FINITE ELEMENT ANALYSIS APPLIED TO SHAPE MEMORY TRUSSES USING ABAQUS

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Abstract. *This contribution deals with the nonlinear analysis of shape memory trusses employing the finite element method. Geometrical nonlinearities are incorporated into the formulation together with a constitutive model with four phases (three variants of martensite and an austenitic phase). Furthermore, the constitutive model considers different material properties for austenitic and martensitic phases together with thermal expansion. An iterative numerical procedure based on the operator split technique is proposed in order to deal with the nonlinearities in the formulation. Numerical simulations are carried out illustrating the ability of the developed model to capture the general behavior of shape memory bars. After that, it is analyzed the behavior of some adaptive trusses built with shape memory allow actuators subjected to different thermomechanical loadings.*

Keywords*: Shape Memory Alloy, finite element, Abaqus, numerical simulation.*

1. Introduction

Shape memory alloys (SMAs) have been found in a great number of applications in different fields of sciences and engineering. Self-actuating fasteners, thermally actuator switches and several bioengineering devices are some examples of these applications. Aerospace technology also uses SMAs for solve important problems, in particular those concerning with space savings achieved by self-erectable structures, stabilizing mechanisms, non-explosive release devices and other possibilities. Micromanipulators and robotics actuators have been built employing SMAs properties to mimic the smooth motions of human muscles. Moreover, SMAs are being used as actuators for vibration and buckling control of flexible structures.

This contribution proposes a finite element formulation to deal with shape memory bars with geometrical nonlinearities. Finite element modeling of SMA structures has been previously addressed by Brinson and Lammering (1993), where a constitutive theory based on Tanaka's model (Tanaka, 1986), and later modified by Brinson (1993), has been employed to describe the SMA behavior. Auricchio and Taylor (1996) have also proposed a three-dimensional finite element model. Savi *et al.* (1998) discuss an iterative numerical procedure that has been developed to deal with both geometrical and constitutive nonlinearities in the finite element model for adaptive trusses with SMA actuators. Lagoudas *et al.* (1997) consider the thermomechanical response of a laminate with SMA strips where the thermomechanical response is based on Boyd-Lagoudas' polynomial hardening model (Boyd and Lagoudas, 1996). Kouzak *et al.* (1998) also treats SMA beams using a constitutive equation proposed by Brinson (1993). Recently, La Cava *et al.* (2004) considers SMA bars with the constitutive model developed by Baêta-Neves *et al.* (2004) exploiting some non-homogeneous situations. Masud *et al.* (1997), Bhattacharyya *et al.* (2000), Liu *et al.* (2002) are other contributions in this field. Moreover, dual kriging interpolation has been employed with finite element method in order to describe the shape memory behavior (Trochu & Qian, 1997; Trochu & Terriault, 1998; Trochu *et al.*, 1999).

Here, the finite element method (FEM) is employed promoting the spatial discretization of bars using a constitutive equation proposed by Paiva *et al.* (2005) to describe the thermomechanical behavior of SMA trusses. This model includes four phases in the formulation: three variants of martensite and an austenitic phase. Furthermore, different material parameters for austenitic and martensitic phases are concerned. Geometrical nonlinearities are also included into the formulation.

An iterative numerical procedure based on the operator split technique (Ortiz *et al.*, 1983) is employed in order to deal with the nonlinearities in the formulation. Newton method is used together with the orthogonal projection algorithm employed for the numerical solution of the constitutive equations. Numerical simulations are carried out showing different behaviors of SMA bars. Results show that the proposed model is able to capture the general behavior of SMAs. The analysis of adaptive trusses with SMA actuators subjected to different thermomechanical loadings is carried out showing some interesting behaviors.

2. Mathematical Modeling

This section presents the mathematical modeling related to shape memory trusses with geometrical nonlinearities. Basically, it is presented a brief introduction to the constitutive model developed by Paiva *et al.* (2005) and its coupling with the finite element method.

2.1. Constitutive Model

There are different ways to describe the thermomechanical behavior of SMAs. Here, a constitutive model presented by Paiva *et al.* (2005) is employed. This model considers different material properties and four macroscopic phases for the description of the SMA behavior. The model also considers plastic strains and plastic-phase transformation coupling, which turns possible the two-way shape memory effect description. Moreover, tensile-compressive asymmetry is taken into account. The following set of equations is used to describe the thermomechanical behavior of SMA. Notice that, for simplicity, plastic effect is not presented here.

$$\sigma = E\,\varepsilon + \left(\alpha^C + E\alpha_h^C\right)\beta_2 - \left(\alpha^T + E\alpha_h^T\right)\beta_1 - \Omega\left(T - T_0\right) \tag{1}$$

$$\dot{\beta}_1 = \frac{1}{\eta_1}\Big\{ \alpha^T\varepsilon + \Lambda_1 + \left(\alpha_h^C\alpha^T + \alpha_h^T\alpha^C + E\alpha_h^T\alpha_h^C\right)\beta_2 - \left(2\alpha_h^T\alpha^T + E\,\alpha_h^{T\,2}\right)\beta_1 + \\ + \alpha_h^T\left[E\,\varepsilon - \Omega\left(T - T_0\right)\right] - \partial_1 J_\pi \Big\} + \partial_1 J_\chi \tag{2}$$

$$\dot{\beta}_2 = \frac{1}{\eta_2}\Big\{ -\alpha^C\varepsilon + \Lambda_2 + \left(\alpha_h^T\alpha^C + \alpha_h^C\alpha^T + E\alpha_h^C\alpha_h^T\right)\beta_1 - \left(2\alpha_h^C\alpha^C + E\,\alpha_h^{C\,2}\right)\beta_2 - \\ - \alpha_h^C\left[E\,\varepsilon - \Omega\left(T - T_0\right)\right] - \partial_2 J_\pi \Big\} + \partial_2 J_\chi \tag{3}$$

$$\dot{\beta}_3 = \frac{1}{\eta_3}\Big\{ -\frac{1}{2}\left(E_A - E_M\right)\left(\varepsilon + \alpha_h^C\beta_2 - \alpha_h^T\beta_1\right)^2 + \Lambda_3 + \left(\Omega_A - \Omega_M\right)\left(T - T_0\right)\left(\varepsilon + \alpha_h^C\beta_2 - \alpha_h^T\beta_1\right) - \\ - \partial_3 J_\pi \Big\} + \partial_3 J_\chi \tag{4}$$

where $E = E_M + \beta_3\,(E_A - E_M)$ is the elastic modulus while $\Omega = \Omega_M + \beta_3\,(\Omega_A - \Omega_M)$ is related to the thermal expansion coefficient. Notice that subscript "*A*" refers to austenitic phase, while "*M*" refers to martensite. Besides, different properties are assumed to consider tension-compression asymmetry, where the superscript "*T*" refers to tensile while "*C*" is related to compressive properties. Moreover, parameters Λ_1, Λ_2 and Λ_3 are associated with phase transformations stress levels and are temperature dependent as follows:

$$\Lambda_1 = \frac{L^T}{T_M}(T - T_M)\,; \quad \Lambda_2 = \frac{L^C}{T_M}(T - T_M) \quad \text{and} \quad \Lambda_3 = \frac{L^A}{T_M}(T - T_M) \tag{5}$$

where T_M is the temperature below which the martensitic phase becomes stable, while L^T, L^C and L^A are parameters related to critical stress for the various phase transformations.

The terms $\partial_n J_\pi$ (n = 1,2,3) corresponds to the sub-differentials of the indicator function J_π with respect to β_n (Rockafellar, 1970). The indicator function $J_\pi\left(\beta_1, \beta_2, \beta_3\right)$ is related to the convex set π, which provides the internal constraints related to the phases' coexistence. With respect to the evolution equations of volumetric fractions, η_1, η_2 and η_3 represent the internal dissipation related to phase transformations. Moreover $\partial_n J_\chi$ (n = 1,2,3) are the sub-differentials of the indicator function J_χ with respect to $\dot{\beta}_n$ (Rockafellar, 1970). This indicator function is related to the convex set χ, establishing conditions for the correct description of internal sub-loops due to incomplete phase transformations and also avoids phase transformations of the type: $M+ \Rightarrow M$ or $M- \Rightarrow M$.

Besides, in order to consider different phase transformation kinetics for loading and unloading processes, the parameters η_n (n = 1,2,3), related to phase transformation internal dissipation, may assume different values. Therefore, $\eta_n = \eta_n^L$ if $\dot{\varepsilon} > 0$ and $\eta_n = \eta_n^U$ if $\dot{\varepsilon} < 0$.

2.2. Finite Element Method

Consider a SMA bar subjected to a constant axial load. From the principle of virtual work (Bathe, 1982),

$$\int_{^tV} {}^{t+\Delta t}_{\;\;t}S_{ij}\,\delta\, {}^{t+\Delta t}_{\;\;t}\varepsilon_{ij}\, d\,{}^tV = {}^{t+\Delta t}R \tag{6}$$

where ${}^{t+\Delta t}_{\;\;t}S_{ij}$ is the second Piola-Kirchhoff tensor, ${}^{t+\Delta t}_{\;\;t}\varepsilon_{ij}$ is the Lagrange strain tensor and ${}^{t+\Delta t}R$ is the work of external forces. Stress tensor can be split into two parts: the increment tensor ΔS_{ij} , and the stress tensor evaluated in the instant t, ${}^t_tS_{ij}$. Therefore,

$${}^{t+\Delta t}_{\;\;t}S_{ij} = {}^t_tS_{ij} + \Delta S_{ij} \tag{7}$$

The second Piola-Kirchhoff tensor in the time instant t is the same as the Cauchy stress tensor in this time instant, ${}^t\tau_{ij}$ Therefore, ${}^{t+\Delta t}_{\;\;t}S_{ij} = {}^t\tau_{ij} + \Delta S_{ij}$.

Since the strain component ${}^{t+\Delta t}_{\;\;t}\varepsilon_{ij}$ is unknown, it is assumed that this strain tensor is approximated by its increments in the time t, $\Delta\varepsilon_{ij}$. Moreover, this increment can be split into two terms, associated with the linear and nonlinear part of strain, denoted respectively, by Δe_{ij} and $\Delta\eta_{ij}$. Therefore,

$$\Delta\varepsilon_{ij} = \Delta e_{ij} + \Delta\eta_{ij} \tag{8}$$

At this point, all these definitions are introduced into the principle of virtual work:

$$\int_{^tV} {}^t\tau_{ij}\delta\Delta e_{ij}\, d\,{}^tV + \int_{^tV} {}^t\tau_{ij}\delta\Delta\eta_{ij}\, d\,{}^tV + \int_{^tV} \Delta S_{ij}\delta\Delta e_{ij}\, d\,{}^tV + \int_{^tV} \Delta S_{ij}\delta\Delta\eta_{ij}\, d\,{}^tV = {}^{t+\Delta t}R \tag{9}$$

Now, for the sake of simplicity, SMA constitutive equation is rewritten by.

$$\Delta S_{11} = E\Delta\varepsilon_{11} + \Delta\Lambda \tag{10}$$

where $\Delta\Lambda = E\left(\alpha_h^C\Delta\beta_2 - \alpha_h^T\Delta\beta_1\right) + \alpha^C\Delta\beta_2 - \alpha^T\Delta\beta_1 - \Omega\Delta\left(T - T_0\right)$. Now, using this constitutive equation into the principle of virtual work and also restricting the general three-dimensional problem to a one-dimensional context, it is possible to write:

$$\begin{aligned}&\int_{^tV} {}^t\tau_{11}\delta\Delta u_{1,1}\, d\,{}^tV + \int_{^tV} {}^t\tau_{11}\Delta u_{1,1}\delta\Delta u_{1,1}\, d\,{}^tV + \int_{^tV} E\Delta u_{1,1}\delta\Delta u_{1,1}\, d\,{}^tV + \frac{1}{2}\int_{^tV} E\Delta u_{1,1}^3\delta\Delta u_{1,1}\, d\,{}^tV + \\ &+ \int_{^tV} E\Delta u_{1,1}^2\delta\Delta u_{1,1}\, d\,{}^tV + \frac{1}{2}\int_{^tV} E\Delta u_{1,1}^2\delta\Delta u_{1,1}\, d\,{}^tV + \int_{^tV} \Delta\Lambda\,\delta\Delta u_{1,1}\, d\,{}^tV + \int_{^tV} \Delta\Lambda\Delta u_{1,1}\delta\Delta u_{1,1}\, d\,{}^tV = {}^{t+\Delta t}R\end{aligned} \tag{11}$$

Notice that, $\Delta e_{ij} = \Delta u_{1,1}$ and $\Delta\eta_{ij} = \dfrac{\Delta u_{1,1}^2}{2}$. Therefore, neglecting higher order terms, the equation can be written in terms of increments,

$$\int_{{}^tV} {}^t\tau_{11}\delta\Delta u_{1,1} d\,{}^tV + \int_{{}^tV} {}^t\tau_{11}\Delta u_{1,1}\delta\Delta u_{1,1} d\,{}^tV + \int_{{}^tV} E\Delta u_{1,1}\delta\Delta u_{1,1} d\,{}^tV + \int_{{}^tV} \Delta\Lambda\delta\Delta u_{1,1} d\,{}^tV + \\ + \int_{{}^tV} \Delta\Lambda\Delta u_{1,1}\delta\Delta u_{1,1} d\,{}^tV = {}^{t+\Delta t}R. \tag{12}$$

The work of external forces can be evaluated by the following equation

$$ {}^{t+\Delta t}R = \int_{{}^{t+\Delta t}V} {}^{t+\Delta t}f_1{}^B \delta\Delta u_1 dV + \int_{{}^{t+\Delta t}S} {}^{t+\Delta t}f_1{}^S \delta\Delta u_1 dS\,, \tag{13}$$

where ${}^{t+\Delta t}f_1^B$ are the body forces and ${}^{t+\Delta t}f_1^S$ are the surface forces acting on the bar.

At this point, the continuous function Δu_1 is discretized with the aid of the finite element formulation. Therefore assuming a two-point element, where Lagrange shape functions are considered, one writes a discrete version of the SMA bar governing equation:

$$[K]\{U\} = \{F\} - \{\hat{F}\} \tag{14}$$

where $\{U\}$ are the nodal displacement increment vector, $[K] = [K_L] + [K_{NG}] + [K_{NC}]$ is the stiffness matrix. Notice that $[K_L]$ is the linear stiffness matrix and the terms related to geometrical nonlinearities are expressed in $[K_{NG}]$, while constitutive nonlinearities are expressed in $[K_{NC}]$. Moreover, $\{F\} = \{R\} - \{F_\tau\}$ is the increment force vector while $\{\hat{F}\} = \{F_\Lambda\}$ is the nonlinear vector force. All these matrixes and vectors are given by:

$$\begin{aligned}
&[K_{NG}] = \int_{{}^tV} [B]^T\, {}^t\tau_{11}\, {}_t[B] d\,{}^tV && \{F\} = \int_{{}^tV} [N]^T f_1^B d\,{}^tV + \int_{{}^tS} [N]^T f_1^S d\,{}^tS \\
&[K_{NC}] = \int_{{}^tV} [B]^T \Lambda\, {}_t[B] d\,{}^tV && \{F_\tau\} = \int_{{}^tV} [B]^T\, {}^t\tau_{11} d\,{}^tV \\
&[K_L] = \int_{{}^tV} [B]^T E\, {}_t[B] d\,{}^tV && \{F_\Lambda\} = \int_{{}^tV} [B]^T\, {}_t\Lambda d\,{}^tV
\end{aligned} \tag{15}$$

In order to deal with nonlinearities in the formulation, an iterative numerical procedure is employed. In terms of constitutive equations, numerical procedure is explained in Paiva *et al.* (2005). Nevertheless, it is necessary to consider Newton method in order to solve the discrete governing equations.

3. Model Verification

The proposed formulation is introduced into ABAQUS as user material procedure. This section presents the model verification through comparisons between the results predicted by the proposed model and experimental data reported in Tobushi *et al.* (1991), which describes tensile tests on Ni-Ti wires at different temperatures. Basically, two different temperatures are here considered: 353K and 373K. The model parameters identified for this simulation are presented in Table 1 (Paiva *et al.*, 2005). After that, some numerical simulations are performed in order to verify the ability of the proposed model to describe the SMA behavior. Since experimental results are related to a tensile test, it is assumed a homogeneous thermomechanical load process, allowing a comparison between the FEM formulation with experimental results, considering a single constitutive point. A single element is used with this aim, assuming a maximum time step of $5\text{x}10^{-3}$.

Figure 1 presents a comparison between numerical and experimental results for two different temperatures. Notice that, even though this is a pseudoelastic test, experimental data presents a residual strain at the end of the loading-unloading process, which is probably related to transformation induced plasticity (Lagoudas *et al.*, 2003). However, it should be pointed out the ability of the finite element model to describe the SMA behavior.

Table 1. Model parameters obtained through comparison between numerical and experimental results provided by Tobushi *et al.* (1991) for a Ni-Ti SMA alloy.

E_A (GPa)	E_M (GPa)	Ω_A (MPa/K)	Ω_M (MPa/K)	α^T (MPa)
54	42	0.74	0.17	330
L^T, L^C (MPa)	L^A (MPa)	T_M (K)	T_0 (K)	ε_R^T
229	44	291.4	295	0.055

η_1^L (kPa.tu)	η_1^U (MPa.tu)	η_3^L (MPa.s)	η_3^U (MPa.s)
1	2.7	1	2.7

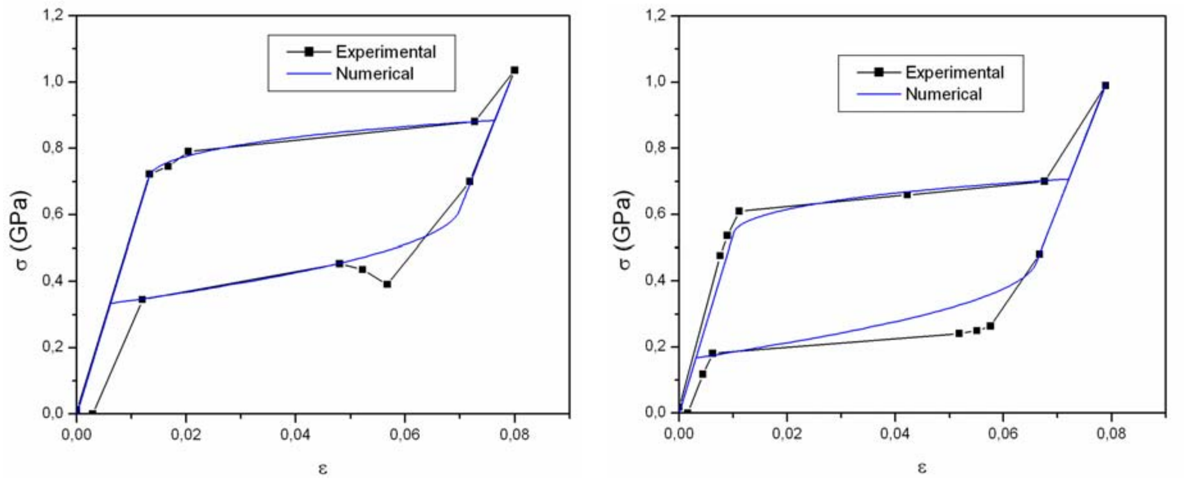

Figure 1. Model verification.

4. Adaptive Trusses

In this section, the proposed procedure is applied to analyze some adaptive trusses with shape memory actuators considering two examples. The first one treats a two-bar truss while the second one considers a truss with nine bars. Both examples treat different thermomechanical loadings.

4.1. Two-bar Truss

At this point, a two-bar truss built by bars with cross-section area 1 cm^2, subjected to thermomechanical loadings presented in Figure 2 is considered.

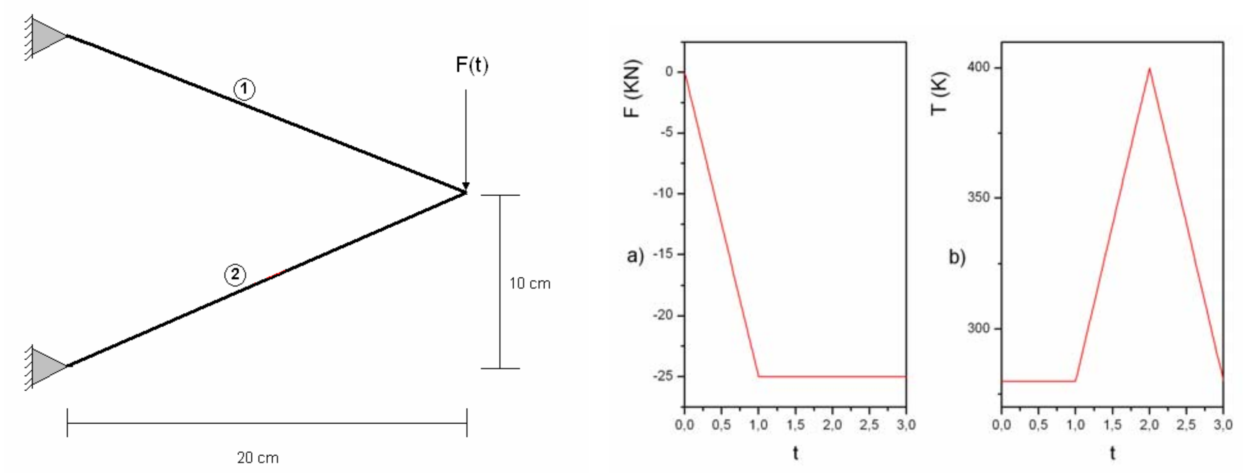

Figure 2. Two-bar truss.

Initially, it is assumed that a single bar is built with SMA ①, while the other one is a typical steel ②. Figure 3 shows deformed configuration compared with the initial one for two different time instants: the first is related to the end of the mechanical process while the second is associated with the end of thermal loading. Notice that the thermal loading changes the position of the truss, although the mechanical loading is not removed.

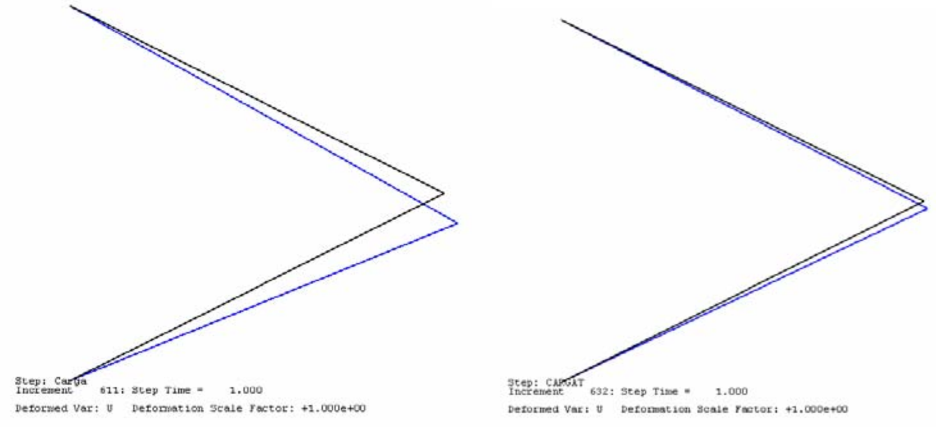

Figure 3. Comparison between deformed and initial configurations of the two-bar truss.

Figure 4 shows the displacement of the end of the bars and also the stress-strain curve. Again it is possible to see the movement of the bar caused by the temperature variation. Notice also that geometrical nonlinearities introduce some characteristics of the response as the non-horizontal curve of the stress-strain relation in the region related to the thermal loading.

At this point, it is assumed that both bars of the two-bar truss are built with SMA. This new truss has smaller stiffness presenting greater displacements. Figure 5 compares results of both problems showing that SMA can also recover displacements even though mechanical force is not removed.

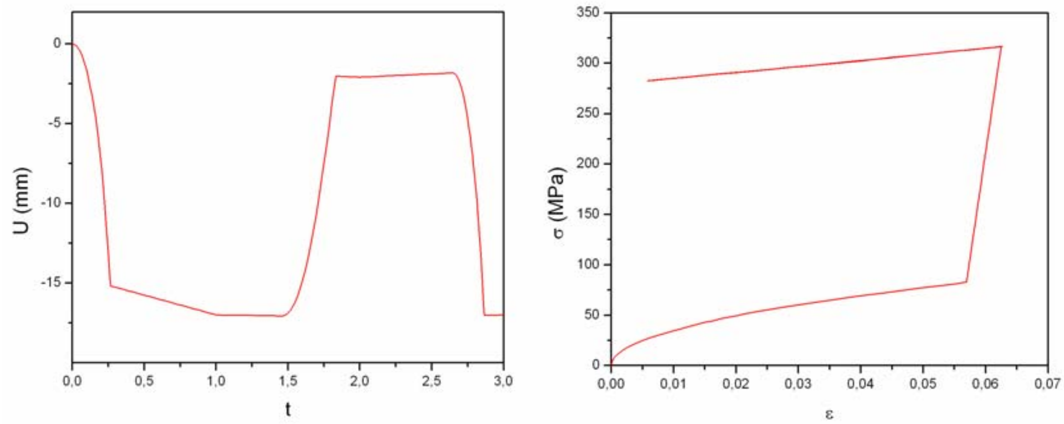

Figure 4. Time history of displacement and stress-strain curve.

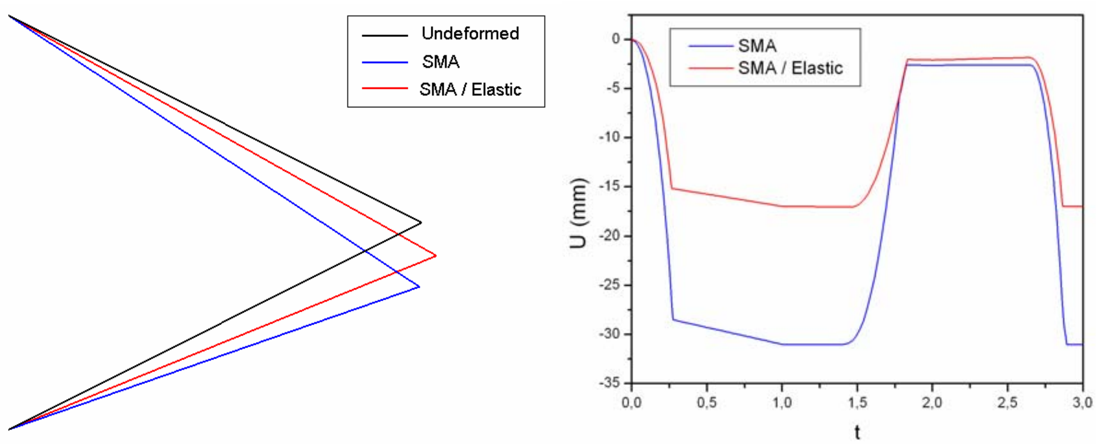

Figure 5. Two-bar truss with two SMA bars.

4.2. Nine-bar Truss

A nine-bar truss where each bar has a cross-section area of 1 cm^2 (Figure 6), subjected to a thermomechanical loadings presented in Figure 3 is now considered. Bars 3 and 5 are built with SMA, while the other are constructed with typical steel. With this assumption, the SMA bars represent discrete actuators of this truss.

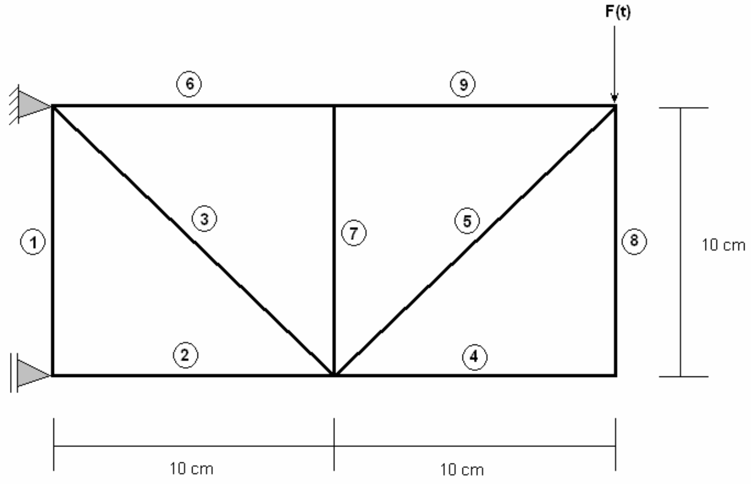

Figure 6. Nine-bar truss.

Figure 7 shows deformed configuration compared with the initial one for two different time instants: the first is related to the end of the mechanical process while the second is associated with the end of thermal loading. Notice that, again, the thermal loading changes the position of the truss, although the mechanical loading is not removed. This behavior is clearly observed in the time history of displacements. Figure 8 shows the displacement of the end of the bars, showing the movement of the truss caused by the temperature variation.

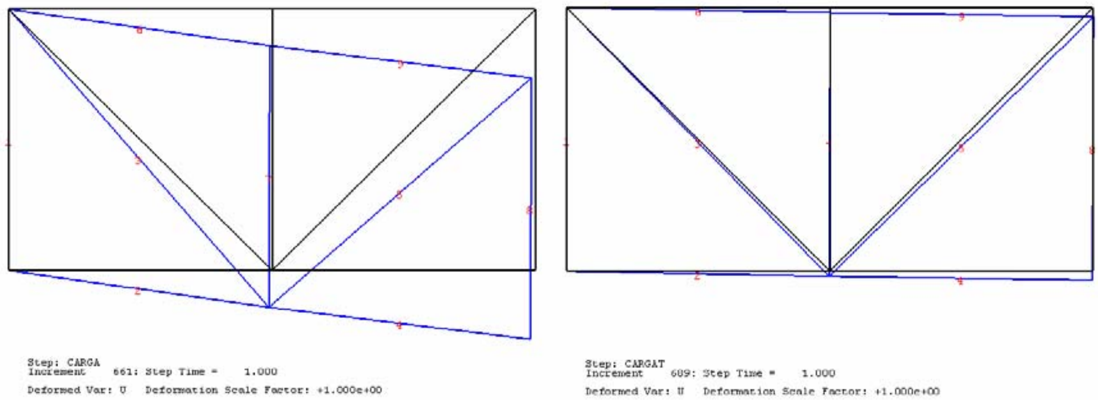

Figure 7. Comparison between deformed and initial configurations of the nine-bar truss.

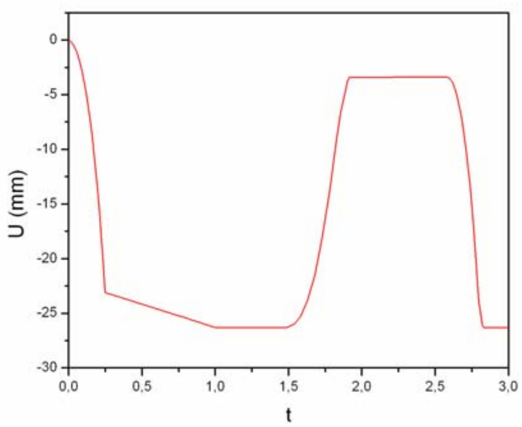

Figure 8. Time history of displacements.

5. Conclusions

This article presents a nonlinear finite element analysis of shape memory adaptive trusses. A constitutive model proposed by Paiva *et al.* (2005) is used to describe the thermomechanical behavior of SMAs. Finite element method is used to perform spatial discretization allowing the analysis of non-homogeneous problems. Geometrical nonlinearities are incorporated into the formulation. An iterative numerical procedure based on the operator split technique is employed in order to deal with nonlinearities. Newton method is used together, with an iterative numerical procedure based on the operator split technique, is employed in order to deal with nonlinearities of the formulation. Numerical simulations show that results from FEM capture the SMA general behavior. Moreover, other simulations show the response of adaptive truss built with SMA actuators.

6. Acknowledgements

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