A Comparative Study of Hot/Cold Wire Anemometer Data Reduction

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Key words: thermal-anemometry, temperature compensation, turbulence.

1. Introduction

In many flows of practical interest the fluid temperature may vary with time or position. This change may be some times very steep, other times very subtle. In all cases, the output signal from a constant temperature hot-wire anemometer (CTA) will be influenced by the variation in temperature. Thus, correction methods must be developed to account for the drift in fluid temperature when one is undertaking measurements of mean and fluctuating quantities in turbulent flows.

Frequently, correction procedures have been developed in literature to deal with small drifts in the ambient-fluid temperature. These methods are very simple, often considering the heat transfer from the hot-wire probe to be linearly proportional to the product of a temperature difference. In an alternative procedure, Lemieux and Oosthuizen(1984) expressed their calibration relationship in such a way that the calibration parameters \( A, B \) and \( n \) were expressed in terms of the ambient temperature. This procedure was reviewed by Loureiro et al.(2002).

In the present paper, three different approaches will be critically tested for the use of thermal anemometry in the simultaneous measurements of velocity and temperature fields in flows with large temperature variations. The work will, in particular, investigate any appropriate method of data curve fitting for measurements on specific conditions of low velocity and large temperature gradients, that is, on flows where buoyancy effects are important. Those conditions were considered in both the velocity and temperature measurements, the main purpose being to compare several curve fitting methods on a common basis, by using the same input data. In fact, the two most common methods for flow measurements on conditions of low velocity and temperature gradients are studied here.

All data presented in this work were obtained in the Laboratory of Turbulence Mechanics of PEM/COPPE/UFRJ. In fact, the entire experimental apparatus, including its instrumentation and data acquisition and treatment software, was developed in the Laboratory. The advantages and limitations of the methods will be discussed. The results are shown in the form of tables, equations and calibration curves for temperature and velocity data. An uncertainty analysis of the results is also given. Some measurements of simulated conditions are presented so as to illustrate the potential of hot wire anemometry for extracting information on various aspects of the turbulent motion.

2. Temperature dependence of CTA signals.

In general, one of the most important aspects of thermal anemometry is the accurate interpretation of the anemometer signal. The main purpose of any sensor calibration is to determine, as accurately as possible, the relationship between the anemometer output voltage and the physical property under consideration, in this case velocity or temperature. However, the direct output from all practical calibration procedures is raw calibration data, which will contain measurements uncertainties. An additional complication is that the true calibration curve of the probe is not known, and furthermore it depends on particular characteristics of each experiment.

Among the several possible methods that can be devised to characterize the velocity and temperature dependence of thermal anemometers signals, it is possible two classify them in three main categories (Freymuth, 1970):

1. The linear correction method where the heat transfer from the probe is assumed to be proportional to a product of the temperature difference \( T_w - T_a \) and a function of the velocity, where \( T_w \) is the temperature of the heated wire and \( T_a \) is the ambient temperature. The output voltage, \( E \), of a constant temperature hot-wire anemometer can hence be represented by:

\[
E^2 = f(U) (T_w - T_a).
\]

2. The convective heat transfer is expressed in a non-dimensional form involving a relationship between the Nusselt number, \( N_u \), the Reynolds number, \( R_e \), and the Prandtl number, \( P_r \).

3. Direct calibration of the variation in the anemometer output voltage, \( E \), with the velocity, \( U \), and the fluid temperature, \( T_w \), for a given hot resistance setting, \( R_w \).
Method 1 is the most simple and is easy to use for data processing. The third method is the most complex to implement but it is also the most accurate. The calibration data obtained by this method can reveal to what an extent the other methods can be used to evaluate the temperature and velocity sensitivity of a hot-wire probe operated in the CT mode. Thus, Method 3 can be used to assess Methods 1 and 2 for flow with large temperature variations.

2.1 Temperature Calibration for the Cold-Wires

Fluid temperature measurements are often performed with a hot-wire sensor operated in the constant-current mode at very low overheat ratio, in order to minimize Joule heating. Ideally, the wire sensor behaves like a resistance thermometer.

Temperature measurements with resistance wires require low drift, low noise constant current anemometers and high quality amplifiers. If the resistance wire is heated by a current \( I = 0.15 \text{ mA} \), then the “hot” resistance \( R_w \) will only deviate from \( R_a \) by \( (R_w - R_a)/R_a \approx 0.0004 \), and the corresponding temperature difference \( (T_w - T_a) \) will be less than 0.1 °C.

Thus, a common practice is to consider \( R_w = R_a \), with

\[
R_a = R_0 [1 + \alpha(T_a - T_0)].
\]

For practical applications, it is recommended that a temperature calibration of the resistance-wire be used to determine the calibration constants in the relationship

\[
R_a = A + B T_a.
\]

2.2 Temperature and Velocity Calibration for Hot-Wires

2.2.1 Direct velocity and temperature calibration of a CT hot-wire probe

The most accurate way of establishing the velocity and temperature sensitivity of a constant temperature hot-wire probe, operated at a fixed hot resistance, \( R_w \), is to measure the anemometer output voltage, \( E \), as a function of the velocity, \( U \), and fluid temperature, \( T_a \). This type of calibration is often carried out by performing a velocity calibration at a number of different fluid temperatures.

The functional dependence of the calibration data may be written as

\[
E = F(U, T_a)_{w=\text{const}}
\]  

(1)

but, more commonly, the calibration data have been interpreted in the form

\[
E = F(U, T_w - T_a)_{w=\text{const}}
\]  

(2)

To evaluate \( T_w \), it is necessary to measure the hot resistance, \( R_w \), and this requires a knowledge of the probe and cable resistance. The mean wire temperature, \( T_w \), corresponding to \( R_w \) is normally determined by the equation:

\[
R_w = R_0 [1 + \alpha_0 (T_w - T_0)]
\]  

(3)

where, the temperature coefficient of resistance, \( \alpha_0 \), is a property that is usually given by the manufacturer of the probe and \( T_0 \) denotes an appropriate reference temperature indicated also by the probe manufacturer.

For single normal probes, in this case the hot-wire probe, the classical convective heat transfer law, King’s law, can be written as

\[
E^2 = A + BU^n
\]  

(4)

where \( A, B \) and \( n \) are empirical constants.

Equation 4 is a very good approximation to the velocity calibration data, obtained at a constant value of \( T_w \), provided the calibration constants are determined by a least-squares curve fit. This procedure has been applied by Koppius and Trines (1976) to their velocity calibration data using the wire-voltage relationship

\[
\frac{E_w^2}{R_w (R_w - R_a)} = A + BU^n
\]  

(5)
This curve-fitting procedure gave the most accurate results, but $A$, $B$, and $n$ were found to be functions of $T_a$. When a constant value of $n (=0.45)$ was selected, $A$ and $B$ also became constants, and the increase in the uncertainty is insignificant for most hot wire anemometry applications.

A similar approach was adopted by Lemieux and Oosthuizen(1984). They expressed their calibration relationship in the form

$$E^2 = A^* + B^*U^n$$

and for each value of $T_a$ they determined the values of $A^*$, $B^*$ and $n$ by a least squares curve fitting procedure. In their subsequent signal analysis the optimum value for $n$ was selected as being the average value from their four calibration curves. The corresponding calibration coefficients $A^*$ and $B^*$ were found to vary linearly with $T_a$:

$$A^* = A_1 + A_2 T_a$$
$$B^* = B_1 + B_2 T_a$$

A linear variation of $E^2$ with $T_a$ has also been reported by other authors (Fulachier(1978), Champagne(1978), Dekeyser and Launder(1983), Bremhorst(1985)). In conclusion, most experimental investigations of this type, covering small or moderate variations in $T_a$, have demonstrated that the output signal from a hot-wire probe operated in the constant temperature mode is directly proportional to a product of the temperature difference ($T_w - T_a$) and a function of the velocity.

### 2.2.2 Analytical compensation technique

The simplest of the analytical compensation techniques recognizes that equation (5) provides an accurate representation of the velocity and temperature dependence of a hot-wire probe operated in the CT mode and exposed to moderate variations in the ambient temperature.

Expressing Eq. (5) in terms of the anemometer output voltage, $E$, one has

$$E^2 = \frac{R_w}{(R_p + R_L + R_w)^2(T_w - T_a)} = A + BU^n$$

where the subscripts $p$ and $L$ indicate the probe and cable resistances respectively.

For a constant temperature anemometer, the ratio involving the resistances in Eq. 9 will be constant, so that it can eventually be included in the calibration constants. It follows that the flow velocity can be given by

$$E^2 = (A + BU^n)(T_w - T_a)$$

where the temperature value must be provided independently.

A second analytical compensation technique has been proposed by Corrsin (1949). The technique uses two dual CT hot-wire probes that are placed so close together that can be supposed to be exposed to the same velocity and temperature fields. Sakao(1973), Blair and Bennet(1987) and Lienhard and Helland (1989) also investigated this technique.

Operating two sensor at two different overheat ratios, one can establish a system of two algebraic equations with two unknown parameters, $U$ and $T_a$, that is

$$\frac{E_{w1}^2}{R_{w1}} = (A + BU^n)(T_{w1} - T_a)$$
$$\frac{E_{w2}^2}{R_{w2}} = (A + BU^n)(T_{w2} - T_a)$$

Taking the difference between equations (11) and (12), it follows that

$$\frac{E_{w1}^2}{R_{w1}} - \frac{E_{w2}^2}{R_{w2}} = (A + BU^n)(T_{w1} - T_{w2})$$

an equation whose only unknown is $U$.

Unfortunately, in practice it is not possible to set up a measuring system where the values of all three calibration constants ($A$, $B$ and $n$) are identical. An alternative solution can be developed if the exponent, $n$, is considered the same from both hot-wires. Optimum values for $n$ are in the range 0.41-0.45.
Thus, Eqs. 11 and 12 can be written as

\begin{align}
E_{w1}^2 &= R_w1(A_1 + B_1U^n)(T_{w1} - T_a), \\
E_{w2}^2 &= R_w2(A_2 + B_2U^n)(T_{w2} - T_a),
\end{align}

which form a system of two non-linear equations for the unknowns \( U \) and \( T_a \).

These equations can be re-written as

\begin{align}
E_{w1}^2 &= a_1 + a_2T_a + (a_3 + a_4 + T_a)U^n, \\
E_{w2}^2 &= b_1 + b_2T_a + (b_3 + b_4 + T_a)U^n,
\end{align}

where,

\begin{align*}
a_1 &= R_w1A_1T_{w1}, & b_1 &= R_w2A_2T_{w2}, \\
a_2 &= -R_w1A_1, & b_2 &= -R_w2A_2, \\
a_3 &= R_w1B_1T_{w1}, & b_3 &= R_w2B_2T_{w2}, \\
a_4 &= -R_w1B_1, & b_4 &= -R_w2B_2,
\end{align*}

and eliminating \( U^n \), gives a second-order equation in \( T_a \)

\begin{equation}
c_1T_a^2 + c_2T_a + c_1 = 0
\end{equation}

with

\begin{align*}
c_1 &= (E_{w2}^2 - b_1)a_3 - (E_{w1}^2 - a_1)b_3, \\
c_2 &= (E_{w2}^2 - b_1)a_4 - (E_{w1}^2 - a_1)b_3 + a_2b_3 - b_2a_3, \\
c_3 &= a_2b_4 - b_2a_4.
\end{align*}

For operational purposes, the wire voltages \( E_{w1} \) and \( E_{w2} \) can be replaced by the output voltages of the anemometer through equation

\begin{equation}
E = \frac{R_p + R_1 + R_w}{R_w} E_w.
\end{equation}

Solving equation (18) for the measured values of voltage will determine the ambient temperature. Then, equations 11 or 12 can be used to find the mean velocity.

3. Experimental Facilities

3.1. Wind Tunnels

The Laboratory of Turbulence Mechanics has two open-circuit wind tunnels. The larger tunnel was specially designed and constructed to emulate environmental flows. It is the low-velocity stratified-flow wind tunnel that has been described in Cataldi et al. (2001). The main purpose of this tunnel is to simulate stratified atmospheric boundary layers. Some improvements have recently been made in the tunnel to achieve a better representation of atmospheric flows and similarity conditions. The test section has now an overall length of 10 m, with a cross section area of 0.67 m x 0.67 m. The position of the roof can be adjusted at will so as to produce different pressure gradients. In this case a special care is taken to set the pressure gradient near zero. The potential velocity of the wind tunnel varies from zero to 3.5 m/s, and the free stream has a turbulence intensity of about 2%.

A stratification section consisting of 10 electrical resistances is able to heat the flow differentially up to 100 °C; each of the resistances can be controlled individually. Following the heating section, the floor temperature can also be raised by 100 °C over a 6 m long surface, by a series of resistances with a controlled variation of 5 °C. The total heating capacity of each panel is about 7 kW/m². The whole facility is capable of developing gradients of up 50 °C at uniform mean speeds in the range 1.5–2.5 m/s, so as to generate different levels of instability.

The second wind tunnel is a low-turbulence wind tunnel with turbulence intensity levels of the order of 0.2%. This wind tunnel can be set to run at velocities that can reach 13 m/s; the test section is 4 m long, the cross section area is
0.30 x 0.30 m. This tunnel was adapted for the calibration of the cold-wire with the inclusion of a new heating section. The heating section was built with four electrical resistances in series, each one of them consisting of strings distributed transversally to the flow.

Both tunnels have honeycombs and screens to control the turbulence levels and to guarantee a uniform flow. The computer-controlled traverse gears are two-dimensional and capable to position sensors with an accuracy of 0.1 mm.

For the present work, only the low turbulence with tunnel was used. The experiments were conducted in a controlled environment, with the laboratory temperature set to 18.0 °C +/- 0.5 °C.

The details of the wind tunnel are shown in Figure 1.

![Wind tunnel and heating section](image1.jpg)

**Figure 1.** Pictures showing wind tunnel and heating section.

The main wind tunnel characteristics are:

- Circuit: open.
- Test section: 0.30 m high, 0.30 m wide and 2 m long.
- Wind speed: continuously variable from 0.5 to 16 m/s.
- Longitudinal pressure gradient: adjustable to zero by means of an adjustable ceiling.
- Turbulence intensity: below 0.2%.
- Incoming flow temperature: variable from 20 to 35 °C.
- Number of resistences used to heat the incoming air: 4.
- Resistances capacity: 7 kW.

### 3.2 Instrumentation

In all experiments, simultaneous measurements of stream-wise velocity and fluctuating temperature were obtained using thermal anemometry. The measurements accounted for any large temperature variation in interpreting the sensor response. To perform the measurements a temperature-compensated Dantec probe, model 55P76, was used. This probe consists of two sensor elements: a hot-wire and a resistance-wire, usually called cold-wire, situated 2 mm below and 5 mm downstream of the former. Both sensors are Pt-plated tungsten wires, 5 µm in diameter, 3 mm in overall length, and sensitive wire length of 1.25 mm. They are copper and gold plated at the ends to approximately 30 µm. They were connected respectively to a constant temperature bridge, Dantec 55M10 and to a constant current bridge, Dantec 56C20. Despite the probe specification, in all our experiments, both wire were used as hot-wires, so that Eq. 14 could be duly implemented.

Reference measurements for velocity was obtained from a Pitot tube connected to an inclined manometer; temperature reference data was obtained from previously calibrated micro-thermocouples.

In getting the data, 10,000 samples were considered. The reference mean temperature profiles were obtained through a chromel-constantan micro-thermocouple mounted on the same traverse gear system used for the hotwire probe. An uncertainty analysis of the data was performed according to the procedure described in Kline (1985). Typically the uncertainty associated with the velocity and temperature measurements were: U = 0.0391 m/s precision, 0 bias (P=0.95); T = 0.2 °C precision, 0 bias (P=0.99).

To obtain accurate measurements, the mean and fluctuating components of the analogical signal given by the anemometer were treated separately. Two output channels of the anemometer were used. The mean velocity profiles were calculated directly from the untreated signal of channel one. The signal given by channel two was 1 Hz high-pass filtered leaving, therefore, only the fluctuating velocity. The latter signal was then amplified with a gain controlled between 1 and 500 and shifted by an offset so as to adjust the amplitude of the signal to the range of the A/N converter.
4. Results

To compare the results provided by the three different methods defined by the systems of Eqs. (6 to 8), Eq. 10 and Eqs. (14 to 18), respectively, the following procedure was adopted.

The low turbulence wind tunnel was fitted with the heating section shown in Figure 1. Next, two probes, one of the type 55P11 and other of the type 55P76, were placed in the center of the test section, 20 cm downstream of the heating section. The probes were separated 3 mm apart from each other and were used together with a Pitot tube and a reference thermocouple. Probe 55P11 was used for redundant measurements. The geometrical arrangement is shown in Figure 2.

![Figure 2. Position of probes in the wind tunnel.](image)

With the probes in position, the output signal of probe 55P76 was routed to two Kauri CTA’s. The output signal of probe 55P11 was routed to a third Kauri CTA.

Under isothermal flow conditions, the calibration constants of Eqs. 10, 14 and 15 were found. The constants in Eqs. 10 and 14 were determined considering an overheating ratio of 1.7, whereas in Eq. 15 the overheat ratio was 1.5. In all experiments \( n \) was considered to be 0.45.

Having found \( A, B, A_1, B_1, A_2, B_2 \), the analytical compensation techniques could be comprehensive implemented.

To find \( A^* \) and \( B^* \), the heating section was turned on and graphs showing the variation of the anemometer output voltage with the fluid velocity and the fluid temperature were constructed. Then by a least-squares curve fitting procedure, \( A^* \) and \( B^* \) were found.

The calibration curves for \( A^* \) and \( B^* \) are shown in Figure 3.

![Figure 3. The variation of the calibration parameters \( A^* \) and \( B^* \) with the fluid temperature.](image)
The values of the calibration parameters are shown in Table 1.

Table 1. Calibration constants for isothermal conditions (21.61 °C).

<table>
<thead>
<tr>
<th>Probe</th>
<th>A</th>
<th>B</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>55P76 Wire 1</td>
<td>5.8749</td>
<td>5.5816</td>
<td>0.45</td>
</tr>
<tr>
<td>55P76 Wire 2</td>
<td>4.2685</td>
<td>4.0751</td>
<td>0.45</td>
</tr>
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</table>

Table 2 shows a comparison between the three different methods for 12 different conditions.

Table 2. Velocity predictions by the three different methods.

<table>
<thead>
<tr>
<th>Condition, °C</th>
<th>Pitot (m/s)</th>
<th>Eq. 6 (m/s)</th>
<th>Eq. 10 (m/s)</th>
<th>Eq. 14 (m/s)</th>
<th>Eq. 14 (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.05</td>
<td>2.91</td>
<td>3.08</td>
<td>2.87</td>
<td>2.82</td>
<td>20.22</td>
</tr>
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<td>21.03</td>
<td>2.17</td>
<td>2.26</td>
<td>2.11</td>
<td>2.04</td>
<td>19.44</td>
</tr>
<tr>
<td>21.01</td>
<td>1.37</td>
<td>1.47</td>
<td>1.38</td>
<td>1.31</td>
<td>18.74</td>
</tr>
<tr>
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<td>1.37</td>
<td>1.47</td>
<td>1.43</td>
<td>1.39</td>
<td>23.34</td>
</tr>
<tr>
<td>24.89</td>
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<td>2.22</td>
<td>2.18</td>
<td>2.14</td>
<td>24.12</td>
</tr>
<tr>
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<td>1.41</td>
<td>1.45</td>
<td>1.47</td>
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</tr>
<tr>
<td>31.95</td>
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<td>1.18</td>
<td>1.22</td>
<td>1.28</td>
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</tr>
<tr>
<td>27.14</td>
<td>2.38</td>
<td>2.46</td>
<td>2.48</td>
<td>2.49</td>
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<td>29.10</td>
<td>1.68</td>
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<td>1.86</td>
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<td>1.22</td>
<td>1.25</td>
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<td>2.96</td>
<td>25.67</td>
</tr>
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</table>

To further illustrate the method, Figure 4 shows the dependence of measured velocities on the ambient fluid temperature.

Figure 4. Dependence of velocity measurements on the ambient fluid temperature.

According to Table 2, Eq. 10 presents a slight advantage over the other Eqs., 10 and 14, as far as accuracy is concerned. This conclusion, however, is not definitive since under the range of velocity and temperature covered by the present research the other methods did not perform badly. Because the velocities involved are very low (below 3 m/s), measurements are notoriously difficult to perform. The indication we have is that for higher velocities and temperature variations of the order of 10 °C, all three methods seem to yield equivalent results. The difficulties in prediction really seems to be for very low velocities profiles. This will have to be further investigated.
5. Conclusion

The present work has reported a critical evaluation on three different correction methods for drift in the fluid temperature when the hot-wire measuring technique is to be applied to the recent progresses made at COPPE/UFRJ to develop packages for the measurement of flows with change in temperature. Results are presented for the longitudinal velocity profile and the temperature profile in stratified environments. The measurement of turbulent quantities will be presented opportually.

Acknowledgements. The authors are grateful to Prof. L. J. S. Bradbury for the many helpful discussions undertaken during the course of the present research. Prof. Bradbury together with Prof. Su Jian have helped to construct the computer code that currently operates the hot-wire units. Both researchers have, in fact, had a leading role in developing the experimental facilities currently in use at COPPE/UFRJ. CNPq through grant 350183/93-7 has financially supported the work. JBRL is grateful to the CAPES (Ministry of Education) for the award of research scholarships. APSF is grateful to CNPq for the award of a research fellowship.

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