figure 7 shows a chaotic-like response when $\delta = 6.75\times10^{-3}$. This analysis shows the richness related to the dynamical behavior of the shape memory oscillator.

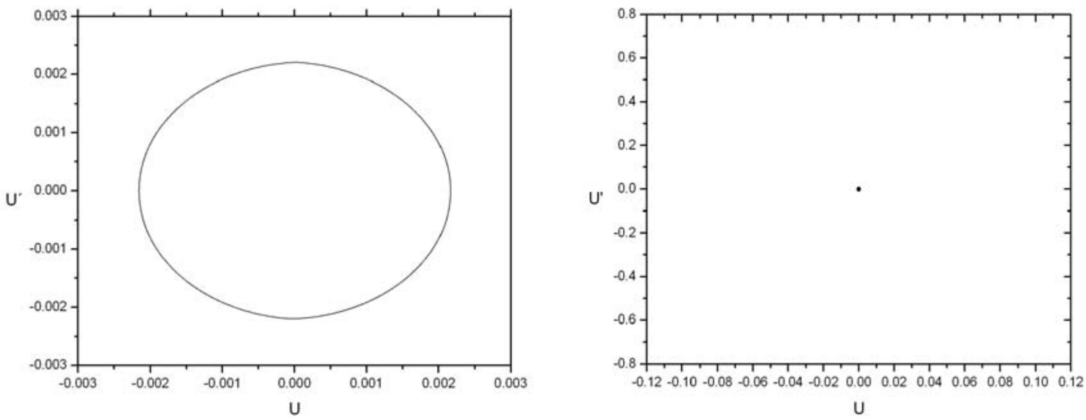

Figure 6. Period-1 response for $\delta = 2\times10^{-3}$.

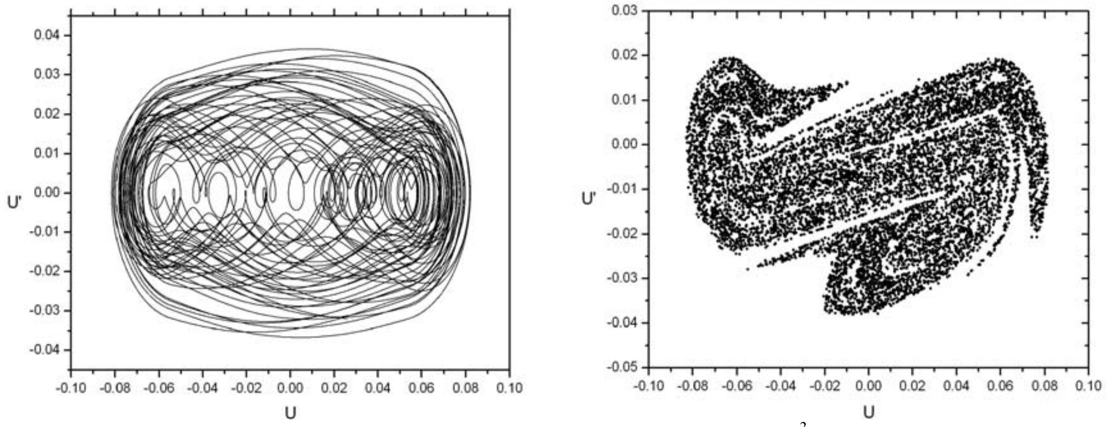

Figure 7. Chaotic-like response for $\delta = 6.75\times10^{-3}$.

5. Conclusions

This contribution analyses the dynamical response of a single-degree of freedom mechanical oscillator where the restitution force is described by a constitutive model with internal constraints. An iterative numerical procedure is developed based on the operator split technique. Under this assumption, coupled governing equations are solved from uncoupled problems, where classical numerical methods can be employed. The fourth order Runge-Kutta method is employed together with the orthogonal projection algorithm, used to solve the constitutive equations. Results of numerical simulations indicate that this system has a rich behavior with different kinds of responses. In general, it should be highlighted the equilibrium points temperature dependence which allows one to imagine changes of system position with temperature variation. Other interesting characteristic of SMA oscillator is the adaptive dissipation due to hysteresis loop. Finally, it should be pointed out the possibility of SMA system to achieve different kinds of behaviors which can be exploited in the sense of confer flexibility to the system. Therefore, the response of SMA devices subjected to dynamic loadings can be very complex being of special interest to be investigated.

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