MODELING TRANSFORMATION INDUCED PLASTICITY IN SHAPE MEMORY ALLOYS

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Abstract. *Transformation Induced Plasticity (TRIP) is widely defined in the literature as the plastic flow arising from solid state phase transformation processes involving volume and/or shape changes without overlapping the yield surface. In the case of the so-called Shape Memory Alloys (SMAs), these two phenomena are present and have significant influence over their macroscopic thermomechanical behavior. Recent studies propose some constitutive theories to describe this phenomenon and its particular features inherent to SMAs. The present contribution presents a macroscopic one-dimensional constitutive model with internal constraints, which accounts for TRIP, plasticity, plastic-phase transformation coupling and tension-compression asymmetry. Comparisons between numerical and experimental results attest the model's capability to capture TRIP phenomenon. Moreover, numerical simulations are conducted in order to evaluate saturation mechanisms during cyclic loadings.*

Keywords*: Shape Memory Alloy, constitutive model, transformation induced plasticity, numerical simulation.*

1. Introduction

Shape Memory Alloys (SMAs) present complex thermomechanical behaviors related to different physical processes. Besides the most common phenomena presented by this class of material, such as: pseudoelasticity, shape memory effect, which may be one-way (SME) or two-way (TWSME), and phase transformation due to temperature variation, there are more complicated phenomena that have significant influence over its overall thermomechanical behavior – for instance: plastic behavior, tension-compression asymmetry, plastic-phase transformation coupling, transformation induced plasticity, among others. All these phenomena take place at a microscopic level but most of them affect SMAs' macroscopic response; therefore, should not be neglected while modeling their phenomenological behavior.

Experimental studies (Greenwood & Johnson, 1965; Magee, 1966; Abrassart, 1972; Desalos, 1981; Denis *et al.*, 1982; Olson & Cohen, 1986) reveal the growth of a nonlinear irreversible (plastic) strain amount while solid state phase transformations take place. This deformation mechanism is known in the literature as Transformation Induced Plasticity (TRIP) and results from internal stresses arising not only from the volume change associated with the transformation but also from the accompanying shape change (Marketz & Fischer, 1994) without reaching the yield surface of the weaker phase involved.

A clear distinction can be made between classical plasticity and TRIP. While classical plasticity arises from applied stress or temperature variation, TRIP is caused by phase proportions variation – even for low constant stresses level (Leblond *et al.*, 1989 ; Gautier *et al.*, 1989 ; Gautier, 1998; Fischer *et al.*, 2000, 1996; Tanaka & Sato, 1985).

TRIP phenomenon is widely attributed to occur due to two distinct physical mechanisms, which have been proposed by Greenwood & Johnson (1965) and Magee (1966). The Greenwood-Johnson effect is admitted to be due an accommodation process of the microplasticity associated with a volume change. The Magee effect, on the other hand, is due to an orientation effect that arises from a shear internal stress state, which favors the thermodynamically preferred orientation direction for martensite formation in the presence of an external stress field, involving shape change.

According to Greenwood & Johnson (1965), transformation plasticity is due to the compactness difference between parent (austenite) and product (martensite) phases' lattice structure. During martensitic transformation, this difference results in a volume change, producing a microscopic internal stress state, responsible for microscopic plasticity in the weaker phase (that with the lower yield stress). Without external applied stress, the average of these internal stresses generally vanishes and, from macroscopic point of view, only global variation of the volume will be observed. In order to understand the Magee effect, consider the martensitic formation process on cooling. During its nucleation, martensite develops plates that grow and generate large amounts of shear (deviatoric) stress in the parent austenitic matrix (Fischer et al. 1998). When no external load is applied, the plate's orientation is generally random, which makes the (macroscopic) resultant of the microscopic internal stresses average out null. An (even low and constant) external applied load, triggers an alignment of these martensitic plates with the loading stress direction. This external load is responsible for an internal stress state increase that no more will macroscopically average out null (Taleb *et al.*, 2001).

Many efforts have been done so far towards adequate TRIP modeling (Leblond *et al.*, 1989; Leblond 1989; Stringfellow *et al.*, 1992; Marketz & Fischer, 1994; Zwigl & Dunand 1997; Cherkaoui *et al.*, 1998; Ganghoffer & Simonsson, 1998; Fischer *et al.*, 1998, Taleb *et al.*, 2001; Taleb & Sidoroff, 2003; Homberg; 2004). The great majority of these models focus on micro scale features of the TRIP. Moreover, some of them discard the Magee effect under some reasonable considerations for particular studies. The present contribution presents a one-dimensional constitutive model with internal constraints, which accounts for TRIP, (linear hardening) plasticity, plastic-phase transformation coupling and tension-compression asymmetry. The target of this work is to describe the macroscopic manifestation of the TRIP phenomenon and also to explore the TRIP importance under cyclic loadings. Comparisons between numerical and experimental results are used to demonstrate the model's ability to capture TRIP phenomenon. Moreover, numerical simulations are conducted in order to evaluate saturation mechanisms during cyclic loadings.

2. Constitutive Model

This work revisits the model with internal constraints previously presented in different references (Savi *et al.*, 2002, Baêta-Neves *et al.*, 2004, Paiva *et al.*, 2005, Savi & Paiva, 2005), which is based on Fremond's proposed model (Fremond, 1996). This one-dimensional macroscopic model considers (linear hardening) plasticity, plastic-phase transformation coupling, tension-compression asymmetry and different material properties. The herein presented model includes a new feature to allow TRIP phenomenon description.

For the thermomechanical description of SMAs behavior, the proposed model formulation considers four volumetric fractions. β_1 is associated with tensile detwinned martensite (*M*+), β_2 is related to compressive detwinned martensite (*M*–), β_3 represents austenite (*A*) and β_4 corresponds to twinned martensite (*M*). A Helmholtz free energy potential (ψ) is adopted for each phase. Each potential consider four state variables: elastic strain (ε_e) and temperature (T) for themoelasticity description and two internal variables (γ and μ) that help the plastic phenomenon description, which are associated with the isotropic and kinematic hardening, respectively.

$$\textbf{\textit{M}+ :}\quad \rho\,\psi_1\left(\varepsilon_e,T,\gamma,\mu\right)=\frac{1}{2}E_M\,\varepsilon_e{}^2-\alpha^T\varepsilon_e-\Lambda_M^T-\Omega_M\left(T-T_0\right)\varepsilon_e+\frac{1}{2}K_M\,\gamma^2+\frac{1}{2H_M}\mu^2 \tag{1}$$

$$\textbf{\textit{M}}\text{–}:\quad \rho\,\psi_2\left(\varepsilon_e,T,\gamma,\mu\right)=\frac{1}{2}E_M\,\varepsilon_e{}^2+\alpha^C\varepsilon_e-\Lambda_M^C-\Omega_M\left(T-T_0\right)\varepsilon_e+\frac{1}{2}K_M\,\gamma^2+\frac{1}{2H_M}\mu^2 \tag{2}$$

$$\textbf{\textit{A}}:\quad \rho\,\psi_3\left(\varepsilon_e,T,\gamma,\mu\right)=\frac{1}{2}E_A\,\varepsilon_e{}^2-\Lambda_A-\Omega_A\left(T-T_0\right)\varepsilon_e+\frac{1}{2}K_A\,\gamma^2+\frac{1}{2H_A}\mu^2 \tag{3}$$

$$\textbf{\textit{M}}:\quad \rho\,\psi_4\left(\varepsilon_e,T,\gamma,\mu\right)=\frac{1}{2}E_M\,\varepsilon_e{}^2+\Lambda_M-\Omega_M\left(T-T_0\right)\varepsilon_e+\frac{1}{2}K_M\,\gamma^2+\frac{1}{2H_M}\mu^2 \tag{4}$$

In the previous equations, subscript *M* is related to martensitic phase while *A* is associated with austenite. Moreover, superscript *T* is related to tensile parameters while *C* is associated with compressive parameters. Observing these indexes, notice that α's are material parameters related to phase transformation, while Λ's are associated with phase transformations stress levels and are temperature dependent (as will be later discussed); *E*'s represent the elastic moduli, Ω's are related to the thermal expansion coefficients, *K*'s are the plastic modulus while *H*'s are the kinematic hardening moduli; T_0 is a reference temperature and ρ is the density.

Here, four more state variables should be added to express the free energy of the whole mixture ($\tilde{\psi}$), which is written weighting each energy potential with the corresponding volumetric fraction. Obviously, it is possible to consider only three volumetric fractions, since $\beta_1+\beta_2+\beta_3+\beta_4=1$. Thus, writing β_4 as a function of β_1, β_2 and β_3, one finds

$$\begin{aligned}
&\rho\,\psi\left(\varepsilon_e,T,\gamma,\mu,\beta_i,\xi_i\right)=\rho\,\tilde{\psi}\left(\varepsilon_e,T,\gamma,\mu,\beta_i\right)+J_\pi\left(\beta_i\right)+J_\tau\left(\xi_i\right)=\\
&=\beta_1\left[-\alpha^T\varepsilon_e-\left(\Lambda_M+\Lambda_M^T\right)\right]+\beta_2\left[\alpha^C\varepsilon_e-\left(\Lambda_M+\Lambda_M^C\right)\right]+\\
&+\beta_3\left[\frac{1}{2}\left(E_A-E_M\right)\varepsilon_e{}^2-\left(\Lambda_M+\Lambda_A\right)-\left(\Omega_A-\Omega_M\right)\left(T-T_0\right)\varepsilon_e+\frac{1}{2}\left(K_A-K_M\right)\gamma^2+\left(\frac{1}{2H_A}-\frac{1}{2H_M}\right)\mu^2\right]\\
&+\frac{1}{2}E_M\varepsilon_e{}^2+\Lambda_M-\Omega_M\left(T-T_0\right)\varepsilon_e+\frac{1}{2}K_M\gamma^2+\frac{1}{2H_M}\mu^2+J_\pi\left(\beta_i\right)+J_\tau\left(\xi_i\right)
\end{aligned} \tag{5}$$

where $J_\pi\left(\beta_i\right)$ is the indicator function of the convex set π, which establishes the constraints associated with the phases' coexistence defined as follows. From now on, the subscript index *i* refers to $i = 1,2,3$ (*M*+, *M*– and *A*, respectively).

$$\pi = \left\{ \beta_i \in \Re \;\middle|\; 0 \le \beta_i \le 1;\; \beta_1 + \beta_2 + \beta_3 \le 1 \right\} \tag{6}$$

while $J_\tau(\xi_i)$ is the indicator function associated with the constraints of the saturation effect during cyclic loadings, represented by the variable ξ_i, that will be later defined. At this point, it is assumed an additive decomposition such that the elastic strain may be written as: $\varepsilon_e = \varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C \beta_2 - \alpha_h^T \beta_1$. Notice that three more internal variables are included: ε_p represents the plastic strain, ε_{tp} is associated to the TRIP deformation and ε is the total strain, while α_h's are material parameters related to phase transformation. As a result, the total free energy in its final form is expressed by:

$$\begin{aligned}
\rho\psi\left(\varepsilon, \varepsilon_p, \varepsilon_{tp}, T, \gamma, \mu, \beta_n, \xi_n\right) = {} & \beta_1 \left[-\alpha^T \left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C \beta_2 - \alpha_h^T \beta_1\right) - (\Lambda_M + \Lambda_M^T) \right] + \\
& + \beta_2 \left[\alpha^C \left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C \beta_2 - \alpha_h^T \beta_1\right) - (\Lambda_M + \Lambda_M^C) \right] + \\
& + \beta_3 \left[\frac{1}{2}(E_A - E_M)\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C \beta_2 - \alpha_h^T \beta_1\right)^2 - (\Lambda_A + \Lambda_M) - \right. \\
& \left. - (\Omega_A - \Omega_M)(T - T_0)\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C \beta_2 - \alpha_h^T \beta_1\right) + \frac{1}{2}(K_A - K_M)\gamma^2 + \left(\frac{1}{2H_A} - \frac{1}{2H_M}\right)\mu^2 \right] + \\
& + \frac{1}{2} E_M \left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C \beta_2 - \alpha_h^T \beta_1\right)^2 + \Lambda_M - \Omega_M (T - T_0)\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C \beta_2 - \alpha_h^T \beta_1\right) + \\
& + \frac{1}{2} K_M \gamma^2 + \frac{1}{2H_M}\mu^2 + J_\pi(\beta_n) + J_\tau(\xi_n)
\end{aligned} \tag{7}$$

State equations can be obtained from the Helmholtz free energy as follows:

$$\sigma = \rho \frac{\partial \psi}{\partial \varepsilon} = E\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C \beta_2 - \alpha_h^T \beta_1\right) + \alpha^C \beta_2 - \alpha^T \beta_1 - \Omega(T - T_0) \tag{8}$$

$$\begin{aligned}
B_1 \in -\rho \frac{\partial \psi}{\partial \beta_1} = {} & \alpha^T \left(\varepsilon - \varepsilon_p - \varepsilon_{tp}\right) + \Lambda_1 + \beta_2 \left(\alpha_h^C \alpha^T + \alpha_h^T \alpha^C + E \alpha_h^T \alpha_h^C\right) - \\
& - \beta_1 \left(2 \alpha_h^T \alpha^T + E {\alpha_h^T}^2\right) + \alpha_h^T \left[E\left(\varepsilon - \varepsilon_p - \varepsilon_{tp}\right) - \Omega(T - T_0)\right] - \partial_{\beta_1} J_\pi
\end{aligned} \tag{9}$$

$$\begin{aligned}
B_2 \in -\rho \frac{\partial \psi}{\partial \beta_2} = {} & -\alpha^C \left(\varepsilon - \varepsilon_p - \varepsilon_{tp}\right) + \Lambda_2 + \beta_1 \left(\alpha_h^T \alpha^C + \alpha_h^C \alpha^T + E \alpha_h^C \alpha_h^T\right) - \\
& - \beta_2 \left(2 \alpha_h^C \alpha^C + E {\alpha_h^C}^2\right) - \alpha_h^C \left[E\left(\varepsilon - \varepsilon_p - \varepsilon_{tp}\right) - \Omega(T - T_0)\right] - \partial_{\beta_2} J_\pi
\end{aligned} \tag{10}$$

$$\begin{aligned}
B_3 \in -\rho \frac{\partial \psi}{\partial \beta_3} = {} & -\frac{1}{2}(E_A - E_M)\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C \beta_2 - \alpha_h^T \beta_1\right)^2 + \Lambda_3 + \\
& + (\Omega_A - \Omega_M)(T - T_0)\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C \beta_2 - \alpha_h^T \beta_1\right) - \frac{1}{2}(K_A - K_M)\gamma^2 - \\
& - \left(\frac{1}{2H_A} - \frac{1}{2H_M}\right)\mu^2 - \partial_{\beta_3} J_\pi
\end{aligned} \tag{11}$$

$$X = -\rho \frac{\partial \psi}{\partial \varepsilon_p} = E\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C \beta_2 - \alpha_h^T \beta_1\right) + \alpha^C \beta_2 - \alpha^T \beta_1 - \Omega(T - T_0) = \sigma \tag{12}$$

$$Y = -\rho \frac{\partial \psi}{\partial \gamma} = -K\gamma \tag{13}$$

$$Z = -\rho \frac{\partial \psi}{\partial \mu} = -\frac{1}{H}\mu \tag{14}$$

$$R = -\rho \frac{\partial \psi}{\partial \varepsilon_{tp}} = E\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C \beta_2 - \alpha_h^T \beta_1\right) + \alpha^C \beta_2 - \alpha^T \beta_1 - \Omega(T - T_0) = \sigma \tag{15}$$

$$S_i \in -\rho \frac{\partial \psi}{\partial \xi_i} = -\partial_{\xi_i} J_\tau \tag{16}$$

where the thermodynamic forces B_i are associated with β_i, X, Y and Z are related to the plasticity phenomenon and R and S_i correspond to the TRIP effect. σ represents the uniaxial stress and $\partial_{\beta_i} J_\pi$ are the sub-differentials with respect to β_i (Rockafellar, 1970). Furthermore, the parameters E, Ω, K and $1/H$ are defined from their correspondent values for austenitic and martensitic phases, as follows:

$$(\,) = (\,)_M - \beta_3 [(\,)_M - (\,)_A] \tag{17}$$

while Λ_1, Λ_2 and Λ_3 depend linearly on temperature and are defined as follows:

$$\Lambda_1 = \Lambda_M + \Lambda_M^T = -L_0^T + \frac{L^T}{T_M}(T - T_M) \quad ; \quad \Lambda_2 = \Lambda_M + \Lambda_M^C = -L_0^C + \frac{L^C}{T_M}(T - T_M) \, ;$$
$$\Lambda_3 = \Lambda_M + \Lambda_A = -L_0^A + \frac{L^A}{T_M}(T - T_M) \tag{18}$$

In order to describe the dissipation processes, it is necessary to introduce a pseudo-potential of dissipation Φ. In general, it is possible to split Φ into an intrinsic part (ϕ) and a thermal part (ϕ_T). Here, the interest is focused on the mechanical part of the potential and, for convenience, is expressed in terms of its dual (ϕ^*).

$$\begin{aligned}\phi^*(B_i, X, Y, Z, R, S_i) = {} & \frac{1}{2\eta_1}(B_1 + \eta_{ci} Y + \eta_{ck} Z)^2 + \frac{1}{2\eta_2}(B_2 + \eta_{ci} Y + \eta_{ck} Z)^2 + \\ & + \frac{1}{2\eta_3}(B_3 - \eta_{ci} Y - \eta_{ck} Z)^2 + R^2 \{(M_{31}\beta_1 + M_{13}\beta_3)\dot{\beta}_1 + (M_{32}\beta_2 + M_{23}\beta_3)\dot{\beta}_2 + \\ & + [M_{43}\beta_3 + M_{34}(1 - \beta_1 - \beta_2 - \beta_3)]\dot{\beta}_3\} + |\dot{\beta}_1| S_1 + |\dot{\beta}_2| S_2 + |\dot{\beta}_3| S_3 + J_\chi(B_n) + J_f(X, Y, Z)\end{aligned} \tag{19}$$

The parameters η_i are associated with the internal dissipation of the material, while η_{ci} and η_{ck} are (isotropic and kinematic, respectively) parameters related to plastic-phase transformation coupling. These parameters are determined as follows where m_p is a saturation parameter and $\bar{\eta}_{ci}$ and $\bar{\eta}_{ci}$ give the magnitude of this coupling phenomenon.

$$\eta_{ci} = \bar{\eta}_{ci} \exp\left[-m_p(\xi_1 + \xi_2)\right] \qquad \eta_{ck} = \bar{\eta}_{ck} \exp\left[-m_p(\xi_1 + \xi_2)\right] \tag{20}$$

Moreover, according to the following relations, where ξ_i are the internal variables associated with TRIP phenomenon, m_i are saturation parameters and $\bar{M}_{13}$, $\bar{M}_{31}$, $\bar{M}_{23}$, $\bar{M}_{32}$, $\bar{M}_{34}$ and $\bar{M}_{43}$ give the magnitude of TRIP deformation.

$$M_{13} = \bar{M}_{13} \exp(-m_1 \xi_1) \qquad M_{31} = \bar{M}_{31} \exp(-m_1 \xi_1) \tag{21}$$
$$M_{23} = \bar{M}_{23} \exp(-m_2 \xi_2) \qquad M_{32} = \bar{M}_{32} \exp(-m_2 \xi_2) \tag{22}$$
$$M_{34} = \bar{M}_{34} \exp(-m_3 \xi_3) \qquad M_{43} = \bar{M}_{43} \exp(-m_3 \xi_3) \tag{23}$$

In order to take into account differences on the kinetics of phase transformation for loading and unloading processes, it is possible to consider different values to the parameter η_i. Moreover, J_χ is the indicator function related to the convex set χ, which provide constraints associated with phase transformations evolution, such as internal subloops due to incomplete phase transformations description and $M+ \Rightarrow M$ and $M- \Rightarrow M$ phase transformations avoidance (Savi & Paiva, 2005). The other indicator function (J_f) is related to the yield surface defined as: $f = |X + HZ| - (\sigma_Y - Y)$.

After these definitions, it is possible to write the complementary equations to describe the internal variables evolution.

$$\dot{\beta}_i \in \frac{\partial \phi}{\partial B_i} = \frac{B_i}{\eta_i} - \frac{\eta_{ci}}{\eta_i} Y - \frac{\eta_{ck}}{\eta_i} Z + \partial_{\beta_i} J_\chi \tag{24}$$

$$\dot{\varepsilon}_p \in \frac{\partial \phi}{\partial X} = \lambda \text{ sign } (X + HZ) \tag{25}$$

$$\dot{\gamma} \in \frac{\partial \phi}{\partial Y} = \lambda + \eta_{ci} \left(\frac{B_1}{\eta_1} + \frac{B_2}{\eta_2} - \frac{B_3}{\eta_3} \right) \tag{26}$$

$$\dot{\mu} \in \frac{\partial \phi}{\partial Z} = \lambda H \text{ sign } (X + HZ) + \eta_{ck} \left(\frac{B_1}{\eta_1} + \frac{B_2}{\eta_2} - \frac{B_3}{\eta_3} \right) \tag{27}$$

$$\dot{\varepsilon}_{tp} = \frac{\partial \phi}{\partial R} = 2R \left[(M_{31}\beta_1 + M_{13}\beta_3)\dot{\beta}_1 + (M_{32}\beta_2 + M_{23}\beta_3)\dot{\beta}_2 + [M_{43}\beta_3 + M_{34}(1 - \beta_1 - \beta_2 - \beta_3)]\dot{\beta}_3 \right] \tag{28}$$

$$\dot{\xi}_i = \frac{\partial \phi}{\partial S_i} = \left| \dot{\beta}_i \right| \tag{29}$$

where λ is the plastic multiplier and $\partial_{\dot{\beta}_i} J_\chi$ are the sub-differentials with respect to variables $\dot{\beta}_i$. The irreversible nature of plastic flow is represented by means of the *Kuhn-Tucker conditions* (Simo & Hughes, 1998). The yield limit σ_Y has different values for austenitic and martensitic phases, and for very high temperatures, this value tends to decrease (Paiva *et al.*, 2005).

In order to control the amount of TRIP deformation at different temperatures, ε_{tp} should be temperature dependent as well. Thus, the parameters $\overline{M}_{13}$, $\overline{M}_{31}$, $\overline{M}_{23}$ and $\overline{M}_{32}$ are assumed to depend linearly on temperature. For instance, for $\overline{M}_{13}$, the following expression is adopted.

$$\begin{cases} \overline{M}_{13} = 0 & \text{if} \quad T < T_{TRIP} \\ \overline{M}_{13} = \overline{M}_{13}^{\,\mathrm{Ref}} \dfrac{(T - T_{TRIP})}{T_F - T_{TRIP}} & \text{if} \quad T \geq T_{TRIP} \end{cases} \tag{30}$$

where $\overline{M}_{13}^{\,\mathrm{Ref}}$ is a reference value of $\overline{M}_{13}$ at $T = T_F$ and T_{TRIP} is the temperature below which no transformation plasticity should exist. Analogous expressions are used for $\overline{M}_{31}$, $\overline{M}_{23}$ and $\overline{M}_{32}$. For the sake of simplicity, this article considers $M_{34} = M_{43} = 0$. These equations form a complete set of constitutive equations. Since the pseudo-potential of dissipation is convex, positive and vanishes at the origin, the Clausius-Duhen inequality is automatically satisfied if the entropy is defined as $s = -\partial\psi / \partial T$. Box 1 summarizes the set of constitutive equations for the proposed model.

Box 1. Constitutive equations.

$$\sigma = E\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^{\;C}\beta_2 - \alpha_h^{\;T}\beta_1\right) + \alpha^C\beta_2 - \alpha^T\beta_1 - \Omega(T - T_0)$$

$$\begin{aligned} \dot{\beta}_1 = \frac{1}{\eta_1}\Big\{ & \alpha^T\left(\varepsilon - \varepsilon_p - \varepsilon_{tp}\right) + \Lambda_1 + \beta_2\left(\alpha_h^{\;C}\alpha^T + \alpha_h^{\;T}\alpha^C + E\alpha_h^{\;T}\alpha_h^{\;C}\right) - \beta_1\left(2\alpha_h^{\;T}\alpha^T + E\,{\alpha_h^{\;T}}^2\right) + \\ & + \alpha_h^{\;T}\left[E\left(\varepsilon - \varepsilon_p - \varepsilon_{tp}\right) - \Omega(T - T_0)\right] - \eta_{ci}K\gamma - \eta_{ck}\frac{\mu}{H} - \partial_{\beta_1}J_\pi \Big\} + \partial_{\dot{\beta}_1}J_\chi \end{aligned}$$

$$\begin{aligned} \dot{\beta}_2 = \frac{1}{\eta_2}\Big\{ & -\alpha^C\left(\varepsilon - \varepsilon_p - \varepsilon_{tp}\right) + \Lambda_2 + \beta_1\left(\alpha_h^{\;T}\alpha^C + \alpha_h^{\;C}\alpha^T + E\alpha_h^{\;C}\alpha_h^{\;T}\right) - \beta_2\left(2\alpha_h^{\;C}\alpha^C + E\,{\alpha_h^{\;C}}^2\right) - \\ & - \alpha_h^{\;C}\left[E\left(\varepsilon - \varepsilon_p - \varepsilon_{tp}\right) - \Omega(T - T_0)\right] - \eta_{ci}K\gamma - \eta_{ck}\frac{\mu}{H} - \partial_{\dot{\beta}_2}J_\pi \Big\} + \partial_{\dot{\beta}_2}J_\chi \end{aligned}$$

$$\begin{aligned} \dot{\beta}_3 = \frac{1}{\eta_3}\Big\{ & -\frac{1}{2}(E_A - E_M)\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^{\;C}\beta_2 - \alpha_h^{\;T}\beta_1\right)^2 + \Lambda_3 + (\Omega_A - \Omega_M)(T - T_0)\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^{\;C}\beta_2 - \alpha_h^{\;T}\beta_1\right) - \\ & - \frac{1}{2}(K_A - K_M)\gamma^2 - \left(\frac{1}{2H_A} - \frac{1}{2H_M}\right)\mu^2 + \eta_{ci}K\gamma + \eta_{ck}\frac{\mu}{H} - \partial_{\dot{\beta}_3}J_\pi \Big\} + \partial_{\dot{\beta}_3}J_\chi \end{aligned}$$

$$\dot{\varepsilon}_p = \lambda \text{ sign } (\sigma - \mu) \qquad \dot{\gamma} = \left|\dot{\varepsilon}_p\right| + \eta_{ci}\left(\dot{\beta}_1 + \dot{\beta}_2 - \dot{\beta}_3\right) \qquad \dot{\mu} = H\,\dot{\varepsilon}_p + \eta_{ck}\left(\dot{\beta}_1 + \dot{\beta}_2 - \dot{\beta}_3\right)$$

$$\dot{\varepsilon}_{tp} = 2\sigma\left[(M_{31}\beta_1 + M_{13}\beta_3)\dot{\beta}_1 + (M_{32}\beta_2 + M_{23}\beta_3)\dot{\beta}_2 + [M_{43}\beta_3 + M_{34}(1 - \beta_1 - \beta_2 - \beta_3)]\dot{\beta}_3\right]$$

$$\dot{\xi}_1 = \left|\dot{\beta}_1\right| \qquad \dot{\xi}_2 = \left|\dot{\beta}_2\right| \qquad \dot{\xi}_3 = \left|\dot{\beta}_3\right|$$

3. Numerical Results

Within this session, numerical simulations are provided to attest the model's ability to capture the thermomechanical features associated to TRIP phenomenon in SMAs. Firstly, the model verification is conceived through comparisons between numerical results and experimental data provided by Tobushi *et al.* (1991). The parameters and properties used to obtain these results are listed in Table 1. Parameters associated with compression are hidden, since this behavior is not focused on in the present work. Parameters are related to classical macroscopic properties of SMAs and plasticity and also some other parameters that are obtained from experimental data fitting.

Table 1. Model parameters obtained through comparison between numerical and experimental results provided by Tobushi *et al.* (1991) for a Ni-Ti SMA alloy.

E_A (GPa)	E_M (GPa)	Ω_A (MPa/K)	Ω_M (MPa/K)	ε_R^T	α^T (MPa)
60	40	0.74	0.17	0.0517	755
L_0^T (MPa)	L^T (MPa)	L_0^A (MPa)	L^A (MPa)	T_M (K)	T_0 (K)
5	161	10	300	293	298
η_1^L (MPa.tu)	η_1^U (MPa.tu)	η_3^L (MPa.tu)	η_3^U (MPa.tu)	$\overline{M}_{13}$ (GPa^{-1})	$\overline{M}_{31}$ (GPa^{-1})
1.8	0.95	1.8	0.68	0.013	0.013
T_A (K)	T_F (K)	T_{TRIP} (K)	σ_Y^M (GPa)	$\sigma_Y^{A,i}$ (GPa)	$\sigma_Y^{A,f}$ (GPa)
323	423	333	1.2	1.5	1
K_A (GPa)	K_M (GPa)	H_A (GPa)	H_M (GPa)	$\overline{\eta}_{ci}$	$\overline{\eta}_{ck}$
1.4	0.4	4	1.1	-0.1	-0.1

m_1	m_3	m_p	$\overline{M}_{34}$	$\overline{M}_{43}$
0.2	0.2	0.1	0	0

Figure 1 brings the comparison between experimental and numerical results for pseudoelastic tests at three different temperatures: 333K, 353K and 373K. For higher temperatures, TRIP effect increases and, notice that, upon unloading there is an amount of irreversible strain, which is not recovered. Another interesting feature about this remaining strain is that it decreases with temperature until it vanishes for T = 333 K. It should be pointed out the close agreement between numerical and experimental results.

Using the parameters identified for Figure 1, cyclic tests are developed to show saturation phenomena in SMAs. In Figure 2(a), a ten cycle pseudoelastic test is conducted. After some cycles, the stress-strain diagram stabilizes and no more TRIP deformation is induced. This feature is experimentally attested by (Lagoudas *et al.*, 2003). During TRIP deformation accumulation, there is a significant decrease on critical stress for transformation and a hysteresis loop area reduction.

In Figure 2(b), another pseudoelastic cyclic test is presented. For this test, the parameters related to TRIP are null ($\overline{M}_{13} = \overline{M}_{31} = 0$) and the mechanical loading is such that the yield limit is overlapped. After some cycles, the stress-strain diagram also stabilizes but due to the plastic-phase transformation coupling saturation. Again, the critical stress for transformation is affected. The correct cyclic tests description is of great importance for SMA training processes.

4. Conclusions

A macroscopic constitutive theory is developed to describe the TRIP phenomenon in SMAs. The numerical results provided by the model are in good agreement with experimental data available in literature. Besides, numerical simulations demonstrate the model's ability to capture some other features associated with TRIP such as saturation processes under cyclic tests, either by TRIP saturation or by plastic-phase transformation coupling saturation. Experimental studies reveal that these two mechanisms are known to be present in SMAs and have great relevance during training processes. All the results here obtained for tensile tests can be obtained for compression as well.

5. Acknowledgements

The authors would like to acknowledge the support of the *Brazilian Research Council (CNPq).*

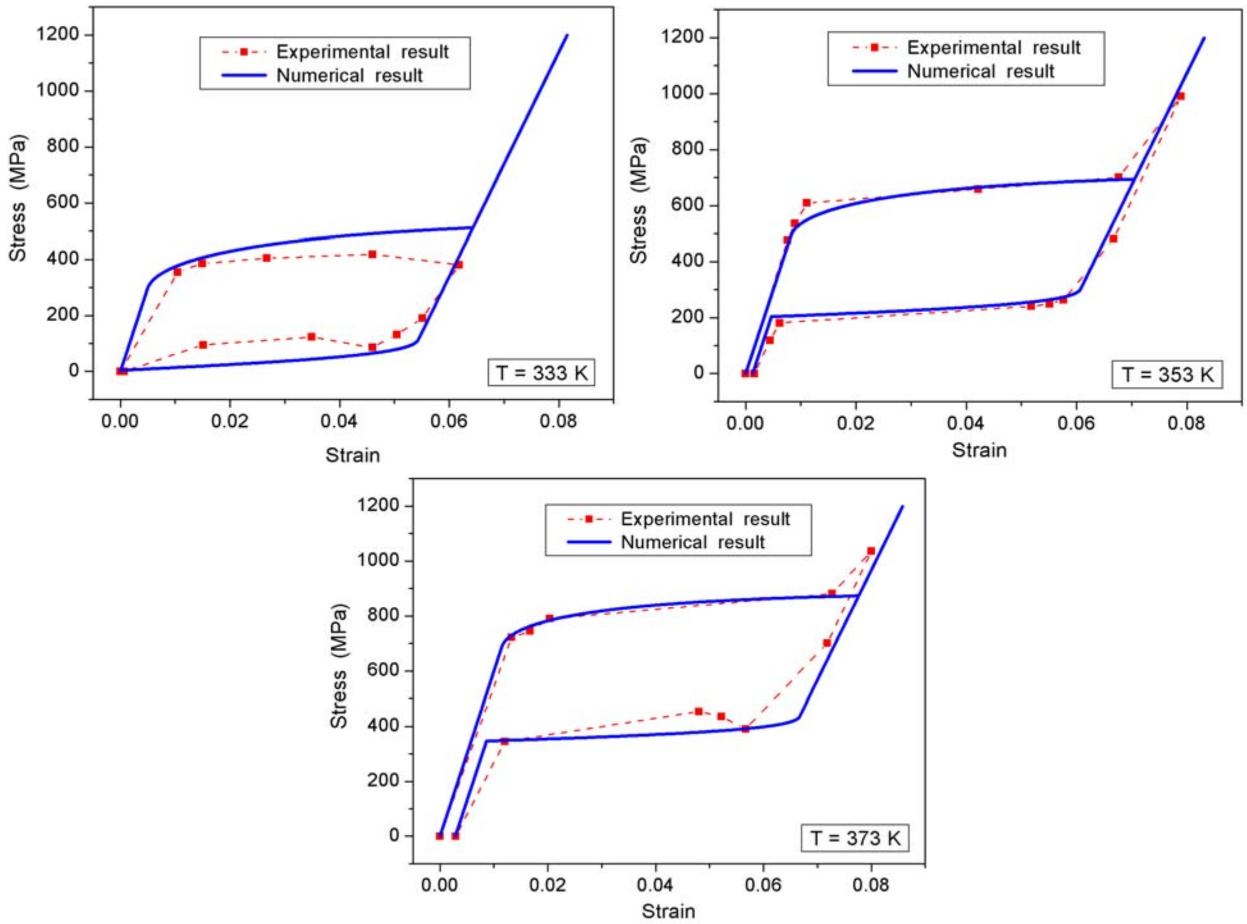
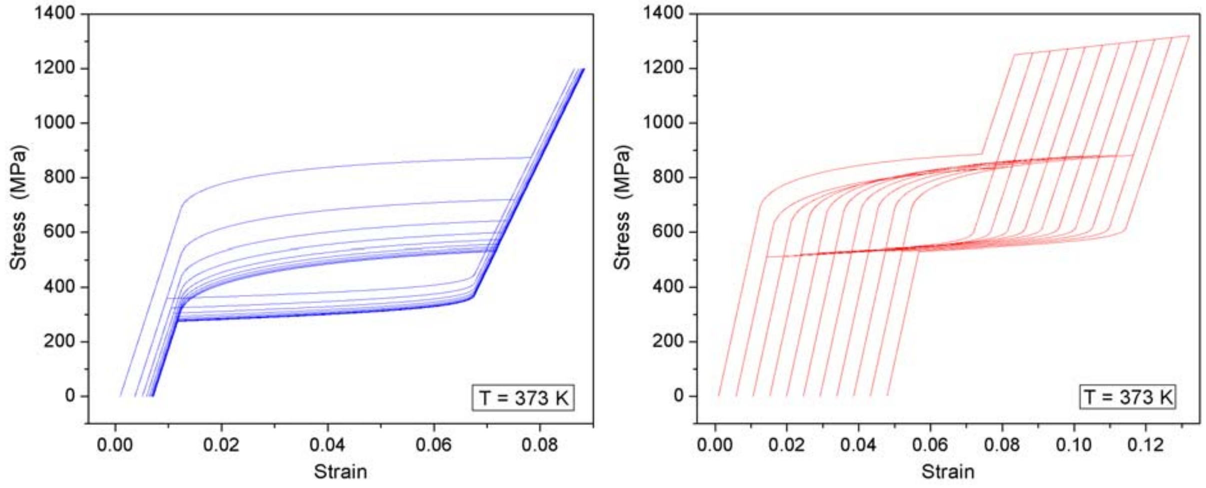


Figure 1. Comparison between numerical and experimental results for pseudoelasticity.



(a) (b)

Figure 2. Saturation effect for pseudoelasticity.

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