ANALYSIS OF SHAPE MEMORY BARS USING THE FINITE ELEMENT METHOD

Carlos A. P. L. La Cava
Volkswagen South America - Truck & Bus
Engenharia Avançada e Protótipos
27.501.970 – Resende – RJ – Brazil
E-Mail: Carlos.LaCava@volkswagen.com.br

Marcelo A. Savi
Universidade Federal do Rio de Janeiro
COPPE - Departamento de Engenharia Mecânica
21.945.970 – Rio de Janeiro – RJ, Caixa Postal 68.503
E-Mail: savi@ufrj.br

Pedro M. C. L. Pacheco
CEFET/RJ
Departamento de Engenharia Mecânica
20.271.110 - Rio de Janeiro - RJ
E-Mail: calas@cefet-rj.br

Abstract. Shape memory and pseudoelastic effects are thermo-mechanical phenomena associated with martensitic phase transformations, presented by shape memory alloys. This contribution concerns with the analysis of nonlinear behavior of shape memory bars employing the finite element method. A constitutive equation based on Fremond’s theory is considered. The proposed model includes four phases in the formulation: three variants of martensite and an austenitic phase. Different material parameters for austenitic and martensitic phases are concerned. Thermal expansion and plastic strains are included into the formulation and hardening effect is represented by a combination of kinematic and isotropic behaviors. A plastic–phase transformation coupling is incorporated into the model allowing a correct description of the thermo-mechanical behavior of SMAs. Moreover, constitutive equations consider horizontal enlargement of the stress-strain hysteresis loop, allowing better adjustments with experimental data. An iterative numerical process based on operator split technique is developed in order to deal with the nonlinearities of the formulation. Numerical simulations are carried out in order to illustrate the general behavior of shape memory bars under different thermo-mechanical loadings. Results show that the proposed model capture the general behavior of SMAs, allowing the description of bars subjected to non-homogeneous themomechanical loads.

Key-words: Shape memory alloys, Finite Element Method, Modeling and simulation

1. Introduction

Shape memory alloys (SMAs) have been found in a great number of applications in different fields of sciences and engineering. Self-actuating fasteners (La Cava et al., 2000; van Humbeeck, 1999; Kibirkstis et al., 1997; Borden, 1991), thermally actuator switches and several bioengineering devices are some examples of these applications (Machado & Savi, 2003; Duerig et al., 1999; Lagoudas et al., 1999). Aerospace technology are also using SMAs for solve important problems, in particular those concerning with space savings achieved by self-erectable structures, stabilizing mechanisms, non-explosive release devices and other possibilities (Pacheco & Savi, 1997; Denoyer et al., 2000). Micromanipulators and robotics actuators have been built employing SMAs properties to mimic the smooth motions of human muscles (Garner et al., 2001; Webb et al., 2000; Fujita & Toshiyoshi, 1998; Rogers, 1995). Moreover, SMAs are being used as actuators for vibration and buckling control of flexible structures (Pietrzakowski, 2000; Birman, 1997; Rogers, 1995). Despite all these applications, the modeling of SMAs is not well established and hence, it is an important task.

This contribution proposes a finite element formulation to deal with shape memory bars. Finite element modeling of SMA structures has been previously addressed by Brinson and Lammering (1993), where a constitutive theory based on Tanaka’s model (Tanaka, 1986), and later modified by Brinson (1993), has been employed to describe the SMA behavior. More recently, Aurichio and Taylor (1996) have also proposed a three-dimensional finite element model. Savi et al. (1998) discuss an iterative numerical procedure that has been developed to deal with both geometrical and constitutive nonlinearities in the finite element model for adaptive trusses with SMA actuators. Lagoudas et al. (1997) consider the thermo-mechanical response of a laminate with SMA strips where the thermo-mechanical response is based on Boyd-Lagoudas’ polynomial hardening model (Boyd and Lagoudas, 1996). Kouzak et al. (1998) also treats SMA beams using a constitutive equation proposed by Brinson (1993). Trochu & Qian (1997), Masud et al. (1997), Bhattacharyya et al. (2000), Liu et al. (2002) are other contributions in this field. Moreover, dual kriging interpolation has been employed with finite element method (FEM) in order to describe the shape memory behavior (Trochu & Qian, 1997; Trochu & Terriaault, 1998; Trochu et al., 1999).

Here, the FEM is employed promoting the spatial discretization of bars using a constitutive equation proposed by Savi et al. (2002) and Baêta-Neves et al. (2003) to describe the thermo-mechanical behavior of SMAs. This model is
based on Fremond’s theory (Fremond, 1987, 1996) and includes four phases in the formulation: three variants of martensite and an austenitic phase. Furthermore, different material parameters for austenitic and martensitic phases are concerned. Thermal expansion and plastic strains are also included into the formulation and hardening effect is represented by a combination of kinematic and isotropic behaviors. A plastic–phase transformation coupling is incorporated into the model allowing a correct description of the thermo-mechanical behavior of SMAs. Moreover, horizontal enlargement of the stress-strain hysteresis loop is considered, allowing better adjustments with experimental data.

An iterative numerical procedure based on operator split technique (Ortiz et al., 1983) is developed in order to deal with the nonlinearities in the formulation. Numerical simulations are carried out showing different behaviors of SMA bars. Results show that the proposed model is able to capture the general behavior of SMAs, including pseudoelastic and shape memory effects and also phase transformations due to thermal expansion.

2. Finite Element Formulation

Consider a composite bar reinforced with a SMA actuator subjected to a constant axial load, as shown in Figure 1. The actuator is assumed to be significantly thinner than the height of the cross section bar and also, built in such a way to preserve symmetry of the axial load, avoiding flexure loads.

![Figure 1 – Composite bar with SMA actuator.](image)

The internal energy increment may be written as follows

\[
\delta \Gamma = \int_{V_m} \sigma_m \delta \varepsilon \, dV + \int_{V_a} \sigma_a \delta \varepsilon \, dV 
\]

(1)

where \( V \) is the volume and the subscripts \( m \) and \( a \) are associated with matrix and actuator, respectively. An elastic relation is considered for the matrix, \( \sigma_m = E_m \varepsilon \), where \( E_m \) is the matrix elastic modulus; \( \sigma_a \) is given by constitutive equation proposed by Savi et al. (2002) and Baêta-Neves et al. (2003). For simplicity, a compact form of the stress-strain relation is here presented,

\[
\sigma_a = E_a \varepsilon + A_\varepsilon 
\]

(2)

where \( A_\varepsilon \) represents nonlinear terms related to phase transformation and plastic behavior,

\[
A_\varepsilon = -E_a \varepsilon^p + (\alpha + E_a \alpha_H) (\beta_2 - \beta_1) - \Omega (T - T_0) 
\]

(3)

\( \varepsilon^p \) is the plastic strain, \( \beta_1 \) and \( \beta_2 \) are the volumetric fraction of martensitic variants, \( M^+ \) and \( M^- \). \( E_a \), \( \alpha \), \( \alpha_H \) and \( \Omega \) are material parameters. A detailed description of the constitutive model may be found in Savi et al. (2002) and Baêta-Neves et al. (2003), where the evolution equations of all internal variables related to the formulation are presented. Kinematics equation, similar to infinitesimal strains hypothesis, is adopted,

\[
\varepsilon = u_{xx} 
\]

(4)

where \( ( \cdot )_{xx} = d( ( \cdot )/dx \). Now, it is possible to consider the principle of virtual work as follows, since the term \( A_\varepsilon \) is assumed to be constant in the actuator and in the length of the bar.

\[
\left( E_m A_m + E_a A_\varepsilon \right) \int_L (u_{xx} \delta u_{xx}) \, dx + A_\varepsilon \int_L (A_\varepsilon \delta u_{xx}) \, dx - \int_L (p \delta u) \, dx = 0 
\]

(5)

where \( A \) is the cross section area and \( p \) is the axial load per length. This model allows one to analyze bars constructed only by SMA considering several actuators.
Spatial discretization is considered by using the finite element method, which establishes the following approximation

\[ u(x) = \sum_{j=1}^{2} U_j^e \vartheta_j(x) \]  

(6)

where \( U_j^e \) are nodal displacements and \( \vartheta_j(x) \) are Lagrange shape functions, presented below (Reddy, 1984):

\[ \vartheta_1 = 1 - \frac{x}{L}, \quad \vartheta_2 = \frac{x}{L} \]  

(7)

From this approximation, Eq.(5) is rewritten as follows,

\[
(E_m A_m + E_u A_u) \int \left[ B_u \right]^T \left[ B_u \right] dx \left\{ U^e \right\} + A_u \int \left[ D_u \right]^T \left[ D_u \right] dx \left\{ U^e \right\} - \int \left( p \left[ N_u \right]^T \right) dx = 0
\]  

(8)

Which follows a discrete version of the governing equation.

\[
\left[ K^e \right] \left\{ U^e \right\} = \left\{ F^e \right\} - \left\{ \hat{F}^e \right\}
\]  

(9)

where \( \left[ K^e \right] \) is the stiffness matrix, \( \left\{ U^e \right\} \) is the displacement vector, \( \left\{ F^e \right\} \) is the force vector and \( \left\{ \hat{F}^e \right\} \) is related to the behavior of the nonlinear shape memory actuator. The definition of these matrices is given by,

\[
\left[ K^e \right] = (E_m A_m + E_u A_u) \int \left[ B_u \right]^T \left[ B_u \right] dx
\]  

(10)

\[
\left\{ F^e \right\} = \int \left( p \left[ N_u \right]^T \right) dx
\]  

(11)

\[
\left\{ \hat{F}^e \right\} = A_u \int \left[ D_u \right]^T \left[ D_u \right] dx
\]  

(12)

After the construction of the global system, an operator split technique (Ortiz et al., 1983) associated with an iterative numerical procedure is applied in order to deal with the nonlinearities in the formulation. At first, the global vector \( \hat{F}^e \) is evaluated assuming that neither phase transformation nor plastic strain has taken place, which means that it has the same value of the previous time instant. Under this assumption, displacements \( U_i \) are calculated by solving a linear system. In the next step, all variables related to the SMA actuator (strain, stress, volumetric fractions of the phases, etc) are evaluated with the aid of constitutive and evolution equations. Afterwards, the matrix \( K_i \) and the vector \( \hat{F}^e_i \) is recalculated. This procedure is repeated to assure a prescribed convergence tolerance.

3. Numerical Simulations

This section considers numerical simulations performed with the proposed formulation. Material properties are the same discussed in Baêta-Neves et al. (2003). A bar of SMA material with a square cross section of 10mm side and 100mm length is analyzed.

At first, a homogeneous thermo-mechanical load process is considered, allowing a comparison between the FEM formulation with results obtained by Baêta-Neves et al. (2003) simulations. These comparisons are used as a validation of the proposed finite element model. Therefore, consider a SMA bar with four elements (Figure 2), subjected to an axial load \( F_x \), Two different effects are treated: pseudoelastic and shape memory.

![Figure 2 - SMA bar subjected to homogeneous axial load process.](attachment:image.png)
Pseudoelastic effect is now in focus regarding a SMA specimen subjected to an isothermal mechanical loading performed at $T = 313$ K ($T > T_A$). Figure 3 shows this load process, the stress-strain curve and the evolution of volumetric fractions of phases. Notice that simulations of FEM and Baêta-Neves et al. (2003) are in agreement. All characteristics of the constitutive model are captured by the FEM analysis. During loading process, the specimen experiences phase transformations from austenitic phase $A$, to positive martensitic variant $M^+$. Afterwards, during unloading process, the reverse transformation is induced.

The shape memory effect is now focused regarding a thermo-mechanical loading depicted in Figure 4. At first, a constant temperature $T = 263$ K ($T < T_M$) is considered, where the martensitic phase is stable. After mechanical loading-unloading process, the specimen presents a residual strain that can be eliminated by a subsequent thermal loading (Figure 4). Notice that the stress-strain curve represents the shape memory effect. Again, FEM and Baêta-Neves et al. (2003) results are in agreement except for small variations on the evolution of volumetric fractions of phases. This small discrepancy is due to convergence criteria employed on both models.

The thermal expansion effect is now considered regarding a thermal loading depicted in Figure 5, with the specimen free of stress. The response of the material under this loading process presents thermal expansion and its phase transformations. Notice the hysteretic characteristics of phase transformation in strain-temperature curve. Again, FEM and Baêta-Neves et al. (2003) results are in agreement.
Figure 4 – Shape memory effect. (a) Thermo-mechanical loading. (b) Stress-strain curve. (c) Volumetric fraction of phases.

Figure 5 – Thermal expansion. (a) Thermo-mechanical loading. (b) Strain-temperature curve. (c) Volumetric fraction of phases.
At this point, a situation where an axial load is applied at the midpoint of the bar clamped at both ends is discussed (Figure 6). The evolution of this loading is similar to the one presented in Figure 3. Nevertheless, it is clear that its distribution through the bar is different. Figure 7 shows stress-strain curves and the evolution of volumetric fractions of each phase for different elements. In elements 1 and 2 there are phase transformations, causing pseudoelastic effect related to martensitic variant $M^+$. Different behavior is observed in elements 3 and 4, where variant $M^-$ is induced. Figure 7, bottom, shows a schematic picture of phase distribution during loading-unloading process.

The thermo-mechanical loading process depicted in Figure 4 is now considered in order to analyze the shape memory effect in a bar subjected to an axial load applied at the midpoint clamped at both ends. Figure 8 shows stress-strain curves and the evolution of volumetric fractions of each phase for different elements. On the left side of the bar (elements 1 and 2), positive variant is induced ($M^+$), while negative variant is induced at the right side (elements 3 and 4) ($M^-$). Figure 8, bottom, shows schematic pictures of phase distribution during loading-unloading process.

Figure 6 – Bar subjected to an axial load at the midpoint and restricted at both ends.

Figure 7 – Pseudoelastic effect for a bar subjected to an axial load at the midpoint and restricted at both ends.
(a) Stress-strain curves. (c) Volumetric fraction of phases. (c) Schematic representation of phase distribution.
Figure 8 – Shape memory effect for a bar subjected to an axial load at the midpoint and restricted at both ends. (a) Stress-strain curves. (c) Volumetric fraction of phases. (c) Schematic representation of phase distribution.
A bar with a non-homogeneous temperature distribution is of concern. Discretization is done considering 20 elements (Figure 9). Figure 10 shows the thermo-mechanical loading process, stress-strain curve, volumetric fractions of phases and plastic strains time histories. The loading process begins with a thermal loading that promotes non-homogeneous temperature distribution through the length of the bar. This induces a situation where austenitic phase ($A$) and twinned martensite ($M$) are distributed through the bar. Afterwards, a mechanical load is applied. The loading process induces the formation of positive martensitic variant ($M^+$). Since temperature distribution is non-homogeneous, different behavior is expected through the bar. Regions with low temperatures present lower values of critical stresses where phase transformations starts. Moreover, yield limit is also smaller and the load level causes plastification. On the other hand, on regions with higher temperatures, phase transformation starts for higher stress levels and plastification do not occur. The subsequent unloading process shows regions with pseudoelastic, partial pseudoelastic and shape memory effects, depending on this position. Moreover, some regions present plastic strains related to all thermo-mechanical process.

Figure 9 – Bar subjected to non-homogeneous temperature distribution.

Figure 10 – Response of the bar under non-homogeneous temperature distribution.
(a) Thermo-mechanical loading. (b) Stress-strain curves. (c) Volumetric fraction of phases. (d) Plastic strain.
4. Conclusions

This article presents a nonlinear finite element analysis of shape memory bars. A constitutive model proposed by Savi et al. (2002) and Baêta-Neves et al. (2003) is used to describe the thermo-mechanical behavior of SMAs. The model considers thermal expansion and plastic strains with hardening. An iterative numerical procedure based on operator split technique is developed in order to deal with nonlinearities of the formulation. Numerical simulations show that results from FEM capture the general behavior of the constitutive equation due to Baêta-Neves et al. (2003). Moreover, other simulations show how non-homogeneous loadings can produce interesting behaviors in shape memory bars. These results indicate that the response of SMA devices subjected to non-homogeneous loadings can be very complex being of special interest to be investigated.

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6. References


